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This Month—

Front Cover

A large electrically driven blooming mill which rolls steel ingots weighing from 15,000 to 50,000 pounds each into slabs a few inches thick.

Photo Courtesy Westinghouse Electric & Mfg. Co.

Special Articles

AIEE Papers

—Turn to next page

In This Issue—

GASEOUS-CONDUCTION lamps of various types have been developed during recent years, some of which have efficiencies of from 30 to 40 lumens per watt. Some of these lamps and their characteristics are discussed in 2 papers in this issue (*pages 1174–80; 1186–90*). These papers supplement the report of the AIEE committee on the production and application of light, and the 2 papers describing some recent advances in the application of light, published in the October issue.

ENGINEERING curricula in 35 educational institutions in the New England and Middle Atlantic States were accredited at the fourth annual meeting of Engineers' Council for Professional Development held recently. In addition to the election of officers for the ensuing year, Council's various committees presented reports of their progress during the past year which are of interest and importance to the engineering profession (*pages 1280–5*).

TENSOR analysis offers new concepts to aid in the visualization of phenomena in complex electrical systems or combinations of systems, according to one of the chief proponents of tensors who discusses in some detail tensor analysis of circuits involving multielectrode electronic tubes (*pages 1220–42*). Another author has applied tensor analysis to the solution of multiwinding transformer circuits (*pages 1214–19*).

DEFINITE progress in insulation research within the past year, in all directions, was reported by Past-President J. B. Whitehead, chairman of the committee on electrical insulation of the National Research Council, at the recent annual meeting of that committee (*pages 1180–5*).

CHAIRMEN of AIEE committees, newly appointed to serve during the year 1936–37, are being introduced to the membership through the "Personals" columns (*pages 1288–94*). Biographical sketches of other chairmen appeared in the October issue.

PERFORMANCE characteristics of polyphase induction motors operating on unbalanced voltages have been analyzed by the method of symmetrical components in a manner that is said to be neither tedious nor highly involved (*pages 1206–13*).

OPERATIONAL analysis has proved to be a convenient tool for the solution of certain electrical circuits. In a paper in this issue, the complete operational solution of a polyphase electrical machine is presented (*pages 1191–1200*).

SELF-REGULATION may be provided for mercury rectifiers by the use of a special grid-control circuit which is practically instantaneous in response, and which may be compounded in a variety of ways (*pages 1200–05*).

ELECTRICAL equipment built during recent years for the steel industry has been largely of the type required for the production of sheet, strip, tin plate, and other flat-rolled steel (*pages 1168–73*).

TO facilitate more effective co-ordination of the work of the Institute's many committees, President MacCutcheon recently held a dinner meeting with the committee chairmen (*page 1276*).

NOMINATIONS of Institute officers for 1937 election are to be made soon. Suggestions are invited from the membership (*page 1278*).

1937 TRANSACTIONS

The AIEE TRANSACTIONS for 1937 will be produced only for those having subscription cards on file at AIEE headquarters not later than Friday, December 18, 1936. Present production methods require determination, in advance of the printing of the January 1937 issue of ELECTRICAL ENGINEERING of the exact number of TRANSACTIONS volumes to be provided for. For the convenience of members desiring to maintain in this permanent form a file of AIEE papers and discussions, important additional information and a convenient order blank are provided on page 8 of the advertising section of this issue.

The Privileges and Emoluments of a President

THIS MORNING as I studied the diploma, hanging on my library wall, which I received from Columbia University, I found that as a graduate I was "entitled to all the rights, privileges, and emoluments thereunto appertaining."

The office of president of the American Institute of Electrical Engineers presents many problems and demands a very large amount of time, but I have had an experience within the last 2 months which demonstrates that there are, in addition to the high honor, many "rights, privileges, and emoluments thereunto appertaining." This was the experience of visiting 3 of the Institute Sections, one of them our youngest but already a sturdy youth.

Immediately following the dinner meeting of the Institute committee chairmen, held at the Engineers' Club in New York, September 14, National Secretary Henline and I left on the night train for Richmond, Va. Upon our arrival in Richmond, we were greeted by the chairman of the Virginia Section, J. E. Jackson; the secretary, W. F. Nimmo; and R. C. Bailey, past-chairman. The afternoon was spent in discussing the affairs of the Institute, followed by a drive around the beautiful city of Richmond. At the dinner meeting in the evening, we were given the "privilege" of addressing the Section on the affairs of our organization. The hearty and enthusiastic welcome extended to us by the large number of Section members present surely can be classed as among the "emoluments" pertaining to our official position in the Institute. The Virginia Section membership is not concentrated as in New York, Boston, Chicago, and Cleveland, but at this meeting were many from Norfolk, Lynchburg, Charlottesville, Roanoke, and Richmond. It is a real demonstration of Institute loyalty when a member drives 50 miles to a meeting.

Mark Eldredge, Institute vice-president from District 4, had investigated and prepared an itinerary permitting us to leave Richmond and, without a change of cars, to travel over 3 railroads so as to reach Knoxville, Tenn., at 3 o'clock the next afternoon. At the train we found a reception committee headed by Chase Hutchinson, chairman of the newly organized East Tennessee Section.

We were accorded the "right" to visit the new Norris Dam, inspect a power house of the latest design, and become acquainted with the engineers under whose direction the project had been carried to completion. At the dinner meeting in the evening, we were really astounded when we were greeted by 88 engineers. It was a real "privilege" to be introduced by Robert W. Lamar, vice-president and general manager of the Tennessee Public Service Company. I have never attended an Institute meeting where there was a greater display of enthusiasm and interest in our organization. Many of the East Tennessee Section members are in Chattanooga. The Section felt that there should be a meeting in Knoxville on Friday night and in Chattanooga on Saturday night.

On Saturday morning came another "privilege"—that of visiting the large and modern plant of The Aluminum Company of America at Alcoa. Particularly was it a "privilege" to be conducted through the plant by J. E. Housley, superintendent of power. Mr. Housley devoted his whole morning to us, and among many other interesting things, we saw the largest d-c air circuit breaker in the world.

Chairman Hutchinson accompanied us through the plant and drove with us to Chattanooga, where we were again accorded the "right" of a reception by Vice-Chairman E. E. George, Paul J. Kruesi, and K. E. Hapgood. These members of the Institute were waiting to greet us upon our arrival at the hotel.

We had the "privilege" of being taken to the top of Lookout Mountain by Mr. Kruesi, president of the Southern Ferro-Alloys Company. Both Mr. Kruesi and his father were closely associated with Thomas A. Edison in all his early electrical work.

Another "privilege" was our introduction by Mr. Kruesi at the dinner meeting in the evening. The meeting was presided over by Vice-Chairman George. To the 88 engineers at Knoxville must be added the 41 at Chattanooga, giving an attendance at the first Section meeting of 129. In proportion to the membership of the Section, this is undoubtedly one of the largest meetings ever held by the Institute. The enthusiasm and interest at Chattanooga fully matched that at Knoxville.

A further "emolument" accrued to the president when he was invited to join the first Detroit Section meeting of the year. This was a dinner meeting at which about 125 were present, coming from Jackson, Ann-Arbor, Lansing, Detroit, and other widely distributed points—another very enthusiastic meeting.

About 2 weeks later, came the "privilege" of attending the first Columbus Section meeting of the year, which also was preceded by a dinner.

This message is being sent to you just prior to a trip including the Dallas convention and a visit to practically all the Pacific Coast Sections.

To take from one Section to another the story of the Institute activities, and the enthusiasm and loyalty so strongly in evidence, is a great "privilege" accorded to the president of the Institute.

A M MacCutcheon

Electrical Developments in the Steel Industry

Electrical developments during the past few years in the steel industry are reviewed briefly here. While the industry did not operate at full capacity throughout that period, there was a good demand for flat-rolled products. Much of the new equipment built recently has been for the manufacture of such products.

By
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AS AN INTRODUCTION to a discussion of electrical developments in the steel industry, it is desirable, for the sake of clarity, to describe briefly the recent trend of development of steel-making processes. During the past 2 or 3 years there has been an unusual demand for electrical equipment for rolling-mill drives. This demand has not been due to electrification of old rolling mills, because most of the few remaining mills not electrically driven will be replaced eventually with entirely new mills having modern electrical equipment. New equipment recently purchased or placed in operation is being used in connection with a new process for producing sheet, strip, tin plate, and other flat-rolled steel.

While the steel industry as a whole did not always operate at full capacity during the past few years, there was a good demand for sheet and tin plate and other flat-rolled steel products. The modern tendency to use sheets or plates for fabrication of an increasing variety of products is responsible for this demand. Users of flat-rolled steel not only have purchased in increasing quantities, but have specified material of greater width, rolled more accurately to gauge and with improved physical properties. Steel makers, therefore, have found it desirable to take advantage of the latest developments in methods of producing flat-rolled steel.

Figure 2 shows schematically the development of the process. In the old method of rolling sheet or tin plate, small bars of steel are heated and passed back and forth by hand through rolls rotating at a surface speed of the order of 250 feet per minute. By combining 2 or more light plates obtained in this manner into a pack and by repeated reheating, rolling, doubling, and rerolling, the individual sheets are reduced to the desired gauge. This process is characterized by the simplicity of the equipment, the large amount of hard manual labor required, and the small output.

A few years ago a great many old-style sheet mills were completely mechanized, thereby increasing the output and reducing the manual effort of the mill crews. Modern heating furnaces with mechanical means for charging and discharging were installed. The mill stands were equipped with electrically driven automatic feeder and catcher tables. Figure 3 shows one stand of a sheet mill so equipped. Stands in some of the heavier mills have been provided also with electrical equipment for automatically adjusting the separation of the rolls for each passage of the metal through the mill. A control station for such an installation is shown in figure 1. Many of the mechanized sheet mills probably will be retained for a long time to come for special work. However, to meet the demand for thinner and wider material, rolled accurately to the specified gauge, it has been necessary to turn to an entirely new method of rolling.

In the most recently accepted process for producing thin flat-rolled stock, heated steel slabs from 3 to 6 inches thick and weighing several tons each are first

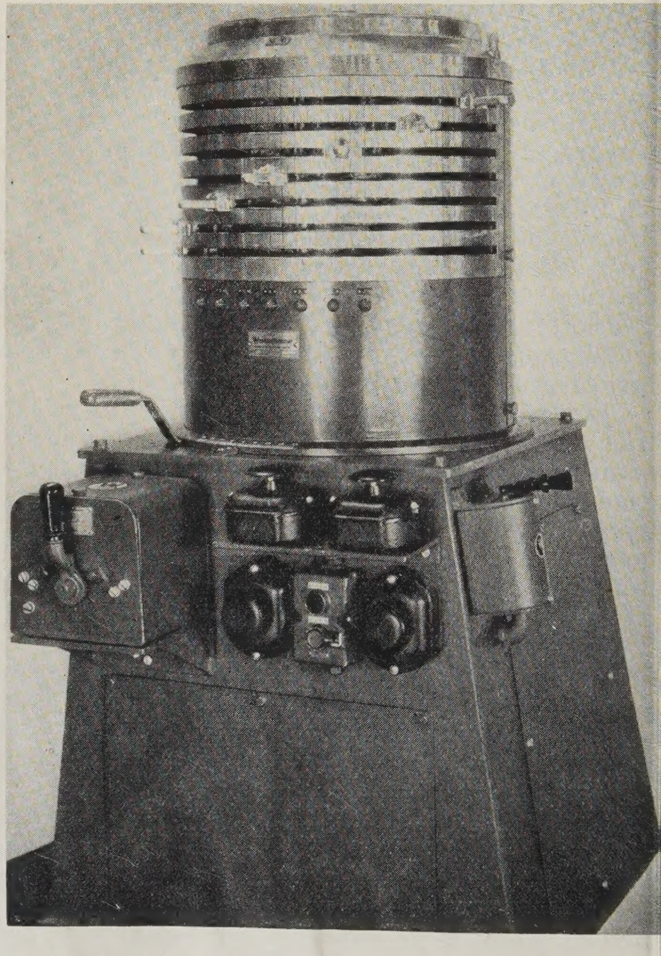
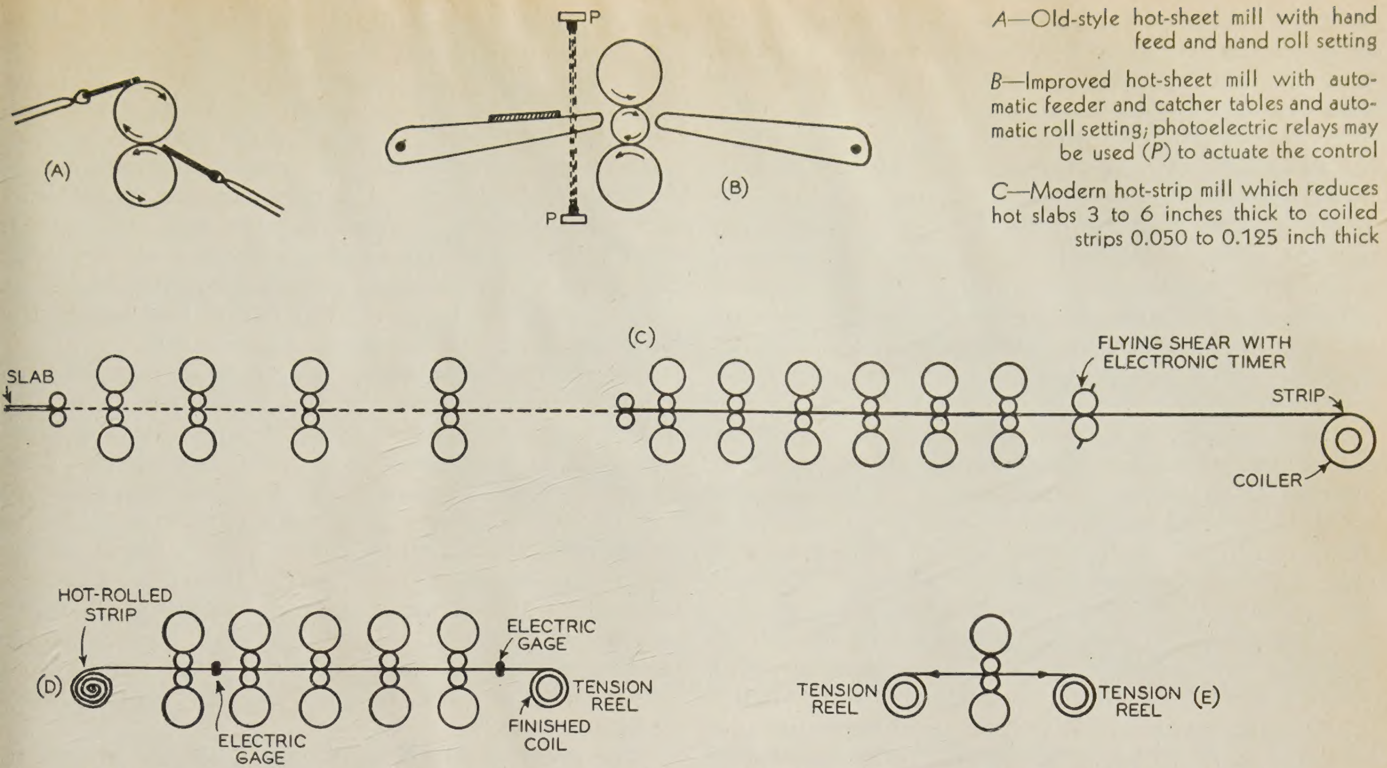


Fig. 1. Automatic-control station for a mill stand heavier than that shown in figure 3

A paper developed from an address presented at the AIEE Great Lakes District meeting, West Lafayette, Ind., October 24–25, 1935; recommended for publication by the AIEE committee on applications to iron and steel production. Manuscript submitted February 20, 1936; released for publication April 27, 1936; brought up to date August 31, 1936.



A—Old-style hot-sheet mill with hand feed and hand roll setting

B—Improved hot-sheet mill with automatic feeder and catcher tables and automatic roll setting; photoelectric relays may be used (*P*) to actuate the control

C—Modern hot-strip mill which reduces hot slabs 3 to 6 inches thick to coiled strips 0.050 to 0.125 inch thick

D—Modern tandem cold-roll mill which reduces coiled hot-rolled strip to coiled strip of sheet and tin-plate gauges

E—Modern single-stand reversing mill which reduces hot-rolled coiled strip to coiled strip of sheet and tin-plate gauges

Fig. 2. Diagrams illustrating development of sheet and tin-plate rolling processes

rolled at high speed in a heavy continuous hot mill. The steel leaves this mill in the form of a strip several hundred feet long and about 0.1 inch thick, and is coiled as it is delivered from the mill. Mills recently installed are designed for maximum delivery speeds up to 2,000 feet per minute. After being pickled to remove the oxide formed during rolling, the coils of strip are reduced to sheet or tin-plate gauges by cold rolling. The cold-roll mills usually deliver at speeds between 500 and 1,200 feet per minute. The finished coil, from 5,000 to 10,000 feet long, is annealed or normalized and cut to desired lengths by automatic shears, ready for shipment or further processing.

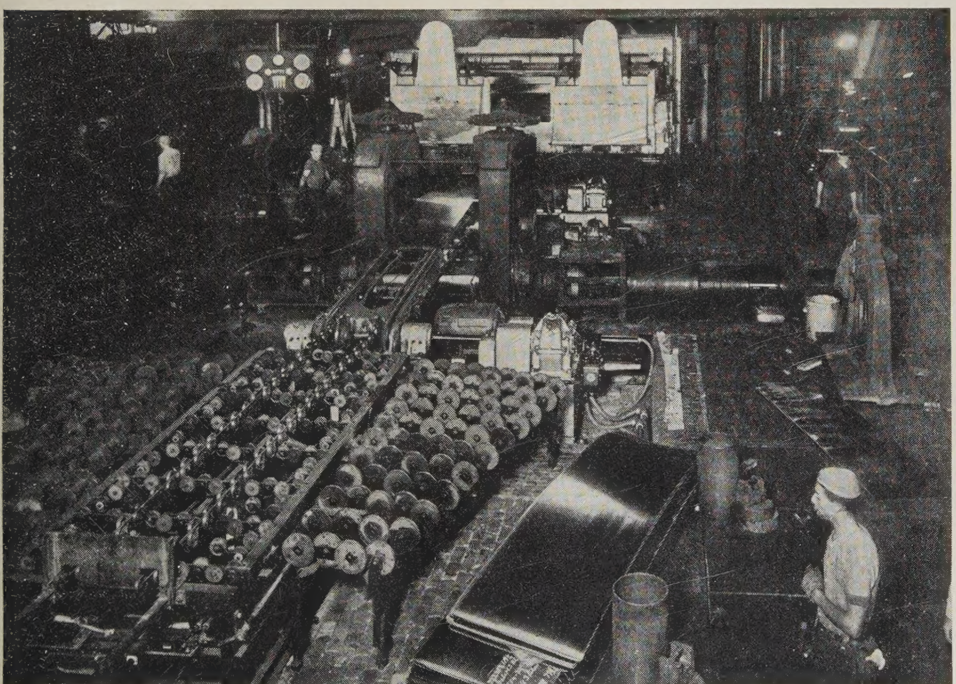


Fig. 3. One stand of an old-style sheet mill equipped with automatic sheet catchers, conveyers, and other labor-saving devices. Heating furnaces and control appear in the background

Slabs for the hot mill are rolled from ingots weighing from 15,000 to 50,000 pounds in blooming mills similar to that shown in figure 4, or in slabbing mills as shown in figure 6. A view of a 10,000-horsepower twin-motor drive for a heavy blooming mill is shown in figure 5.

A modern hot-strip mill usually consists of 10 main roll stands arranged in tandem, as shown in figure 2. Four are roughing stands, separated so that metal is in only one stand at a time. The roughing stands usually are driven at constant speed by induction motors ranging in capacity from 2,500 to 5,000 horsepower. The 6 finishing stands are

mounted as close together as possible and each stand is driven by a 600-volt adjustable-speed d-c motor; these motors range in capacity from 2,500 to 5,000 horsepower. Power for the finishing train is supplied by 2 or more synchronous motor generators each having a capacity of from 4,000 to 6,000 kw. The complete installation occupies a building some 1,500 feet long and 100 feet wide. About once every minute a steel slab weighing several tons passes in a straight line through the mill. With the passing of each piece of steel through the mill, the electrical input to the motor room will rise from a few thousand kilowatts to a peak of from 15,000 to 30,000, depending on the width of the finished strip.

Since January 1, 1935, 6 hot-strip mills have been placed in operation. One mill is 43 inches wide, another is 56 inches, 2 others are 79 inches, another is 80 inches, and another is 96 inches. A somewhat similar mill, 100 inches wide, which will be used to roll wide plate, is now under construction. Each installation with auxiliaries and finishing mills requires an investment of from 8 to 20 millions of dollars. Figure 7 gives a view of the finishing train of a typical 79-inch mill. The motor room of this mill, with 6 3,500-horsepower 175/350-rpm finishing-train motors and 2 6,000-kw synchronous motor generators in the foreground, is shown in figure 8. The complete list of main equipment for 2 typical mills is given in table I.

The modern hot-strip mill, in spite of its size, is a assembly of precision machinery. The rolls of th finishing stands are supported in sleeve or rolle bearings of very special design which are fitted wit extreme accuracy. In some mills special pressur indicators are installed to prevent overloading c bearings so that the accuracy can be maintaine over long periods. Temperatures and combustio conditions in the slab-heating furnaces are recorde continuously by precision metering equipment. Th temperature of the steel as it enters and leaves th finishing train is controlled and recorded.

Description of a simple electrical device used t transmit signals between the inspection beds an the roller's station will serve to illustrate the clos supervision exercised over mill operation. Thi device permits the inspectors to signal the roller t change the thickness of the strip by from 0.001 t 0.010 inch, the width by from $^1/_{16}$ to $^3/_8$ inch, or th length cut by the flying shear by from 2 to 4 inches or to indicate an unsatisfactory condition of eithe surface of the steel. To the layman, such accurat control of a steel strip delivered from a mill at speed of from 1,000 to 2,000 feet per minute ma seem uncanny.

The receiver of the signaling device is shown i figure 10. This unit is located at the roller's station One or more transmitters with the same dial mark ings are mounted at points convenient to the inspec tors. When the pointer on any transmitter i placed in a certain position, the receiver and othe transmitters assume the same position and a bell o horn on the receiver gives an alarm. The rolle observes the position of the receiver pointer, reset it to the "OK" position to indicate receipt of th message, and makes the desired change on the mill.

The individual a-c motor drives for run-out table for conveying finished strip and for hot-strip coiler have become an important part of the modern hot strip mill. A few years ago individual drive wa

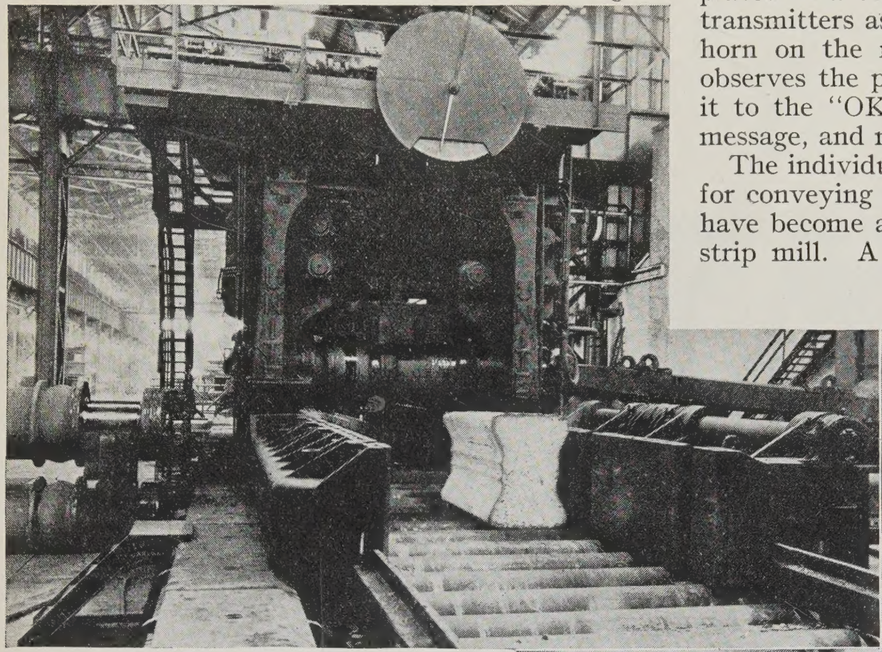


Fig. 4 (above). Large blooming mill

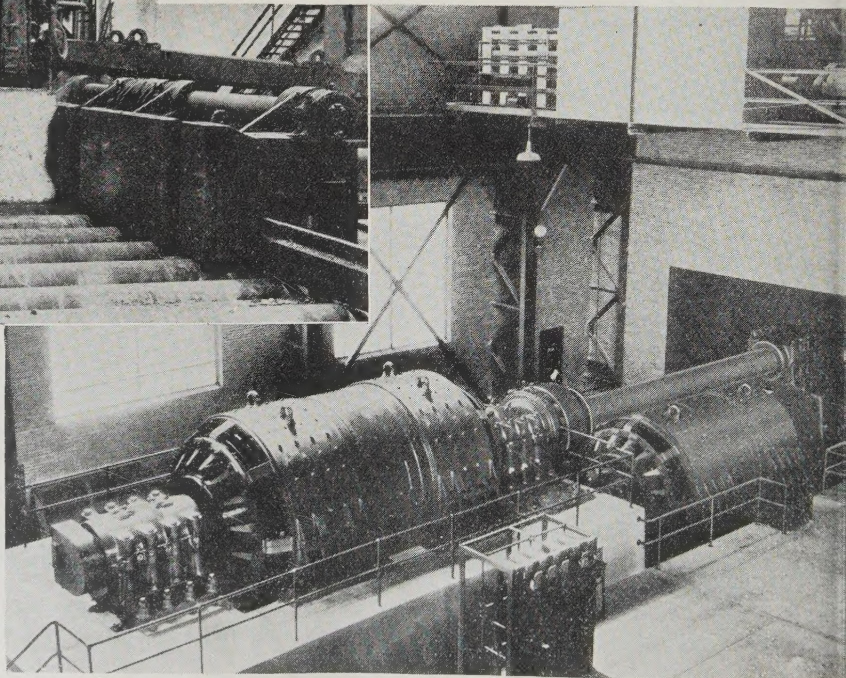


Fig. 5 (right). A 10,000-horsepower twin motor drive for a large blooming mill

ıpplied principally to continuously running conveyor ables with light rollers. The notors usually were direct-connected. Mill builders have een quick to utilize the development of the gear-motor and recent improvements in he electrical design to widen he field of application of individual drive. In recent installations, tables with relatively heavy rollers and designed for frequent starting and stopping have been equipped with individual motors. Three of the strip mills placed in operation during the past year require a total of 2,100 such motors supplied with power through 20 variable-frequency motor generators. Approximately 1,200 such motors are equipped with thermal overload switches which serve as disconnecting devices and which automatically disconnect a motor should a roller become jammed. Many of the switches are installed in waterproof cast-iron or steel switch boxes. This reflects the care now being exercised in the installation of such auxiliary drives.

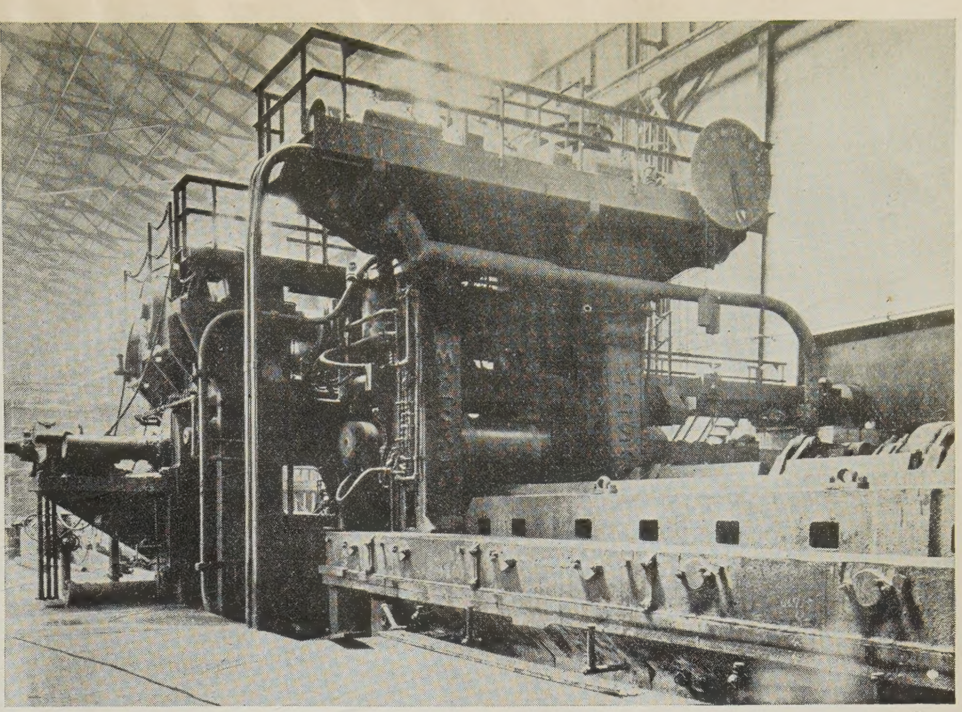


Fig. 6. Large slabbing mill

Cold rolling to sheet and tin-plate gauges is accomplished in tandem mills or in single-stand reversing mills having a tension reel on either side of the working rolls. In either arrangement, d-c adjustable-speed drives are used. Mills of this type up to 93 inches in width are now installed. A typical reversing mill is shown in figure 11. Such a mill consists of a heavy 4-high roll stand and 2 reels. As shown by the schematic diagram in figure 9, the working rolls and reels are driven independently. The most powerful mill of this type, regardless of size, has a 4,000-horsepower motor for the working rolls and 2 1,500-horsepower reel motors.

In operating a reversing cold mill, a coil of hot-rolled strip is placed in the reel on one side of the mill and the end of the strip is threaded through the working rolls and into the jaws of the second reel. Electrical adjustments are made which place the strip under tension on both sides of the mill, and the mill and reels are brought up to the desired speed. Several passes are required to reduce hot-rolled strip having a thickness of the order of 0.1 inch to tin-plate gauge. The finishing passes may be made at speeds up to 1,100 feet per minute. The running tension on both sides of the mill can be adjusted independently, and the tension is maintained automatically within very close limits. Because of the decreasing diameter of the coil on the entry side, the speed of the entry reel must increase gradually throughout the duration of each pass. The back tension on the entry side is obtained by operating the entry-reel motor as a constant-current generator, feeding power into the main generator. The delivery-reel drive functions as a motor with constant-current input and with gradually decreasing speed as the diameter of the coil increases. Tension on either side is proportional to armature current, and the desired currents are maintained by vibrating regulators which control boosters in the shunt-field circuits of the reel-motor. Successive passes are made in opposite directions, so the functions of the reel drives are reversed after each pass.

The speed range of the reel motors must be sufficiently wide to cover the speed range of the mill and the variations in diameter of the coils. In addition, the speed of the entry reel must be reduced to compensate for mill draft, which may be as much as 50 per cent. The total range required is often as much as 6 to 1 in addition to the range secured by variable voltage control. Usually it is undesirable to build large 600-volt motors for 6 to 1 speed range by field control. Figure 9 shows an arrangement often used to permit the use of reel motors with not

Table I—Main Motor and Generator Equipment for 2 Typical Hot-Strip Mills

	43-Inch Mill			79-Inch Mill		
	Horsepower	Speed, RPM	Drive	Horsepower	Speed, RPM	Drive
Scale breaker 1	500	500		1,000	375	
Stand 1	2,500	500		3,000	150	
Stand 2	2,500	500		3,000	500	
Stand 3	2,500	500		3,000	500	
Stand 4	2,500	500	Geared	3,000	500	
Scale breaker 2	500	250/750		500	150/600	
Stand 5	3,000	200/400		3,500	175/350	Geared
Stand 6	3,500	200/400		3,500	175/350	
Stand 7	3,500	200/400		3,500	175/350	
Stand 8	3,500	135/270		4,500	125/250	
Stand 9	3,000	170/340	Direct	4,500	125/250	
Stand 10	2,500	185/370		2,500	175/350	
D-c generators	6	2,000 kw		4	3,000 kw	

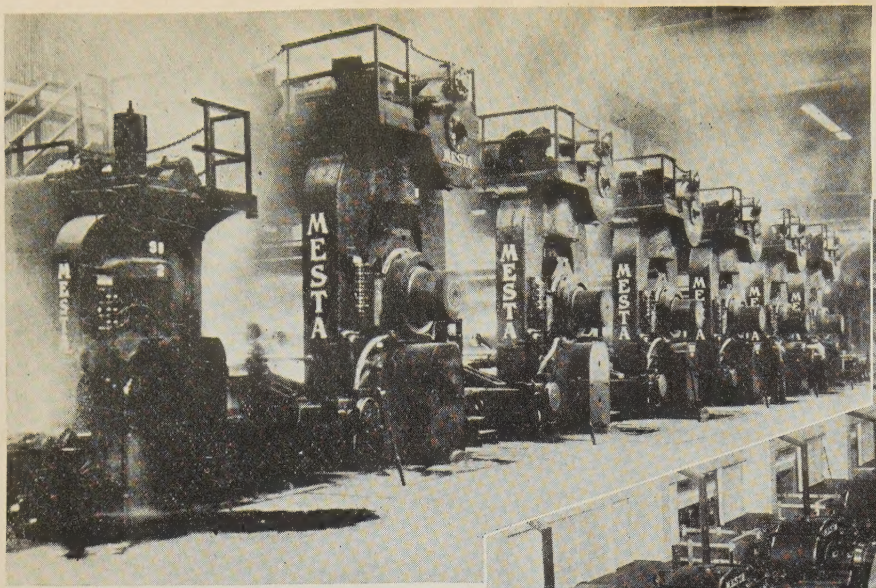


Fig. 7 (left). Finishing train of a typica
79-inch continuous hot-strip mil

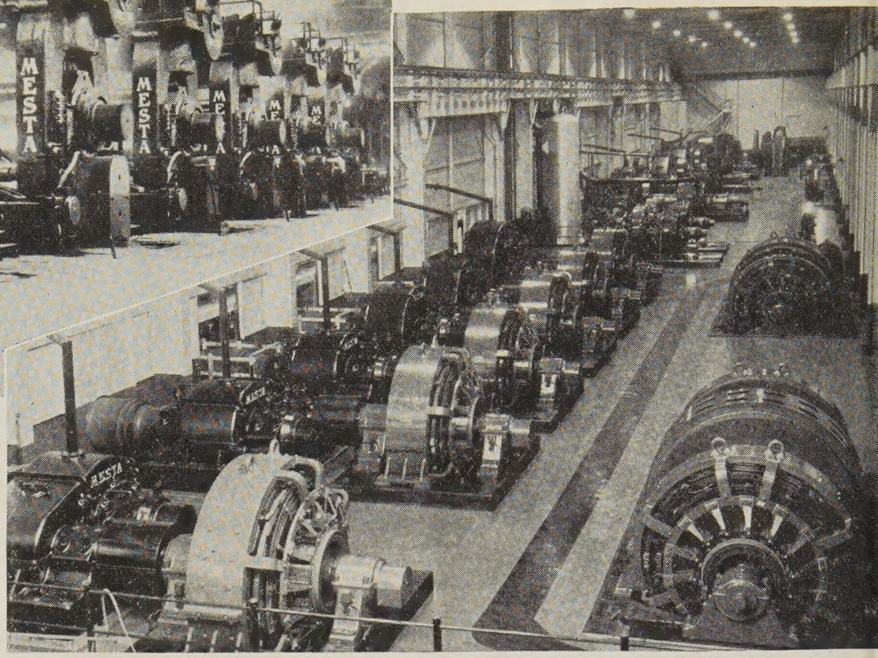


Fig. 8 (below). Motor drives for the
mill shown in figure 7

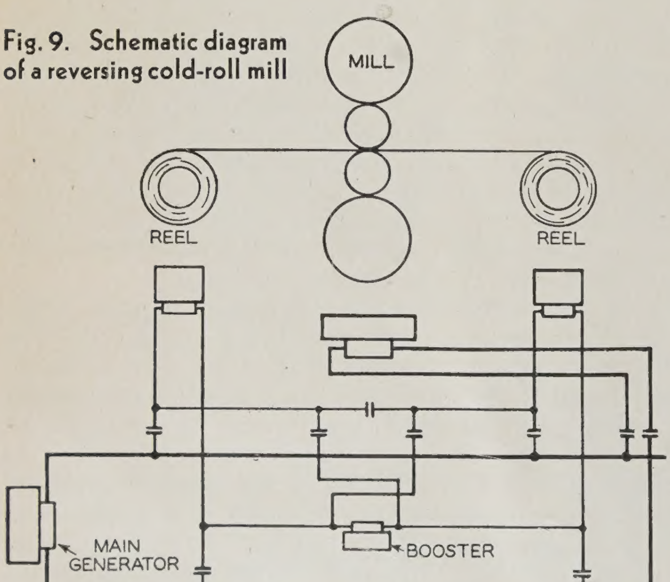
more than 3-to-1 speed range by field control. A booster generator is used, and connections are made so that the booster can be connected in series with whichever reel motor is on the entering side of the mill.

Tandem cold-roll mills may have from 3 to 5 stands, depending on the product to be rolled. For rolling the heavier sheet gauges 3 stands are used, and for rolling tin plate 4 or 5 stands are used. Delivery speeds of mills of this type vary from 700 to 1,300 feet per minute. A recent 5-stand tandem mill 42 inches wide has one 500-horsepower and 4 1,250-horsepower main motors and a 250-horsepower reel motor. All motors are d-c adjustable-speed machines.

In operating a tandem mill, the field rheostats of the mill motors are adjusted to suit the schedule of drafts and the desired delivery speed. Each strip is threaded through the mill at low speed and the end is entered in the reel. The strip is placed unde
tension between stands and between the last stand
and the take-up reel, and the entire mill is brought
up to speed by voltage control. Tension is maintained by means of automatic regulators or by
manual control.

Since the beginning of 1935 about 30 cold-roll mills
of the 2 types described have been ordered or placed
in operation. Each of these installations is equipped
for precision control of the product. Electronic
micrometers indicate deviations of less than one ten-
thousandth of an inch from the desired thickness
and adjustments are made immediately to correct
any variations.

Fig. 9. Schematic diagram of a reversing cold-roll mill



To utilize the equipment at his disposal to the
maximum extent, the supervisor of a modern rolling
mill must be supplied with raw steel of the best
possible quality. This makes it necessary to exercise
close control of every steel making operation from
the ore pile to the final heat-treating furnace
The steel maker, therefore, is obliged to use specia
equipment which a few years ago would have been
classed as unnecessary luxuries or at least too com-
plicated for use in steel plants. For example, a few
years ago the use of electronic devices was not con-
sidered necessary or desirable. Today the stee
maker is becoming accustomed to the use of appa-
ratus having electronic control. Some typical ex-
amples of electronic equipment applications are:

1. Photoelectric relays are used to replace mechanical flag switche
for automatic sheet catchers and similar applications where it i
desirable to avoid the use of mechanical limit switches actuated b
contact with steel in motion.

2. Photoelectric relays are used to actuate furnace or soaking-pi
doors from the cranes that handle the steel. This greatly simplifie

the electrical circuits of the cranes, since the operator controls the doors by flashing a light on the crane and no other control circuits are in the crane.

3. Electronic timers are used in connection with flying shears and other devices that must operate in synchronism with the movement of metal through a mill.

4. Electronic high-voltage rectifiers are used in connection with dust precipitators for cleaning blast-furnace gas.

5. Electronic amplifiers are used in gauging and recording apparatus to secure greater sensitivity.

6. Electronic timers are used for special resistance welding work.

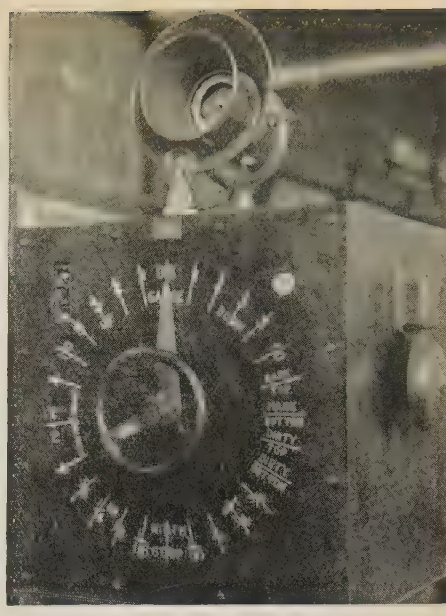


Fig. 10. Receiver of signaling system used to transmit signals between the inspection beds and roller's station

In recent years the mercury-arc rectifier, which is a high-capacity electronic device, has been developed for heavy power work, and some installations have been made in railway substations. To date, only one rectifier has been sold for steel mill service in the United States. Steel plant engineers have been slow to use large rectifiers for 2 reasons. First, American mills have standardized on 250 volts for d-c auxiliary service, and until recently rectifiers have not been attractive from the standpoint of efficiency for this voltage. The recent development of the rectifier utilizing an igniter rod provides a means of improving the efficiency at low voltage and widens the field of application. Second, for large 600-volt d-c installations it is usually necessary to provide variable voltage for motor starting and constant voltage under varying load, and at times regenerative braking peaks occur. To provide for these features it is necessary to use considerable auxiliary apparatus, and so far steel makers have preferred synchronous motor generators.

To secure the utmost advantage from the improvements in rolling processes, steel makers are utilizing heat-treating equipment of the latest type. A steel maker in Detroit, Mich., has installed 12 annealing furnaces and 36 bases and metal hoods to be used in connection with a new strip mill. Each furnace is rated at 230 kw, and all units are designed to anneal 2 16,000-pound coils of strip 52 inches in outside diameter, 30 inches in inside diameter, and 48 inches wide. In the same plant, a continuous electric normalizing furnace for cold-rolled strip was placed in operation. This furnace has a capacity of 5 tons per hour of 19-to-20-gauge cold-rolled strip 48 inches wide. The normalizing is done without affecting the finish on the steel.

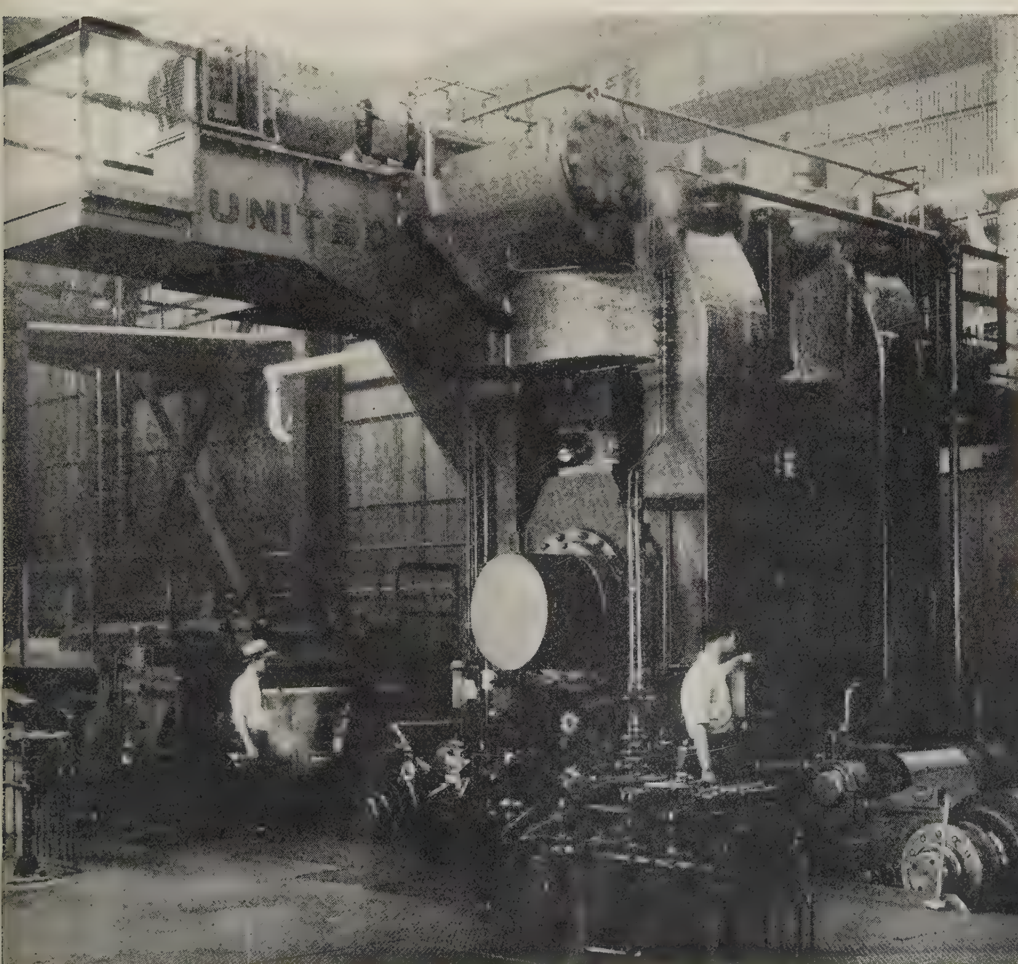



Fig. 11. Typical reversing cold-roll mill

High-Efficiency Gaseous-Conduction Lamps

By
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Lamp Co., Hoboken, N.

High-efficiency gaseous-conduction lamps, as exemplified by the recently developed sodium, fluorescent, and high-pressure mercury lamps, are discussed in this paper. The relationship between the light energy radiated at different wave lengths and the efficiency of the lamps is treated in some detail.

THE remarkable efficiency of the recently developed sodium, fluorescent, and high-pressure mercury lamps has stimulated interest in the factors determining the efficiency of lamps in general. In this paper the relative energy distribution in the spectra of light from such sources is related to their efficiency, with emphasis on the use of fluorescent materials to convert invisible radiation into light to produce mercury lamps of unusual color and efficiency. There is reference to recent improvements in the operating circuits and the thermal insulation of sodium lamps. There is a general discussion of the effect of increased energy density in the mercury arc on its efficiency, and reference to the operating characteristics of recently developed lamps that supply from 30 to 40 lumens per watt.

Sodium Lamps

If by some process unknown today, it were possible perfectly to convert electricity directly into radiation of the yellow-green color to which human eyes are most sensitive, the result would be an efficiency of some 620 lumens per watt. The best that has been done by electric heating of a tungsten wire is to convert from about 15 to 30 per cent of the electricity into radiation of the full range of color to which the eye is sensitive, from a maximum in the yellow-green range to zero at the ultraviolet and infrared ends of the spectrum, and which the eye sees with an average effectiveness of about 23 per cent; the resulting efficiency is from 15 to 40 lumens per watt, depending upon the temperature. The best that has been done by electric excitation of gases or vapors is, in the case of sodium, to convert from about 10 to 12 per cent of the electricity into radiation of a yellow color to which the eye is about 75 per cent sensitive, with an efficiency of from about 45 55 lumens per watt.

While these efficiencies may seem low, it must remembered that with a perfect conversion of ele tricity into visible radiation about 220 lumens p watt is the highest possible without some su sacrifice of light color as accounts for the efficien of the sodium lamp.

Since the efficiency of a sodium lamp is depende upon a rather close maintenance of a critical vap pressure, or of the electrical characteristics corr sponding to it, the sodium arc must be enclosed an insulating vacuum jacket that will insure a wa temperature for the glass arc tube of about 2 degrees centigrade. So long as the optimum wat per unit of activated sodium vapor is maintaine the actual arc current and voltage may be varied suit various design conditions as represented by t 6.6-ampere 32-volt series lamp of American desi and manufacture, and a possible 1.4-ampere 150-vo lamp of the type made in Holland.[1]

The American lamp was designed primarily f street-lighting service where the economies possib with constant-current series operation are desirabl It naturally took the high-current low-voltage for in which oxide electrodes of adequate capacity a located a few inches apart in an arc tube of a diamet about 3 times the arc length. The character of t electric discharge is dominated by the cathod functioning of the electrodes and nearly half of t arc voltage drop is at the electrodes. This is contrast with the low-current high-voltage type arc being made abroad for multiple operation constant-voltage lines. It has a positive colum type of electric discharge, the electrode drop being small proportion of the total arc drop and the di tance between the electrodes some 30 times t diameter.

The design and operating characteristics of t sodium lamp have been discussed so fully els where[2–10] that only the recent series constructi and operation will be discussed here. Because the high-current low-voltage operation, a separate heated type of cathode has been used which h required an elaborate delayed action switch to insu about a minute of preliminary electrode heati before the starting of the arc discharge. This heati was provided by 2 low-voltage windings on a tran former required for each lamp unit.

In the new construction and circuit, the heate are designed to take the same normal current as t arc itself and are always in series with the arc, th providing some of the ballasting. In figure 1 a shown the new and old series circuits in schemat form. A delayed action switch is still required

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1. For numbered references see list at end of paper.

the new circuit, but the heater transformer has been eliminated.

To provide the essential thermal insulation, the practice has been to enclose sodium lamps in double-wall vacuum flasks whose exhaust has been expensive and whose operating temperatures have been such as to encourage leakage. Recently, it has been found that the evacuation can be dispensed with by the insertion of a third cylindrical glass wall between the flask and the lamp as shown schematically in figure 2.

While there is still some controversy as to whether visual acuity as ordinarily measured is a factor in highway illumination, there is some evidence that sodium light provides a heightened contrast making for improved definition of most of the objects to be seen on a typical highway.[11,12] Improvements still are to be made in the design of reflector and diffusing equipment, as is to be expected from the radical changes being made in these accessories for the incandescent lamp.

Sufficiently extensive installations of sodium lamps now are being made to insure a decisive test of them during the next few years. It would seem that where color discrimination is not important, so efficient a lamp should have a place to be disputed only by the high-intensity mercury lamp.

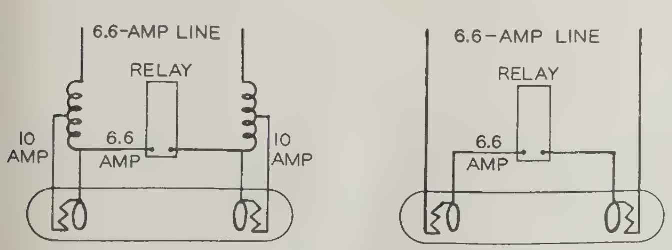


Fig. 1. Series constant-current sodium-lamp circuits with multiple (left) and series (right) electrode heaters

Fluorescent Lamps

The possibilities and limitations of fluorescence as a secondary source of light have long been known,[13] and recently the perfection of fluorescent screens[14,15] for experimental and laboratory use in television was reported.

The 2 important factors in a fluorescent lamp are the fluorescent material and the source of radiation to activate it. The characteristics of these 2 factors have dominated the design of the fluorescent lamps now being developed for practical use, and it is surprising how well lamps resulting from so definitely mechanical, electrical, and chemical limitations fit practical and artistic needs.

The immediate aim in the addition of fluorescent material to a lamp is to secure increased efficiency in the production of approximately white light or of colored light of unique quality. Fluorescent materials may be divided conveniently into 2 classes: those materials activated by ultraviolet radiation of longer wave lengths than about 330 millimicrons, and hence applicable to the outside of lamp bulbs and tubes of glass transmitting this radiation; and those activated by ultraviolet radiation of shorter wave lengths than about 330 millimicrons or by cathode rays, and hence applicable only to the inside of bulbs or tubes that do not ordinarily transmit this activating radiation but are perfectly transparent to the resulting fluorescent radiation. Practical materials of the former class are sulphide and uranyl salts of alkali metals, the much used sulphides of zinc, and a great variety of organic materials, some of which are familiar dyestuffs such as eosin, fluorescein and rhodamine; and of the latter class typical examples are zinc silicate in the form of natural or artificial willemite, and calcium tungstate. In so far as the activating source may contain visible radiation useful because of either its luminosity or its color, it is desirable that these materials be applied to the arc tube in such a manner as to absorb as little as possible of the radiation. Since the materials often are finely divided but opaque particles, there is a simple relation between the absorption of activating radiation and the transmission of visible radiation by the screen formed when the material is uniformly distributed on the surface of a glass tube or bulb. In any case the preparation of this screen is more difficult than the preparation of a comparable fluorescent surface or reflector of the familiar variety as all the activating radiation, in effect, must pass through the body of the screen and, to some extent, reach the effective side of the particles by diffusion. A glazed screen surface providing for internal reflection of activating radiation may defeat its own purpose by also imprisoning the fluorescence, as observed years ago.

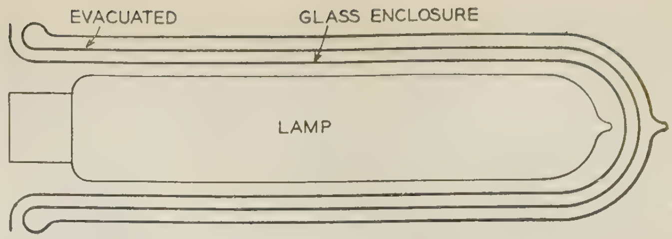


Fig. 2. Thermal insulation of the sodium lamp

As a matter of fact, nearly the whole development problem of the lamp centers around the manner of applying materials, themselves well-known except for recent chemical variations of a nature foreign to this paper. The screen may be applied to either the outer or the inner surface of the lamp tube or bulb, depending upon the nature of the activating radiation, the transmission of the glass, and the response characteristics of the material, as will be shown graphically later. If placed on the outside the screen must resist abrasion, moisture, and cleaning agents. The outside film makes possible the use of a single type of lamp tube proper as the activating source for a variety of fluorescent materials—an important advantage from the manufacturing standpoint, paralleled by the common practice of dipping standard clear incandescent-lamp bulbs into paints and lacquers to secure a variety of colors. The inside film, while well-protected from external injury, must resist reaction with the vapor or gas used and the destructive action of the electrical discharge, in

order that the lamp as a whole may not depreciate too rapidly. The inside film requires the original fabrication of a variety of types as great as the number of fluorescent materials used—an important manufacturing disadvantage, paralleled by the practice of using inside colored coatings only on a few incandescent lamps of the types made in large quantities.

Obviously this screen problem is largely one of the vehicle used to carry the fluorescent material. This vehicle has varied from the use of the glass itself, as represented by the practice of fusing the material onto the surface of the glass or into the composition of the glass, to the use of ordinary lacquers. This use of glass seems ideal from the mechanical and practical, and from the chemical standpoints, as many of the fluorescent materials are of the same chemical nature as standard glass ingredients. A great variety of such glasses has been made and one, a uranium glass, is widely used in molded glass articles because of its striking yellow-green "canary" fluorescence even by sunlight. The limitations of the method are that the glass must transmit the activating radiation, the usable materials are relatively inefficient, and the process is inflexible in manufacture as is the inside screen. The use of a sodium silicate vehicle also seems natural, as many of the best fluorescent materials are silicates, but the limitation here is the hydrolysis by water vapor. Lacquers, shellac, and similar substances have poor heat resistance, develop color or selective absorption for the fluorescent light, and they are supposed to decrease and inhibit the fluorescence of some materials. Just what are the most practical vehicles today is too new to disclose and probably too transient to record.

The most important recent advance in materials is in those of the sort used for cathode-ray-oscillograph and television screens, as represented by zinc silicate and calcium tungstate. These materials are generally fluorescent to the 254-millimicron resonant radiation of the mercury arc as well as to cathode rays or to electron bombardment. For practical purposes, these materials must be placed on the inside surface of the lamp tube and are limited in their efficient use to an activating source that is rich in radiation having a wave length of 250 millimicrons or less. Some of the efficiency of these materials in lamps may be attributed also to a response to electrons present in the mercury arc, even if of too low a velocity to be called cathode rays. Materials of this type have been studied more carefully than any other because of their importance in cathode-ray tubes and as intensifying and fluoroscopic screens for X-ray practice. When used in fluorescent lamps, however, they are exposed to the activating radiation more completely and continuously than in cathode-ray devices. Some of their depreciation is attributable to obscure changes in their physical makeup and is a function of the intrinsic intensity of the activating radiation. This is to be expected as fluorescence generally is associated with unstable compounds and reversible reactions.

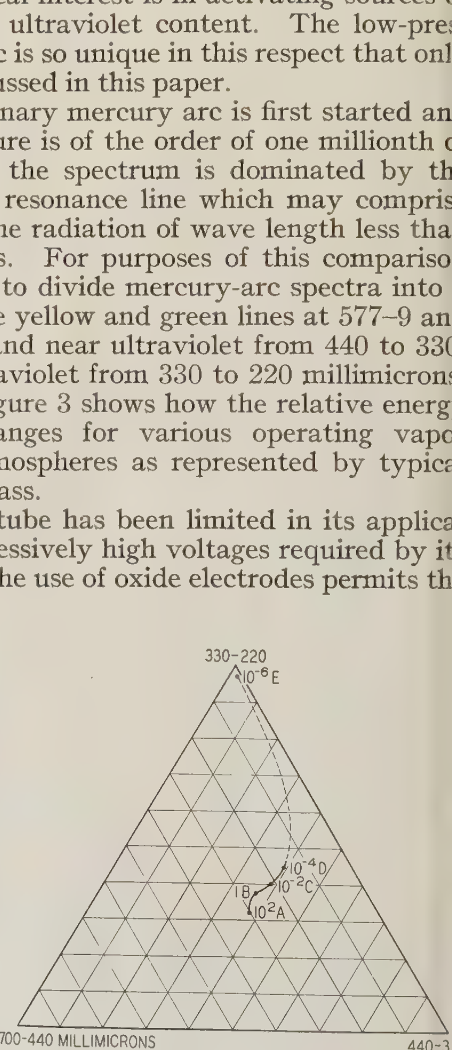
Only minor refinements have been made in materials responsive to radiation passing through ordinary glass. Their limitations as to efficiency, life, and variety have long been known. They tend to de-preciate for the same reason as do the previousl discussed materials and, in the case of the dyes, ma be entirely fugitive in the color and luminosity (their fluorescence. Thus with these materials, a with those activated by radiation of shorter wav length, the fluorescent surface of a lamp should b as extensive as possible for a given size of lamp t secure the lowest possible intrinsic intensity of th activating but destructive energy consistent with useful intensity of fluorescence.

The source of radiation for the activation of thes materials is comparable with them in importance, bu is simpler in the development because of the uniqu superiority of one type of mercury arc. Since fluo-rescent light nearly always requires radiation (shorter wave length for the activation of its sourc material, practical interest is in activating sources (high violet and ultraviolet content. The low-pres-sure mercury arc is so unique in this respect that onl this type is discussed in this paper.

When an ordinary mercury arc is first started an the vapor pressure is of the order of one millionth (an atmosphere, the spectrum is dominated by th 254-millimicron resonance line which may compris 95 per cent of the radiation of wave length less tha 330 millimicrons. For purposes of this compariso it is convenient to divide mercury-arc spectra into parts, the visible yellow and green lines at 577–9 an 546, the violet and near ultraviolet from 440 to 33(and the far ultraviolet from 330 to 220 millimicrons On this basis, figure 3 shows how the relative energ distribution changes for various operating vapo pressures in atmospheres as represented by typica arcs in quartz glass.

The Geissler tube has been limited in its applica-tions by the excessively high voltages required by it electrodes, but the use of oxide electrodes permits th

Fig. 3. Change in relative energy distribution with a decrease in vapor pressure from 100 atmospheres to one millionth of an atmosphere, due largely to the increase in the 254 - millimicron resonance line



A—Super-high-pressure arc
B—High-intensity arc
C—Intermediate-pressure arc
D—Cooper Hewi lam
E—Geissler tube

design of lamps of similar radiation characteristic and operating at ordinary voltages. They take th form of tubes about one inch in diameter and 12 to 2 inches long, and the wattage must be kept low, of th order of 0.5 watt per square inch of arc tube surfac to retain a sufficiently low vapor pressure for th production of 254-millimicron radiation.

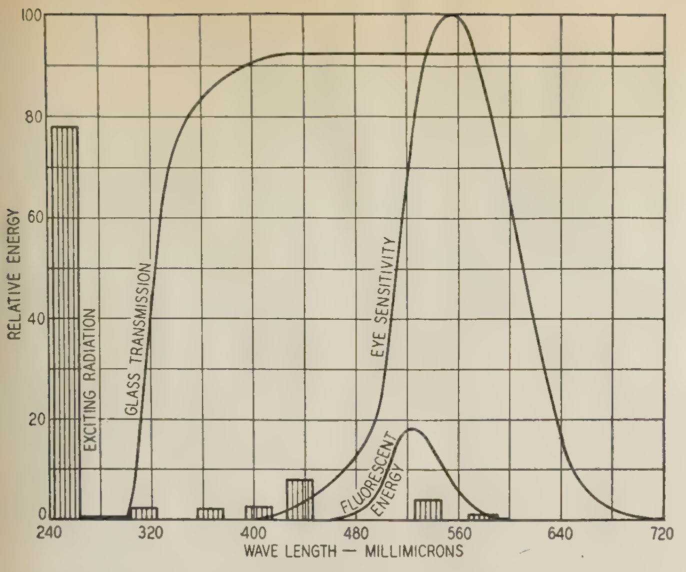


Fig. 4. Energy distribution, glass transmission, and fluorescent component for a typical fluorescent mercury lamp

Figure 4 shows the relationship between the energy distribution of the primary source of radiation, the transmission of the glass arc tubing and the energy distribution of the fluorescent light, for a typical combination. It is obvious that the quality of the light from such a lamp is then a property of the fluorescent material used rather than of the source of primary radiation. One of the virtues of this type of lamp is in the delicate pastel colors characteristic of the various materials themselves, and these unsaturated colors are somewhat incompatible with high brightness. One of the principal applications of these lamps is for assembly into low-wattage lines of light for use in decorative and architectural lighting.

Thus the apparent limitations of these lamps prove not to be limitations at all. The desirable low wattage and low brilliancy permits optimum electrical conditions and minimizes depreciation. The surface of the lamp is the source of light and no further diffusion is needed. The efficiencies obtainable are comparable with the best of the largest lamps and are from 5 to 10 times as great as those of small incandescent lamps of comparable wattage in clear or colored bulbs. There should be a large field for such lamps in specialty lighting work.

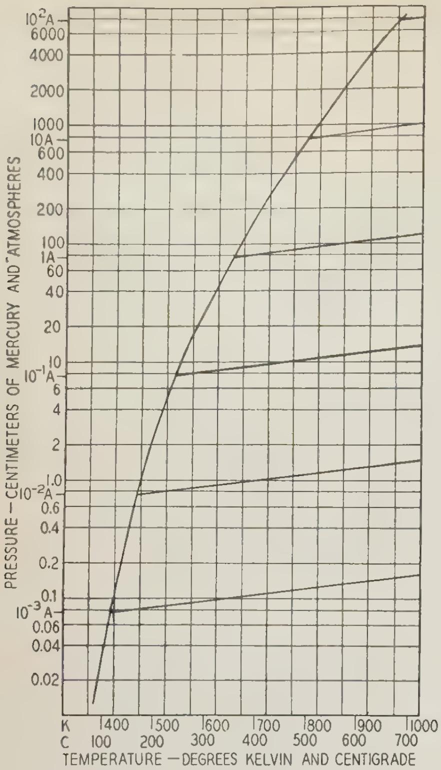
High-Intensity Mercury Lamps

The prime reason for the introduction, in 1902, of the mercury arc as a light source was its then phenomenal efficiency of about 18 lumens per watt for the bare tube of the d-c lamps as compared with from 3 to 4 lumens per watt for the comparable carbon-filament incandescent lamp of that time. Since then the incandescent-filament lamp has been improved to nearly the theoretical limit of efficiency, still barely comparable with the mercury arc. Only recently have any outstanding changes been made in the mercury arc through the use of new electrode materials.

It has been known for years, since the work of Küch and Retschinsky[16] in 1906, that efficiencies of the order of 50 lumens per watt in the arc may be secured by operating a mercury arc at increased vapor pressures. It has been obvious that this increased efficiency is a function not of increased vapor pressure but of the current density, the potential drop, and the amount of conducting mercury vapor resulting from the heating of the liquid mercury cathode. The latter factor, the rôle of the mercury vapor, was difficult to determine until Krefft and Pirani,[17] Elenbaas,[18,19] and others constructed oxide-cathode mercury arcs containing definitely measured amounts of mercury to be completely vaporized during arc operation. This at once simplified both the theory and the operation of the mercury arc and led to such generalizations, applicable to all mercury arcs, as are presented by J. W. Marden elsewhere in this issue.[22] In this paper the discussion is limited to the practical operating characteristics of the limited-pressure arc, as exemplified in the high-intensity mercury arc recently standardized for manufacture and general use.

The mercury arc, of necessity, has always been as truly an enclosed and evacuated a device as the incandescent lamp. This complete enclosure of the arc introduces vapor and gas pressure as a factor so profoundly influencing the familiar voltampere characteristics of the ordinary arc as to make temperature control the dominant problem in lamp design and operation. While vapor density may be more interesting than vapor pressure from the standpoint of the radiation characteristics of an arc, it is variable, with high gradient in the arc itself, and difficult to measure or calculate. Vapor pressure, which can be calculated from temperature measurements, limits the choice of arc tube materials, and can be closely correlated with the arc voltages. From the standpoint of practical lamp operation, it is a convenient basis for the comparison of various types of mercury arcs.

In all arcs of the open type, the pressure is relatively constant and corresponds to that of the surrounding atmosphere. With enclosure, the vapor pressure becomes at once a function of temperature. In a gaseous-conduction arc such as the hot-cathode neon lamp, the gas pressure, at the constant volume defined by the enclosure, varies simply and directly with the absolute temperature. In a vapor-conduction arc such as that of the Cooper Hewitt mercury lamp, the vapor pressure is a complicated function of the absolute temperature of the condensed vapor, but is independent of the volume of the enclosure. With the quantity of conductive materials limited to an amount that can be completely vaporized at the temperature and pressure permitted by the enclosure, as in the recently perfected high-intensity mercury-vapor arc, the vapor pressure, at the constant volume defined by the enclosure, is the vapor tension or pressure of the material of the arc up to the temperature of complete vaporization. From this critical point the pressure varies directly with the absolute temperature as in the neon lamp. This theoretical relation between the minimum arc-tube temperature and the vapor pressure is shown graphically in figure 5.



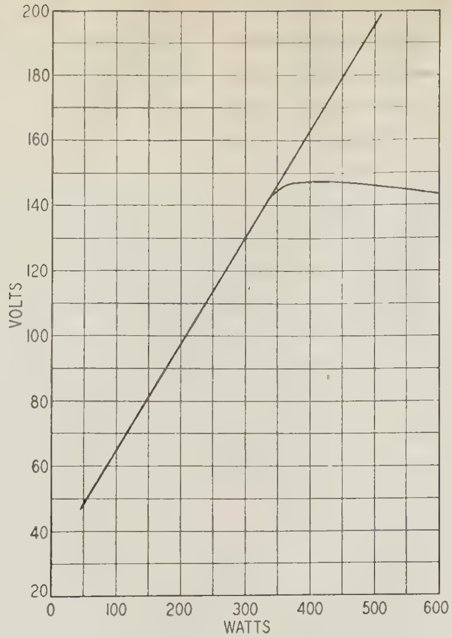


Fig. 6. Static arc volt-watt characteristic curve

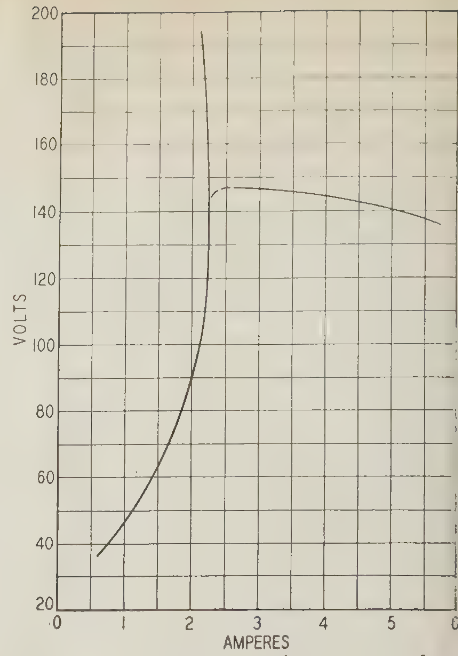


Fig. 7. Static arc volt-ampere characteristic curve

Fig. 5 (left). Mercury-vapor pressure as a function of temperature with complete vaporization from various pressures

For any one mercury arc, such as the 400-watt high-intensity arc, the minimum arc temperature follows closely the energy input to the arc, or the arc watts, while the arc voltage is determined largely by the mercury-vapor pressure. Hence this watts-volts curve, figure 6, is very similar to the temperature-pressure curve, figure 5. It represents either the earlier quartz so-called Uviarc or the latest high-intensity arc up to the point of complete vaporization of the mercury in the latter, from which point it shows the latter's unique performance as a limited-pressure limited-voltage device.

If a mercury-arc is operated in series with an adjustable reactor and is allowed to reach thermal equilibrium at various values of arc current and voltage, a static volt-ampere curve such as that shown in figure 7 will be obtained. If excess or unvaporized mercury always is present so that the arc space is in a so-called saturated condition, the vapor pressure, and hence the arc current, becomes very sensitive to slight changes in any of the factors affecting the rate of heat dissipation from the arc, and transient blasts of mercury vapor caused by the mechanical passage of unvaporized mercury into the hotter parts of the arc cause the arc voltage to rise and the current to drop to a point of arc extinction. Limitation of the quantity of mercury provides a desirable thermal stability, and enclosure of the arc in a stable surrounding atmosphere by the use of an outer enveloping bulb provides an added factor of thermal stability.

The normal heat insulation and energy input to the high-intensity lamp is so chosen that, for the lowest supply voltages encountered, mercury never condenses and the arc voltage remains nearly constant for various values of arc current, even when the changes are so slow that there is thermal equilibrium at every point. Since constant-current sources are not ordinarily available for the operation of these arcs, a series ballast or control unit is used whose value, fixed by experience, is such as to absorb about 40 per cent as much voltage as the arc itself. In its simplest form this ballast is a series reactor, and in a preferred form a reactive transformer with tapped primary winding. The static electrical characteristics of a typical arc and its series reactor are shown in figure 7.

For rapid changes of arc current the arc voltage remains practically constant, and in a-c operation the difference between the arc voltage and the sinusoidal supply voltage is absorbed in the series ballast. The wave form of the arc voltage is of the square type giving the arc alone an apparent power factor of about 0.90. The sinusoidal supply voltage and the distorted arc current result in an apparent power factor of similar value in case resistive ballast is used, but of much lower value if reactive ballast is used. In the latter case a 50-degree lag of the current minimizes the importance of the distorted current wave. Figure 8, traced from oscillograms, shows fairly accurately these relationships for resistive ballast, reactive ballast, and power-factor modification by means of a capacitor. As is shown graphically in figure 8*C*, the apparent power factor of a reactively ballasted arc may be raised to a value limited by the current form factor from which value it decreases as a leading power factor with the further addition of capacitance.

Since the 10-microfarad capacitor indicated by these characteristics is rather impractical, a reactive transformer usually is used instead of a series reactor; this transformer has an extended winding on its primary to provide 570 volts for a 3-microfarad capacitor.

The light output of the high-intensity arc follows approximately the arc current, as shown in the arc-current and photoelectric-cell oscillograms of figure 9. Since persistence of vision tends to prolong the effect of the hemicyclic periods of radiation and to shorten the interval of minimum radiation, the apparent flicker of the light source varies inversely with the frequency of the electrical supply. At 60 cycles it is such as to be undetectable, even in the flicker-sensitive peripheral vision, except through a stroboscopic effect on moving objects. In this connection it is interesting to refer back to reactive and resistive ballasting, figure 8, and note that while the arc currents are in each case comparable at the beginning of the half-cycle, the current is prolonged at the end of the half-cycle by the reactive ballast, and to an extent of some practical importance. While the flicker of an a-c arc may disappear at the higher frequencies, reflection from a moving object will cause it to reappear at a frequency equal to the difference between the light source and the reflector frequencies, the effect generally being to produce a false appearance of movement of the reflector. In practice, this has proved of little importance, as most periodically rotating and reciprocating machine parts already are enclosed as a safety measure.

The high-intensity mercury lamp owes its over-all efficiency, as compared with the familiar Cooper Hewitt lamp, to the method of ballasting and to a change in both the intensity and the relative distribution of the radiation, with increased energy density in the conducting vapor.

While the quartz mercury arc introduced for industrial lighting in 1905 had an efficiency in the arc equal to that of the new high-intensity arc, it operated only on direct current and the resistive ballast absorbed 30 per cent of the total energy input. As oxide cathodes were not available the only means of operation on alternating current was as a full-wave rectifier, in which case the transformer and ballast losses were of the order of from 30 to 40 per cent of the total energy input. The introduction of oxide cathodes at once permits operation with a simple series reactor in which the losses may be from 5 to 10 per cent of the total energy consumed. Thus there is an operating gain in over-all efficiency of from 30 to 35 per cent as compared with all previous practical mercury arcs.

Küch and Retschinsky[16] showed that as the vapor pressure of a mercury arc is increased, the efficiency is increased. It now is recognized that vapor pressure is an unsatisfactory modulus of efficiency and that the energy density per unit of active mercury can be related much more closely to efficiency. How the change in efficiency may result from a change in both the amount and the relative distribution of the radiated energy will be clear from the graphic comparison in figure 10 of the Cooper Hewitt so-called low-pressure (0.002 atmosphere) lamp with the high-intensity so-called high-pressure (1 atmosphere) lamp, on the basis of equal total arc and auxiliary wattages.[20,21] It seems from this comparison that about 65 per cent of the gain in luminous efficiency is attributable to an increase in the amount of visible energy and 35 per cent to a more favorable distribution of this energy under the visibility curve.

The net result of this is an over-all luminous efficiency of a ballasted high-intensity bare lamp of

Fig. 8. Effect of reactance ballast and a multiple capacitor on power factor

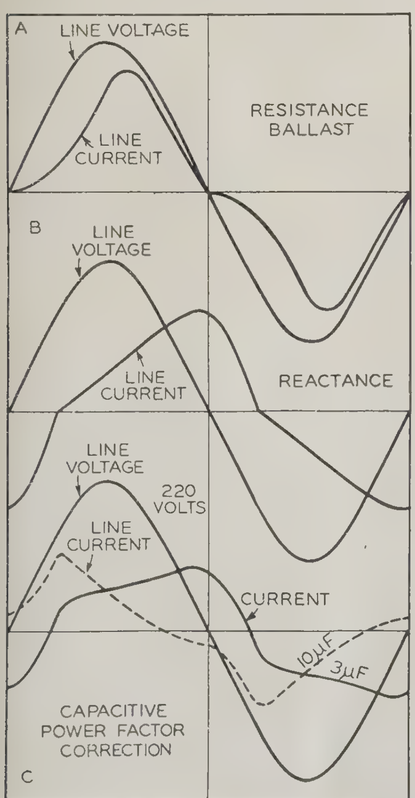


Fig. 9. Effect of the ballasting method on arc current and light

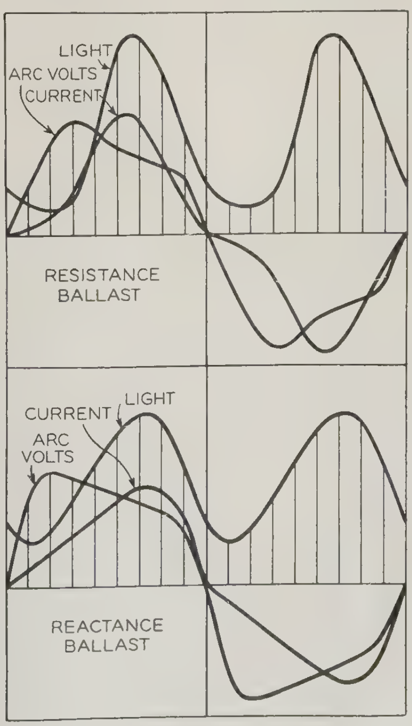
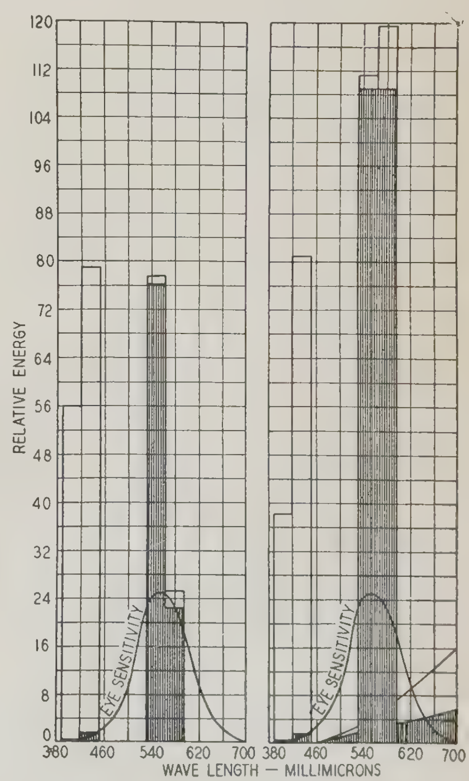


Fig. 10 (right). Energy distribution and luminosity on an equal-total-wattage basis in a Cooper Hewitt (left) and a higher-pressure (right) type of arc



about 40 lumens per watt as compared with about 25 lumens for the d-c quartz mercury arc and 15 for the Cooper Hewitt arc.

In addition to the increase in the yellow mercury lines, there appears in the high-intensity lamp red radiation amounting to a few per cent of the total but enough to make a noticeable improvement in the discrimination of colors by the light.

These changes in relative energy distribution with operating conditions are much more complicated than those of ordinary incandescent sources, and there are indications that the radiation characteristics of the mercury arc may be subject to much more design control than was once thought possible.

The variations in color, brightness, wattage, size, shape, and method of operation, leave open innumerable interesting possibilities for new lamps in the immediate future.

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Recent Progress in Dielectric Research

Research in the field of dielectrics, of fundamental importance to the electrical industry, continues, with results of far-reaching significance. An authority on the subject, Doctor Whitehead here reviews the salient points as reported in the technical literature during the past year.

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THE VOLUME of research, pure and applied, in the field of dielectrics and insulation, as reported in current journals and reviews, is increasingly great. A complete survey is quite impossible not only because of the volume of the literature, but because its highly diversified character requires correct appraisal and intimate acquaintance with both theory and experimental advance in the fields of chemistry, physics, and electrical engineering, which few individuals can claim to possess. A necessary limitation is therefore imposed upon the character of this review. It is based upon a survey during the year of some 50 or 60 papers that have seemed of special interest for those who are concerned with the theory of electrical insulation and with its improvement.

As convenient broad subdivisions of the survey may be considered: (a) dielectric strength and breakdown; (b) stability; (c) dielectric loss and phase difference; (d) dielectric constant and capacitance; (e) new applications and materials.

Dielectric Strength and Breakdown

Gases. The breakdown of gases between parallel plates may now be said to be fairly completely understood. With increasing voltage and some free electrons always present, ionization by collision increases geometrically the number of free electrons resulting eventually in an initial electronic avalanche. This leaves behind it a positive space charge which increases the gradient at the cathode, facilitating the emission from the cathode of further avalanches and the passing over into complete breakdown. Discussion for some time has centered around the part

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1. References listed at end of paper.

played by the positive ions. The collision ionizing power first attributed to these by Townsend is now questioned, and their most important function is thought to be the liberation of electrons from the cathode as they fall into it under the strong field caused by the space charge left by the avalanche. An admirable review of existing theory and discussion of this subject, together with experimental results, has been published recently by von Hippel,[1] an enthusiastic worker in this field. Additional data recently have been reported by M. Messner[2] in a study of the breakdown in wide gaps with an improved cathode ray oscillograph permitting observations at voltages up to 100 kilovolts. Static and impulse voltages are recorded on the same film. The measured values of spark-lag for voltages up to 60 per cent above static value are between 0.2 and 3.5 $\times 10^{-7}$ seconds. Upon breakdown the voltage falls to zero in time intervals from 2 to 3 $\times 10^{-8}$ seconds with one fairly marked intermediate step indicating the passage from spark to arc. Rogowski's conclusion, based upon this and other work, that the positive ions play no part in breakdown is disputed by von Hippel and Franck[3] who state that because of the positive space charges left by the electron avalanches, positive ions may reach the cathode within the observed short time intervals, and at velocities sufficiently high to liberate the further electrons necessary for the avalanches of breakdown.

Study of the sphere-gap as a high-voltage measuring instrument continues. These studies have been successful in uncovering the causes of the variation among different observers. However, as shown by Sorensen and Ramo[4] with the sparkless sphere gap, because of the variations of the dielectric strength of air it will never be possible to rely upon the sphere gap as a standard instrument. Variations of the sphere-gap caused by surroundings, dimensions, and the like have been carefully examined by W. Dattan[5] with spheres up to 75 centimeters in diameter, and with critical comparison of the results of other workers, including Meador, Bellaschi, and Weicker. Based upon results for both impulse voltages and normal frequencies, conclusion is reached that most of the differences among different observers are attributable principally to the geometry and dimensions of gaps and surroundings, and that polarity effects are probably caused by space charges as related to field intensities.

Liquids. For the most part, recent studies of liquids are directed to questions of stability and loss without special reference to breakdown. Of interest as related to breakdown is a study of the properties of semiconducting liquids for the suppression of edge-effect in breakdown tests of solids (A. M. Thomas).[6] No outstanding new contributions to the theory of breakdown of liquid dielectrics have been noted. Such literature as has appeared apparently accepts a pure electric breakdown for sufficiently pure liquids at sufficiently short impulses. For pure liquids and continuous voltages a secondary ionization, as in gases, is indicated. For liquids of less purity the picture becomes hazy as among the liberation of gas from any one of several causes, increase of ionic content through internal ionization, and the actual presence of foreign matter, including water, leading to bridge formation.

Solids. An unusually large amount of important work has appeared concerning breakdown in solids. The range covered extends from the most carefully studied theory to routine performance tests on commercial materials. In a brief report it is possible to select for comment only those of outstanding interest.

There is increasing evidence that the thermal type of breakdown is very much less frequent than commonly supposed. Breakdown at lower temperatures is undoubtedly of different character, certainly in its initial stages, although the final stages of breakdown may at times partake of thermal character. This type of breakdown is independent of temperature as shown by Inge and Walther and others. Although recent contributions from Germany (R. Becker)[7] support the Wagner thermoelectric theory, several others, notably those of S. Whitehead and W. Nethercot,[8] and of von Hippel and Franck[3] presenting very convincing experimental data with attendant analysis, reach the contrary conclusion. Based upon a review of this material the following is indicated:

Provided a sufficiently wide range of external conditions is available, a dielectric will exhibit at least 2 types of breakdown, a thermal and a nonthermal type. Thermoelectric strength depends only upon electrical processes that generate heat. For thermal breakdown the electric strength corresponding to a long time of stress may be computed from the electric losses when external and internal thermal constants are known. Joint electrical and thermal effects as suggested by Rogowski are generally absent in carefully controlled laboratory tests.

A transition temperature exists for any given time of application of electric stress and given external conditions. Below this temperature nonthermal breakdown occurs. Above it the electric strength obeys the theory of thermal instability. The transition temperature for several well dried materials and long time, according to Whitehead and Nethercot,[8] is from 80 to 100 degrees centigrade. The range of application of the thermal theory is confined largely to higher temperatures, thicker specimens, poorer dielectrics, longer times of application, and electrode surroundings having high thermal resistance. The Wagner thermoelectric theory, starting from a single path of high conductivity, is to be considered as a highly special case of thermal breakdown not subject to general thermal theory.

It remains then to explain the nonthermal breakdown. For this there is an extensive analysis by von Hippel[1] based upon the results of other workers, and upon some of his own carried out with especially careful provision for eliminating disturbances, for limiting the volume of discharge, and thus permitting intimate study of the discharge path. He worked with various crystalline solids of well recognized lattice structure, and from his results is able to draw astonishingly definite conclusions concerning the relation between the crystal structure, the direction of breakdown, the value of breakdown strength, and thus the breakdown mechanism. This extensive paper[1] should be read by every student of dielectric phenomena. A very brief picture of von

Hippel's recent experimental results and his conclusions therefrom are as follows:

Electric breakdown is to be understood as an electron collision phenomenon originating through an excess number of electrons in the lattice. The frictional losses of these electrons are due to the oscillations which they excite by electrostatic influence in passing the ions of the lattice. This friction may be expressed as a function of the electron energy. Beyond the maximum value of this function the frictional retarded motion of the electrons passes over into an accelerated movement down the potential gradient. Electric breakdown thus occurs primarily through the setting up of electron collision ionization channels. The "directional breakdown" noted in crystals is a result of the shape of the excitation function, which is dependent upon the direction of the path with reference to the lattice and also upon the high gradients that result from the accumulations of space charge.

The foregoing picture is supported by von Hippel's work on a series of crystals in which the inherent potential wave between lattice elements is known and which therefore afford a means of testing the conclusions indicated. The behavior of the directional studies and of the breakdown field strength in a series of alkali halogens especially confirms the theory. Moreover other experiments show that by synthetic changes of crystals of definite type it is possible to increase the dielectric strength in accordance with the principles indicated.

Other work on breakdown in solids worthy of note is as follows:

Baker and Boltz[9] give a convenient method of measurement of breakdown and current voltage characteristics of liquids with continuous potential. R. Becker,[7] in discussing the measurement of dielectric strength of solid insulators in the frequency range from 1 to 15 $\times 10^6$ cycles, shows that in all the materials investigated there is a marked decrease of dielectric strength with increase in frequency. A further development of the Wagner theory is added. G. Keinath,[10] with reference to the control of high-voltage tests of dielectric strength by simultaneous registration of the values of dielectric loss, describes a highly developed and accurate direct-reading instrument for high-voltage loss. This instrument has the advantage of continuous registration, permitting study of continuous time variation of power factor and capacitance not heretofore possible.

A particularly interesting group of papers concerning the breakdown of impregnated paper insulated cables has appeared. L. G. Brazier[11] attempts to predict the limiting current capacity of oil-filled cables in terms of conductor current, dielectric loss, thermal conductivity, temperature, and others. Using his own experimental results he finds only a rough agreement between observed life and that computed on the theory of thermal breakdown. This is ascribable to the uncertainty as to loss-stress relations and other internal conditions. He concludes that in oil-filled cables with the thicker walls required above 66 kilovolts, thermal instability may be expected if present specific dielectric losses and current densities are exceeded. D. M. Robinson[12] attributes the beginning of breakdown in cables t a coring action in an oil space next to the conducto together with an oil channel in the second pape layer, thereafter spreading into the generation c "tree" patterns. Gaseous ionization in larger void is seen as only a subordinate factor aiding in the ex tension of tree patterns and carbonization of o spaces.

The 2 foregoing papers, together with that of Dun sheath[13] on internal ionization, in a preceding yea constitute an admirable review of present knowledg of the working properties of impregnated paper insu lation. M. R. Laroche[14] presents experience from French company leading to his conclusion that the mal breakdown in cables never occurs until ther has been a substantial carbonization of a large pro portion of the insulating wall by the Brazier process The experiments of F. Quittner[15] seem to confir Robinson's picture of breakdown, his findings bein that in impregnated paper up to a certain density breakdown occurs in the oil-filled pores, the break down voltage increasing with decrease of pore diame ter. Beyond a certain density a stage is reache where the breakdown strength of the weakest chan nel exceeds that of the paper fibers, which thereupo break down instead. E. Kirch[16] reports data on th properties of impregnated paper insulation for cable under high continuous voltage, with special referenc to relative values of breakdown strength of continu ous, alternating, surge, and pulsating voltages These data and the current literature, particularl the results of J. Delon, permit the conclusion that i continuous-current cables, a maximum stress of from 20 to 25 kilovolts per millimeter, or 5 or 6 times th value usually accepted for a-c cables, is possible.

Stability

A great deal of work is at present under way on th influence of oxidation or other chemical change i oils on their electrical properties, and on their elec trical stability. Of some of this work there wer preliminary reports last year. More complete re ports of several of these are now available in pub lished papers. Among these may be mentioned tha of J. G. Ford[17] showing that although an overrefine oil is oversensitive to oxidation, certain such oil give oxidation products that form an inert gas ove the oil and so may be used with advantage in trans formers. F. M. Clark[18] gives a clear exposition o the development of chlorinated diphenyls for th synthesis of a stable dielectric liquid (called Pyranol of noninflammable and other properties highly de sirable for transformers and capacitors. Pipe Thomas, and Smith[19] report results indicating th relative unimportance of oxidation products, whe added to paraffin oils, in increasing power facto However, it should be noted that the oxidation prod ucts were added to the oil and not brought about a the result of oxidation of the oil itself. More com plete record of the work of R. W. Dornte[20] on th laws of absorption of oxygen by oils is also note H. F. Ornstein[21] reports evidence that changes in di pole moment may be used as an indication of the ex tent and progress of oxidation in transformer oils.

Dielectric Loss and Phase Difference

Studies of dielectric loss have been largely in the upper range of frequency, and the development of methods therefor. L. Rohde[22] reports studies by a substitution method in the range from 1 to 5 $\times$ 10^8 cycles on several crystalline, vitreous, and fibrous materials. He finds no difference in the value of phase difference within this range, the values varying from 0.3 $\times$ 10^{-4} to 4 $\times$ 10^{-4} for vitreous and crystalline substances, 2 $\times$ 10^{-2} for hard rubber, and 9 $\times$ 10^{-2} for hard paper. J. Kritsch[23] reports tests up to 10,000 volts in the frequency range from 1 to 6 kilocycles on complex commercial insulation with special reference, however, to a new high-voltage high-frequency method of which the principal feature is a capacitive voltage divider. H. Schwartz[24] shows the importance of the relative humidity of the atmosphere on dielectric loss measurements in the upper range of frequency. At 2 $\times$ 10^5 cycles a 7-fold increase is found between vacuum and 90 per cent relative humidity. It is shown that the result is not a surface effect, but is attributable to volume absorption of moisture. W. Jackson[25] reports a series of experimental tests on the liquid known as "perimitol" the English trade name of a non-inflammable chlorinated diphenyl compound. Measurements of phase difference at 50 cycles and from 50 to 107 kilocycles with temperatures ranging from -20 to $+80$ degrees, show families of curves with the familiar maxima of dipolar orientation, and the author claims general agreement with the Debye theory. J. Lahousse[26] proposes a new bridge method for loss measurements in which the heating effects of the capacitor current and of a current proportional to the voltage are utilized for balancing the bridge. Arman and Starr[27] report the measurement of the high-frequency components of the charging current of dielectrics caused by the presence of internal gaseous discharge as in impregnated paper. The method involves principally the use of a high-pass filter passing frequencies of 2,000 cycles or higher. It is claimed that the resulting data permit separation of the total increase in power factor with voltage into the components attributable to gaseous ionization and to other causes.

Dielectric Constant and Capacitance

Studies of dielectric constant have been numerous. Variations of dielectric constant of dilute liquid solutions and latterly of crystalline solids over wide temperature ranges continue to be used by physicists and chemists for the information afforded as to the basic polar molecules and to solid crystal structure. Particularly striking are the abrupt changes in the values of dielectric constant of solids when carried to very low temperatures, indicating abrupt changes in molecular structure rather than the more gradual elastic rotation of dipoles. Substances showing this phenomenon may pass from the liquid to the solid condition without abrupt change of dielectric constant, thus indicating that something other than simple molecular orientation is involved. The literature referred to must be consulted for further information concerning recent progress in these highly interesting and important fields. It is contributing largely to the advance of knowledge of the molecular structure of solids. The knowledge so gained is already pointing to the directions in which insulating materials for special purposes may be selected and controlled.

Temperature and frequency changes in dielectric constant also have been shown to occur as a result of the separation of space charges both in liquids and in solids and also of the polar orientation of aggregates much larger than molecules. As bearing on the former, W. O. Schumann[28] has supplemented his theoretical analysis with experiments on both solids and liquids, calling attention to the fact that absorption phenomena attributable to a Maxwell layer dielectric may be considered as an extreme case of space-charge separation. P. Böning[29] presents a study of the space charges separated when liquid dielectrics are solidified under electric stress, and shows that in many instances the "electret" behavior is not caused by molecular polarization, but by the separation of space charges. The evidence of polar aggregates larger than molecules is chiefly found in the large values of relaxation time. Argue and Maass[30] have studied the variation of dielectric constant of dry cellulose to which water is adsorbed. The methods permit measurement of the dielectric constant of water in the initial stages of adsorption beginning with the value of 20, and indicating that the first layer of adsorbed molecules is less free to turn than subsequent layers; the dielectric constant slowly increases with increasing amounts of water. Figures for the weight of adsorbed water and values of dielectric constant are in fair agreement with those reported by J. B. Whitehead and E. W. Greenfield.[31]

A new method of mixtures for measurement of dielectric constant is reported by E. Kleinke,[32] especially adapted to complex solid substances in which the dielectric constant is quite high and beyond the values of the liquids commonly used. This and other special types of measurement are described in the study of special materials. Examples of high dielectric constant of solids are reported by Handrek[33] and by Kritzsch,[23] the latter giving the values between 34 and 87 for the material called Condensa, no other description being given. On the other hand, a material called Styroflex developed for the insulation of high-frequency low-loss communication cables has a dielectric constant in the neighborhood of 1.2. A new type of radio-frequency bridge, open at one corner for external voltage, is reported by Gross and Hausser.[34]

New Applications and Materials

New insulating materials are being developed in large number. They cover a wide range of physical properties extending from the rigid ceramic type, through the complex solids susceptible to molding and working into special forms, and into the liquids. Some of these have been mentioned in the foregoing. Materials of high dielectric constant are titanium and tin oxide (Schusterius),[35] rutil and seignettesalz

(Kleinke)[32] and the liquids, Pyranol (Clark)[18] and Permitol (Jackson).[25] The interesting substance Styroflex (Kieser)[36] developed for the insulating of high-frequency communication cables is derived from a brittle material known as Polystyrol, has in its final form the properties of a very flexible glass, and is applied to the insulation of cables in the form of spirals filling the radial distance between conductor and sheath. The low values of dielectric loss and dielectric constant result in very low values of surge impedance and damping constants, with consequent advantages for high-frequency transmission. A. Meissner[37] reports mixtures of dielectric solids with finely divided interspersed crystals as a method for improving thermal conductivity of dielectrics without serious lowering of dielectric constant.

An important series of papers by A. A. New[38] has appeared concerning the physical and chemical structures of cellulose in its various forms and as treated physically and chemically for modification of properties desirable for electrical insulation. The development of the material called Cotopa is reported with special value for the insulation of telephone conductors. Reference also is made to paper fiber for power conductors in cables.

High-Voltage Insulation

The impregnated paper power cable continues to receive intensive study. The chief problems are the reduction in wall thickness through increased dielectric strength, and permanence or stability as inherent in the properties of the basic materials and in the suppression of gaseous ionization. The pressure principle at present is receiving extensive trial in the service performance of noteworthy installations in this country and abroad. The advantages of pressure on the dielectric are increased dielectric strength and the suppression of internal gas voids. Open questions are the most reliable methods of applying the pressure, whether by an outside gas, or an outside liquid medium, or by hydrostatic pressure inside the cable, and the proper ranges of pressures for best results. Underlying all these is the oil-filled principle for the higher ranges of voltage, which seems to be firmly established. Unfortunately divergent competitive interests so far have prevented careful comparative study of the relative merits of the several plans for applying the pressure principle. Technical literature is scarce and the test of time on existing installations and experience therewith must, as usual, be relied upon. There is nevertheless a substantial volume of literature concerning allied subjects that is of both interest and importance.

A 220-kilovolt oil-filled cable installation 18 kilometers long near Paris, is described by M. Laborde.[39] A new type of high-voltage oil-filled cable impregnated after installation, and using a segmented or type *HH* conductor for improving the oil channel is described by J. Borel.[40] Limitations and advantages of drying and impregnating after sheathing are examined, and installations of 3-conductor and single-conductor cables up to 132 kilovolts are described. For the last named, a maximum gradient of 10.5 kilovolts per millimeter is reported with 1-hour test at 43.6 kilovolts per millimeter. C. F. Proos[41] reports interesting accelerated life tests on both solid and oil-fille insulation under voltage and temperature cycle separately and jointly. A striking result is the experimental evidence of the necessary combination o high voltage with temperature load cycles in bringing about the elevation of power factor usually attributed to gaseous ionization.

Dunton and Muir[42] and W. H. Anderson[43] discus insulating papers in general, with special referenc to impregnation. G. B. Shanklin,[44] in a brief descriptive paper, gives results of recent studies of th power factor improvement of impregnated paper resulting from treatment of paper during drying an impregnating process with carbon dioxide. Thoug lead is not an insulating material, the lead sheath is vital element in preserving the inherent propertie of cable insulation. Improvements of lead sheathin continue. Especially noteworthy are the vacuu press (Atkinson and McKnight)[45] for limiting oxidation and gas inclusions during leading, the hydroge press (Shanklin) for similar purposes and other measures for greater uniformity of the resulting metal Much interest also is being given to the continuou extrusion machines under development in Englan and in Italy. Metallurgical and metallographica studies of lead and lead alloys continue, but there i little available literature.

Several interesting auxiliaries are reported. A new hydraulic cutter for the direct reeling of a submarine cable into a narrow trench on a river bottom i described by K. Hesse,[46] and statement is made tha it can lay a cable 20 centimeters in diameter, 6 meter below the bottom of a river 18 meters deep. Improvement in methods for joints and terminals fo aluminum cables are described by G. Kramer[47] an F. Brinkmann.[48] H. Schiller[49] describes a metho for reducing the high stress at the end of the lea sheath of a high-voltage cable and of equalizing th stress distribution over the usual taped insulation The principle is the introduction of interspersе layers of paper impregnated with glycerine, dielectri constant 56.

Some of the larger utilities companies have unde way research programs specifically directed to th improvement of the performance and life of high voltage cable. These programs usually are divide into studies of complete cables, properties of the insulation, properties of the sheath, and operatin characteristics. Unquestionably the year-to-yea continuance of these programs contributes largely t the improvement of high-voltage cable and to th operating characteristics of high-voltage insulation Published reports of work of this character have bee few within the past year, but this report should include a reference to this important work.

Definite progress in improving the electrical an mechanical properties of porcelain insulators is indicated in a paper by D. H. Rowland[50] reporting careful studies of the stresses in the surface layers an glaze.

Pauthenier and Moreau-Hanot[51] report the performance of an ionic generator for high continuou potentials in which the charged particles of dust o powdered dielectric are carried by a draft of ai

through an insulated metallic sphere there discharged and returned in a circulating air system. Preliminary experiments on a laboratory setup reached 450 kilovolts. The possibilities of the principle are examined analytically. Similar experiments with dust as charge carrier are reported by P. Böning.[52]

It is abundantly evident from the foregoing that the problems of insulation in all its phases are under continued and aggressive attack. Definite progress within the year in all directions is indicated, and with much promise of further advance. There is some suggestion that in recent years greater attention has been devoted to the advance of theory. This is perhaps due to our recent industrial depression. There is already good evidence, however, of renewed attention to the larger problems of practice and that electrical engineers may continue to look to research as a powerful aid to further progress.

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High-Intensity Mercury-Arc Lamps

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The general relationship between wattage and light output of high-intensity mercury-arc lamps, some of the effects of voltage and pressure on their efficiency, and a tabulation of the actual amount of energy they produce in various portions of the spectrum are discussed in this paper. The practical aspects of these lamps is discussed particularly in the fields of industrial lighting and where very high intensities of light are needed, as in searchlights, locomotive headlights, and, under certain conditions, for projection work.

IN EARLY experiments using mercury pools for electrodes,[1] the operation of mercury arcs proved quite difficult because the arc would wander around on the pool surface. When the wattage was increased the voltage of the lamp would rise until it approximated the supply voltage and extinguished the arc. Cooper Hewitt's work,[2] which resulted in preventing the mercury pressure from becoming too high, represented a distinct advance in overcoming this difficulty. Küch and Retschinsky,[3] 30 years ago, showed that mercury-pool quartz lamps increased in efficiency as the energy in the discharge was increased, indicating that high efficiencies were possible. Manufacturers of mercury-vapor lamps for ultra-violet radiation largely overcame the instability of mercury-pool lamps by the use of transformers having high magnetic leakage. Starting difficulties of the lamps were overcome by tilting devices or relays. Oxide-coated emitting cathodes were used later.

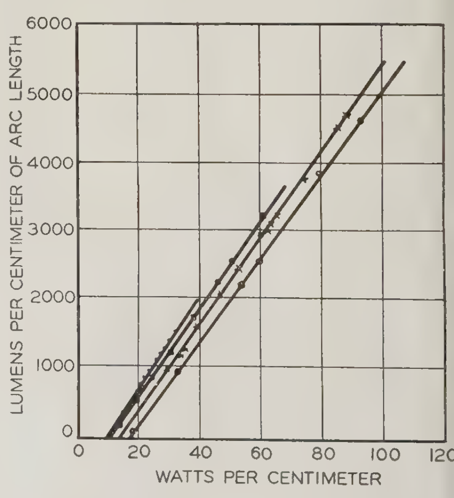
The difficulties to be overcome in order to make commercial lamps for illumination purposes consist of (1) the control of the mercury pressure, (2) the construction of lamps that start and operate on a simple transformer or reactor, (3) the finding of suitable glass or the working of quartz to withstand the high temperatures of highly efficient lamps, and (4) the making of color corrections where these are needed for commercial reasons.

Ryde[4] has described a lamp similar to the present 400-watt high-intensity mercury lamp now produced in the United States as well as abroad. The commercial 400-watt lamp is the result of developments made by various inventors wherein the electrodes are oxide-coated to permit low-voltage a-c operation. A gas like argon is added to facilitate starting and to carry the discharge until the mercury pressure accumulates. Only a limited quantity of mercury is used. It is completely vaporized during operation, causing the lamp to burn at a relatively constant vapor pressure even under widely different external temperatures.

A capillary quartz lamp producing 50 to 60 lumens per watt has recently been developed by C. Bol[5] in the Philips laboratories in Holland and is being distributed commercially by that organization. Its adaptability for use in projectors and searchlights is at once apparent, since this lamp has been operated with an intrinsic brilliancy about $^1/_4$ that of the sun.

Largely because a transformer or choke-coil equipment is necessary to their operation on various line voltages, there has been only a gradual development of high-intensity mercury arcs for general illumination purposes. Another obstacle has been the ease of use and simplicity of tungsten-filament lamps. Because of the added expense and the space required for the transformer equipment, the illuminating engineer has been willing to use gas-discharge lamps only within the last few years. As time will show, however, some effects can be produced with such devices that cannot be duplicated with tungsten-filament lamps.

Fig. 1. Relationship between wattage and light output for high-intensity mercury-arc lamps having arc lengths of 100, 40, 15, and 7 millimeters (left to right)



In spite of handicaps, research laboratories have not been idle. As a result, the authors are able to

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1. For all numbered references see list at end of paper.

set forth briefly certain general laws covering the variations between the lumen output of constricted arcs, wattages, arc lengths, and pressures.

Producing High Efficiency in Mercury Arcs

The relationship between wattage and light output was developed from a large number of tests with various lamps (figure 1). From left to right, the curves represent lamps having arc lengths of 100, 40, 15, and 7 millimeters.

Energy losses in lamps of various arc lengths are shown by the ordinates of the curve in figure 2. They represent the watts lost at the electrodes from which no light is obtained. These ordinates are the intercepts on the "watts per centimeter" axis of figure 1 and vary from 10 to 17 watts for arc lengths of 100 to 7 millimeters, respectively. Hence, a long lamp should be more efficient than a short one because the proportion of wasted or lost energy is smaller for the long lamp.

The straight lines of figure 1 can be expressed by simple mathematical formulas of the type

$$L = K(W - a)$$

where

L = lumens per centimeter
W = watts per centimeter
a = intercept with "watts per centimeter" axis
K = a constant

The efficiency reaches a maximum value when the intercept becomes a negligible proportion of the wattage, but the maximum attainable varies to some extent with the arc length. If one neglects the electrode losses (that is, the value a in the formula), the maximum theoretical efficiencies vary from 62 to 68 lumens per watt for the lamps represented in figure 1. It may be shown[6] that the efficiency also can be calculated from the logarithm of the watts per centimeter of arc length. The above method is simple, however, and if the calculations are held within useful ranges, they seem to be fairly satisfactory for estimating lumen output.

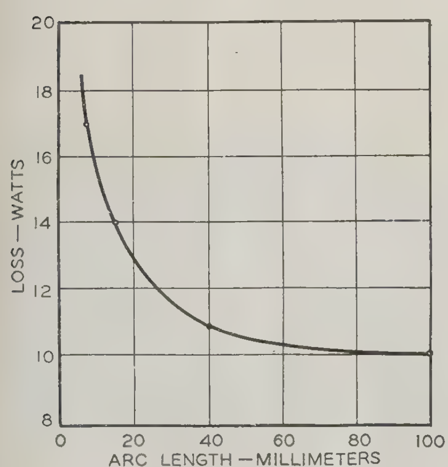


Fig. 2. Relationship between arc length and energy loss in a high-intensity mercury-arc lamp

Ordinates are the intercepts of the curves in figure 1 with the "watts per centimeter" axis

There are practical limitations to the amount of light generated because, with a short arc length, the diameter of the tube can be only about half the arc length at fairly high voltages without the arc's bowing caused by gas currents in the tube. This definitely limits the size of the container, which in turn limits the wattage that can be used without decomposition of the glass or quartz envelope. Of course, cooling of the envelope allows a higher wattage to be used.

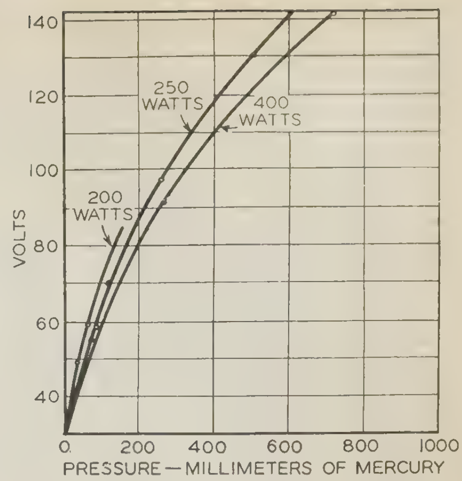


Fig. 3. Relationship between pressure and voltage for a high-intensity mercury-arc lamp

The light energy measured here is the total light in the visible part of the spectrum approximately in proportion as it affects the eye. The intensity of the light was measured with a photoelectric cell having a wave-length sensitivity corresponding very closely to that of the eye. The visible radiation from the mercury arc consists of strong lines of green and yellow with a relatively small proportion of blue and red. As the wattage is increased, each of these main lines increases; therefore, if a fundamental relationship between efficiency and energy consumed by the arc is to be determined, these changes in spectral distribution should be taken into consideration. It is necessary to consider also the amount of radiant energy in the ultraviolet and infrared regions and to measure or estimate accurately the losses at the electrodes.

The diameter of the tube also appears to affect the efficiency of the lamp. In general, tubes of large diameter have somewhat higher efficiencies than those of smaller diameter.

Voltage-Pressure Relationship of Mercury Arcs

At the present time, the belief is quite general that high-voltage or high-pressure lamps are high-efficiency lamps. As shown by the wattage-lumen output formula given in the previous section of this paper, high-pressure lamps are highly efficient because high wattage is utilized per unit length of arc stream. Data show that, after a pressure of a few hundred millimeters (of mercury) is obtained, the efficiency of mercury-arc lamps changes but little with either voltage or pressure so long as the wattage is held constant.

The change of voltage with pressure for several wattages is shown by the curves in figure 3. A

thermocouple was attached to a small side tube at the lower end of this particular lamp. By operating the lamp in an electric oven with the thermocouple at the coldest part of the lamp, the vapor pressure of the mercury could be determined accurately. The arc stream was 15 centimeters long, and the internal

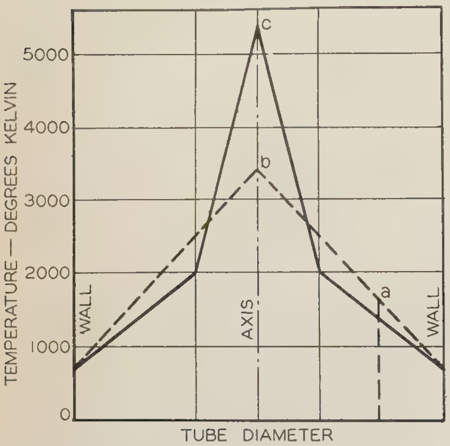


Fig. 4. Temperature distribution across the axis of a high-intensity mercury-arc lamp

tube diameter was 3.2 centimeters. It is evident that the voltage of the lamp depends upon wattage and pressure, because higher wattages at a given pressure result in higher-current and lower-voltage conditions. Constricted arcs are affected very little by glass walls that are some little distance from the arc stream. The decrease of voltage with increasing wattage may be in part the result of increased heating and electron emission (thermal electrons) from the cathodes and of increased temperature or decreased vapor density in the arc stream. The voltage-pressure curve for quartz lamps at very high pressures has been measured by De Groot[7] who gives the formula

$$P \text{ (atmospheres)} = \frac{V \text{ (volts per centimeter)} - 100}{3}$$

for the conditions of his experiments.

Temperature of Arc Stream

The average temperature of the arc stream is of interest to the engineer, as it can be used in calculating the quantity of mercury required for a lamp of known dimensions. Data[6] accumulated from the work of Güntherschultze and DeBruyne[8] indicate that approximate temperatures at the axis of the arc stream can be calculated by the empirical formula

$$\text{Temperature (Kelvin)} = \sqrt{\text{watts per centimeter}} \times 1{,}000$$

Using this formula, temperatures of about 2,000 degrees Kelvin at the edge of the arc stream and about 5,150 degrees at the axis of the arc stream are calculated for the 400-watt lamp. From theoretical considerations, Elenbaas[9] gives a temperature of the same order of magnitude for this type of arc lamp.

The temperature distribution in a lamp can be determined from a knowledge of the volume and diameter of the tube, the pressure of operation, the weight of mercury used, and the temperature of the glass walls. In a present commercial type of arc lamp, the volume is 130 cubic centimeters, the tube diameter 32 millimeters, the pressure about one atmosphere, the weight of mercury about 210 milligrams, and the temperature of the glass walls about 400 degrees centigrade. This temperature, however, varies at different parts of the bulb.

In a curve showing the temperature distribution across the axis of the lamp (figure 4), a dashed line passing through the average-volume point *a* indicates a temperature of about 1,500 degrees Kelvin if the center of the tube is the hottest and the temperature gradient from the axis to the walls is uniform. However, it is believed that the temperature at the axis of the arc stream, indicated at *c*, is 5,000 to 6,000 degrees Kelvin. Since the arc stream is found by measurement to be about 10 millimeters wide, the volume of the arc stream is only about 10 per cent of the total volume and the temperature at *a* can be lowered only a relatively small amount. According to this data, the temperature at the edge of the arc stream is about 2,000 degrees Kelvin or a little higher.

When the arc stream is projected through a lens system and the intensity of light is measured through a narrow slit across the arc stream, the intensity appears much higher at the center; but when the arc stream is divided into concentric cylindrical zones and the intensity is calculated per cubic millimeter, the intensity at the axis appears to be about 5 times the average intensity near the edge of the arc stream.

Air-Cooled and Water-Cooled Mercury Lamps

A quartz lamp designed to operate in a housing at 100 watts can be operated at 300 watts or more with air blasts, for at least a short time. Lamps have been burned with water cooling up to about 1,500 watts per centimeter. The total amount of light

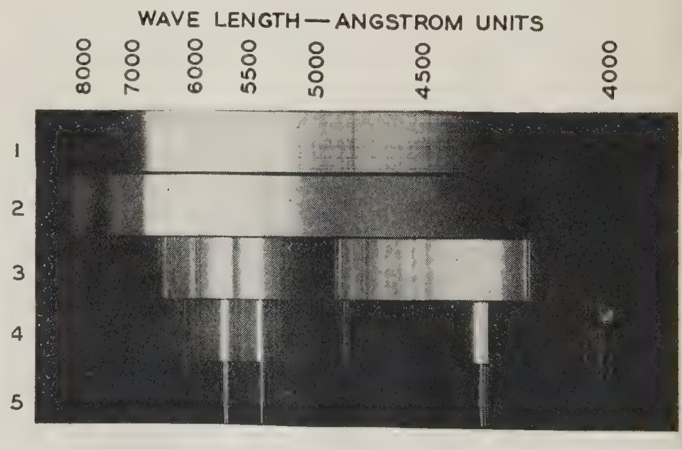


Fig. 5. Spectrograms of light from high-intensity mercury-arc lamps and from other sources

1—Solar spectrum
2—Tungsten-filament lamp
3—High-intensity mercury-arc quartz lamp
4—400-watt mercury-arc glass lamp
5—Mercury Uviarc (quartz) lamp

increases with increased wattage, but the efficiency approaches a limiting value (figure 1). It has often been observed that the efficiency actually decreases with extreme wattages, probably as a result of the arc stream's spreading and of the loss of energy as the distance between the arc stream and the walls of the lamp is decreased.

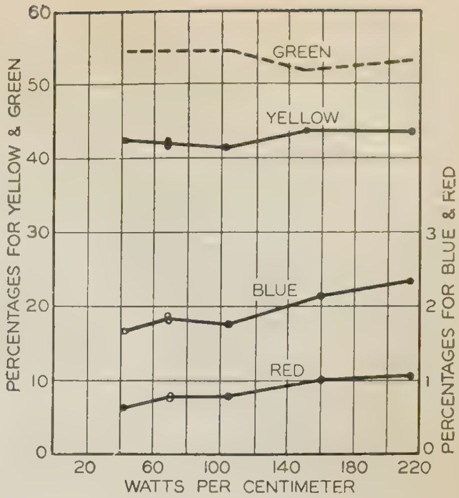
Spectra of High-Intensity Mercury Arcs

Several spectrograms were taken on a red-sensitive photographic plate (figure 5). Although these plates do not accurately show the relative intensities, they give a general idea of what might be expected if these high-intensity quartz lamps can be made to burn a reasonable length of time.

The quartz lamp was operated at about 200 watts per centimeter of arc length and 300 volts per centimeter. The 400-watt mercury glass lamp (a commercial high-intensity lamp) was operated at 27 watts per centimeter and 10 volts per centimeter. The spectra from these lamps are compared with those from the sun, a tungsten filament lamp, and a commercial Uviarc (quartz) lamp. The "continuous" spectrum of the high-intensity mercury lamp is especially pronounced. The absorption bands above 7,000 angstrom units are due to the photographic emulsion. (One angstrom unit equals 10^{-8} centimeters.)

Many lines other than mercury lines are recorded on spectrogram 3. Most of these lines are very faint, but a few are relatively strong. Practically all the extraneous lines are due to barium, calcium, and strontium. While this spectrogram was made at the extreme operating temperature of the lamp, some of the electrode coating material was volatilized and ionized; yet the actual light contributed by these "impurity" lines is insignificant.

Fig. 6. Relationship between wattage and the proportions of the various colors in the light from the high - intensity mercury arc



Color Value of High-Intensity Mercury Arcs

The relative energies in the various wave-lengths of the mercury-arc spectrum were tabulated with values for 5,461 angstrom units set equal to 100 so that comparisons may be made with data obtained by other observers (table I). Lamp *A* was of the high-pressure air-cooled quartz type consuming 100 watts in an arc length of 15 millimeters, and having an internal diameter of approximately 6 millimeters. Measurements were made with a large quartz monochromator, thermopile, and sensitive galvanometer.

The lamp described by Hoffman and Daniels[10] was a water-cooled quartz capillary arc operating at high pressure and consuming about 700 watts. Data given by Pirani[11] refer to a large-sized quartz lamp operating at a pressure of only one atmosphere. The differences in lamp structure, their operating conditions such as pressure, voltage, current density, and type of cooling, account for variations in the relative intensities as recorded in this table. Since all these lamps are made of quartz, large quantities of ultraviolet radiation are emitted unless the lamps are surrounded by glass bulbs.

An estimate of the color composition of the light from such lamps can be obtained by multiplying the energies in the various spectral lines by the eye sensitivities for those wave-lengths. This has been done in table II for lamp *A* of table I.

The output from a small high-intensity quartz lamp was measured with colored filters, the transmissions of which had been determined. The percentages of red and blue were found to increase slightly with wattage (figure 6). The curves, however, do not show large or rapid changes in the yellow-to-green ratio. The percentage curve of red, as measured by means of a red filter, plotted against watts per centimeter of arc length was extended to water-cooled lamps operating at extremely high wattages per unit of arc length (figure 7).

Table I—Relative Energies in the Various Wave Lengths of the Mercury-Arc Spectrum

Wave length in Angstrom Units	Lamp A*	Hoffman and Daniels[10] Lamp	Pirani[11] Lamp
6,234	5.3		
5,770 and 90	88	87.5	123
5,461	100	100	100
4,916	6.2	8.3	2
4,358	74	75	77
4,047	38	43	41.5
3,650	105	66	155
3,341	9	11	13.5
3,132	47	29	99
3,023	22	17	51
2,967	11	15	25
2,925	4.7		3.9
2,894	4.7	5.5	9.7
2,804	13	5.3	20

* See text for description.

Table II—Color Composition of Light From Lamp A of Table I

Wave-length in Angstrom Units	Relative Energy	Eye Sensitivity	Energy × Sensitivity	Per Cent of Total Energy	Color
6,234	5.3	0.33	1.8	1	Red
5,770 and 90	88	0.88	77.4	42.8	Yellow
5,461	100	0.98	98	54.4	Green
4,916	6.2	0.22	1.4		
4,358	74	0.02	1.5	1.8	Blue
4,047	38	0.005	0.2		

The commercial 400-watt and 250-watt high-intensity mercury-arc lamps are shown as A and C in figure 8. Lamp B is an experimental high-intensity quartz lamp similar in design to the lamp proposed by the Philips Lamp Company.[12]

Future of High-Intensity Lamps

Although tungsten-filament lamps are approaching a point where a radical increase in efficiency is difficult to achieve, the filament type of lamp has the advantages of cheapness and simplicity for the consumer because it requires no special equipment on present circuits. In many places, however, a mercury lamp, with an efficiency of 35 or 40 lumens per watt, would be more economical, as cost figures on power, auxiliary equipment, and replacements readily will disclose. Increased production will lower the actual cost of such lamps so that the time may come when they will be used extensively for the illumination of highways, city streets, and especially industrial buildings.

Because of their concentrated arc stream, however, high-intensity lamps (especially quartz lamps) are particularly desirable in fields such as floodlighting where the reflectors and housings may be made smaller and the lamp used almost as a point source of light. Another suggested use is in projection work where small high-intensity sources are needed. Compared with tungsten-filament lamps for this service, short-arc-length lamps are particularly suitable because only about $^1/_3$ as much energy is needed for the production of a given amount of light and, as a result, there is less heat to dissipate. Many other applications have been proposed, such as in searchlights and locomotive headlights.

As has been indicated hereinbefore, lamps can be cooled by air blasts or by water, in which event the

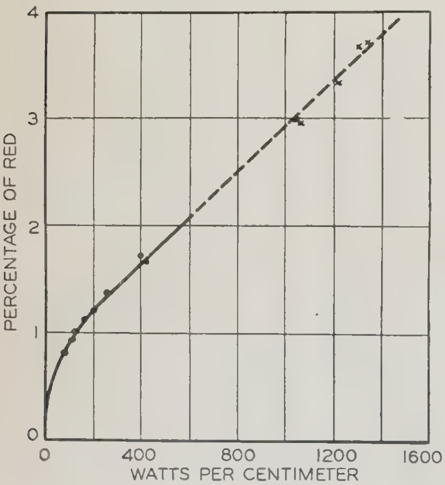


Fig. 7. Relationship between wattage and proportion of red in the light from a high-intensity mercury arc

Solid portion of curve is for an air-cooled lamp; dashed portion for a water-cooled lamp

wattage per centimeter and intensity can be greatly increased. Lamps have been burned for a short time in the laboratory at as much as $1^1/_2$ kw per centimeter, or about 4 kw per inch of arc length. Materials of construction will limit this kind of burning. Although quartz is more stable than glass, it also has temperature limitations, and a substance capable of withstanding a higher temperature than even quartz would be desirable. These lamps are in compact form and can produce intensities not to be equalled by any other inclosed lamps.

The spectral energy distribution of the quartz high-intensity lamp does not match daylight or skylight exactly, for it is deficient in the blue and red ends of the spectrum. Since cadmium gives both

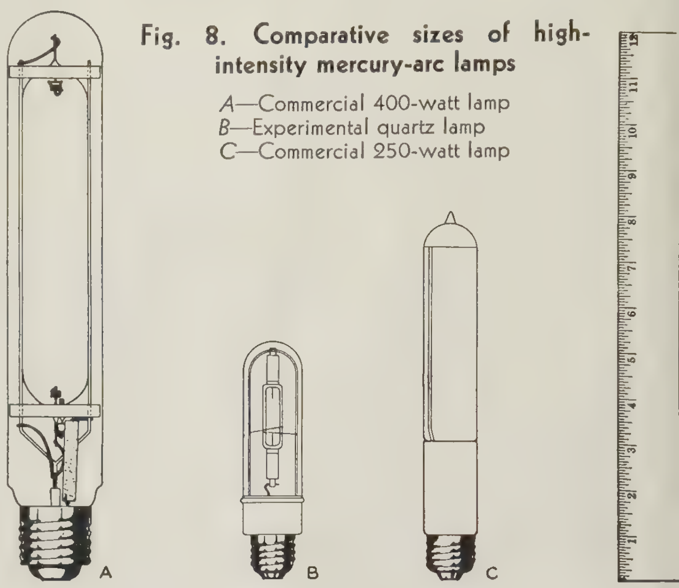


Fig. 8. Comparative sizes of high-intensity mercury-arc lamps

A—Commercial 400-watt lamp
B—Experimental quartz lamp
C—Commercial 250-watt lamp

blue and red lines it has been added to mercury arcs, but some loss of efficiency results. If, however, mercury lamps can be used with cadmium or other similar materials, or with fluorescent materials, to give good color effects with high efficiency, it is likely that their use will be quite widespread in de luxe lighting equipment for the illumination of stores, show rooms, and, under certain conditions, even of the home.

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Operational Solution of A-C Machines

In this paper the complete solution of a polyphase machine is developed using the 2-reaction theory, and applications are made quantitatively to a particular machine, showing the application of the solution under several conditions.

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THE SOLUTION for currents and voltages in the alternator has gradually become more and more fully developed during the past 10 years. While it is true that Steinmetz gave an approximate solution some years ago, the solution used at present was developed by Doherty and others by the introduction of the "constant flux linkage theorem." This scheme has been developed and extended to such a satisfactory state that it is now largely used in engineering practice. These methods are approximate, in that resistance is not directly included in the solution.

During this interval of time, Park introduced an operational solution which was rigorous under its assumptions. Fortunately, these, assumptions agree very well with practical conditions, for the most part. By this method resistance was not neglected; at the same time the alternator was reduced to an equivalent circuit, resulting in simplicity in its application to electrical circuit theory and in its solution. His solution dealt only with symmetrical conditions, but can be extended to unbalanced alternator solutions. Perhaps because few illustrative examples of this type of solution have been given heretofore, its application has not been as extensive as it should be, considering its merits. In this paper it is the intention to extend this solution by way of illustrative examples, using the constants of a machine and circuits arranged for this particular purpose, showing how those constants are measured, converted, and applied. The calculated results are compared with measured values and oscillograms, to aid further in the introduction of this rigorous solution. In this paper only symmetrical alternator conditions are considered; it is the intention to introduce the unsymmetrical solutions at a later date.

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The general theory is developed in the following; it is essentially a collection and a rearrangement of the complete analysis of this problem, found scattered through engineering literature. In the development as given here only the 3 armature phases and a field winding are assumed in order to simplify the algebraic manipulations involved and thus aid in developing the physical background.

By this scheme, the 3 phase circuits as a unit are resolved into 2 armature circuits; a circuit in which the direct axis current flows and another in which the quadrature axis current flows, and which are mutually related to each other through the rotational characteristics of the machine. There is also a field circuit inductively coupled to the direct axis. These units of armature current in the 2 axes finally must be decomposed into the actual armature currents. (Corresponding units of flux linkages and voltages exist as well as currents in these 2 axes.) Mathematically, the purpose of the scheme is to avoid the introduction of variable parameters in the equations for the various phases and the field of the machine, as these would make their solution impossible. The method also reduces the algebraic manipulations to a minimum.

List of Symbols

The following is a list of the symbols used in this paper:

e_0 = zero-sequence voltage
e_a = phase a voltage
e_d = direct-axis armature voltage
e_q = quadrature-axis armature voltage
E_{fd} = field circuit voltage
i_0 = zero-sequence current
${}_1i_2$ = negative-sequence fundamental current in the armature
${}_3i_1$ = positive-sequence third-harmonic current in the armature
i_a = phase a current
i_d = direct-axis armature current
i_q = quadrature-axis armature current
I_d = current flowing in the direct-axis rotor circuits
I_q = current flowing in the quadrature-axis rotor circuits
I_{fd} = main field current
I_{1d} = direct-axis additional rotor-circuit current
I_{1q} = quadrature-axis additional rotor-circuit current
M_d = armature magnetomotive force along the direct axis
M_q = armature magnetomotive force along the quadrature axis
p = d/dt
p_1 = direct-axis permeability
p_2 = quadrature-axis permeability
R = armature circuit resistance per phase
R_{fd} = field circuit resistance
R_{1d} = direct-axis additional rotor-circuit resistance
R_{1q} = quadrature-axis additional rotor-circuit resistance
t = per unit time
x_0 = zero-sequence reactance
x_2 = negative-sequence reactance
x_d = synchronous reactance, direct axis
x_q = synchronous reactance, quadrature axis
x_l = leakage reactance per phase
x_m = mutual reactance between phases
x_d' = transient reactance, direct axis
x_q' = transient reactance, quadrature axis
X_{afd} = mutual reactance between direct-axis armature circuit and main field circuit
X_{fad} = mutual reactance between main field circuit and direct-axis armature circuit
X_{ffd} = reactance of main field circuit
X_{11d} = direct-axis additional rotor-circuit reactance
X_{11q} = quadrature-axis additional rotor-circuit reactance

X_{a1d} = mutual reactance between direct-axis armature circuit and direct-axis additional rotor circuit
X_{a1q} = mutual reactance between quadrature-axis armature circuit and quadrature-axis additional rotor circuit
X_{1ad} = mutual reactance between direct-axis additional rotor circuit and direct-axis armature circuit
X_{1aq} = mutual reactance between quadrature-axis additional rotor circuit and quadrature-axis armature circuit
X_{f1d} = mutual reactance between main field circuit and direct-axis additional rotor circuit
X_{1fd} = mutual reactance between direct-axis additional rotor circuit and main field circuit
θ = angle between the direct axis and the axis of phase a
θ_0 = angle between direct axis and axis of phase a at $t = 0$
ψ_0 = zero-sequence flux linkages
ψ_a = phase a flux linkages
ψ_d = direct-axis flux linkages
ψ_q = quadrature-axis flux linkages
ψ_{fd} = direct-axis field-circuit flux linkages
ψ_{1d} = direct-axis additional rotor-circuit flux linkages
ψ_{1q} = quadrature-axis additional rotor-circuit flux linkages
ψ_d' = direct-axis air-gap flux linkages
ψ_q' = quadrature-axis air-gap flux linkages
ω = angular velocity = $2\pi f$

General Theory

In per unit values, if a sinusoidal distribution of armature windings is assumed, the magnetomotive force along the direct axis is

$$M_d = \frac{2}{3}\,[i_a \cos\theta + i_b \cos(\theta - 120) + i_c \cos(\theta + 120)] \qquad (1)$$

and along the quadrature axis

$$M_q = \frac{2}{3}\,[i_a \sin\theta + i_b \sin(\theta - 120) + i_c \sin(\theta + 120)] \qquad (2)$$

where θ is the angle between pole axis and axis of phase a.

Let p_1 be the permeability in the direct axis, and p_2 be the permeability in the quadrature axis. Then the per unit flux linkages in each axis, due to the air gap flux produced by the armature current, are

$$\psi_d' = \frac{2}{3}\,p_1[i_a \cos\theta + i_b \cos(\theta - 120) + i_c \cos(\theta + 120)] \qquad (3)$$

$$\psi_q' = \frac{2}{3}\,p_2[i_a \sin\theta + i_b \sin(\theta - 120) + i_c \sin(\theta + 120)] \qquad (4)$$

The total flux linkages with phase a, then, result from field circuits, phase leakage, mutual induction between phases, and the air gap fluxes in the direct and quadrature axes,

$$\psi_a = I_d \cos\theta + I_q \sin\theta + \frac{2}{3}\,p_1 \cos\theta\,[i_a \cos\theta + i_b \cos(\theta - 120) + i_c \cos(\theta + 120)] + \frac{2}{3}\,p_2 \sin\theta\,[i_a \sin\theta + i_b \sin(\theta - 120) + i_c \sin(\theta + 120)] + i_a x_l - (i_b + i_c)\,x_m \qquad (5)$$

$$= I_d \cos\theta + I_q \sin\theta + i_a x_l - (i_b + i_c)x_m + \frac{p_1 + p_2}{3} \times \left(i_a - \frac{i_b + i_c}{2}\right) + \frac{p_1 - p_2}{3}\,[i_a \cos 2\theta + i_b \cos(2\theta - 120) + i_c \cos(2\theta + 120)] \qquad (6)$$

Similar expressions can be written for the other phases. In general,

$$\theta = t + \theta_0 \qquad (7)$$

where θ_0 is the angle between phase a and field at $t = 0$.

The quantities x_d and x_q are 2 common constants of synchronous machines, and are measured by rotating the machine at synchronous speed in a positive direction and applying positive-phase-sequence current to the armature. Let this current be of unit value, maximum in phase a at $t = 0$, then

$$i_a = \cos t \qquad i_b = \cos(t - 120) \qquad i_c = \cos(t + 120)$$
$$\theta = t + \theta_0 \qquad I_d = I_q = 0 \qquad (8)$$

Then, if the axes of phase a and the field coincide at $t = 0$, which is the position for measuring x_d, $\theta_0 = 0$ and

$$\psi_a = \left(x_l + x_m + \frac{p_1 + p_2}{2}\right)\cos t + \frac{p_1 - p_2}{2}\cos t$$
$$= (x_l + x_m + p_1)\cos t$$

However, it may be written

$$\psi_a = x_d \cos t$$

Thus

$$x_d = (x_l + x_m + p_1) \qquad (9)$$

To measure x_q, the axis of the field poles is at right angles to the axis of phase a at $t = 0$, that is, $\theta_0 = 90$ degrees, and then

$$\psi_a = \left(x_l + x_m + \frac{p_1 + p_2}{2}\right)\cos t - \frac{p_1 - p_2}{2}\cos t$$
$$= (x_l + x_m + p_2)\cos t$$

Whence, by the procedure just given,

$$x_q = (x_l + x_m + p_2) \qquad (10)$$

The quantities x_d and x_q correspond to the self-impedances in their respective axes.

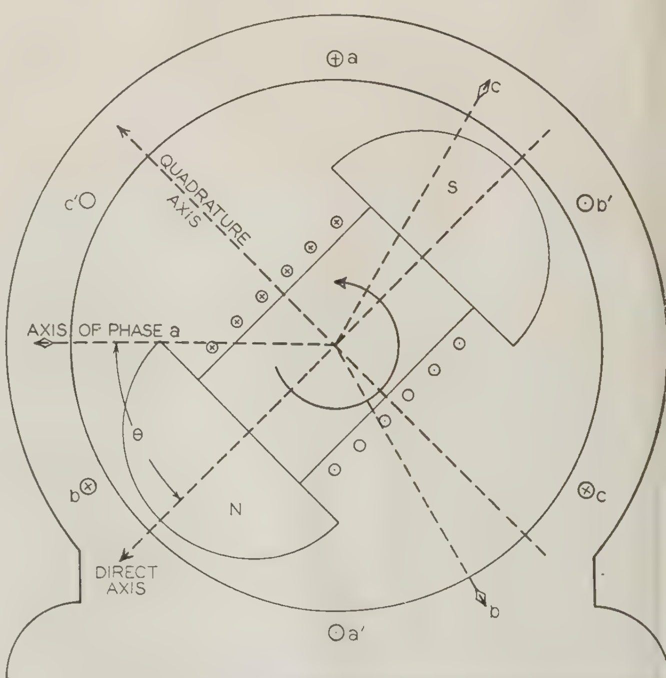

Fig. 1. Elementary diagram of 3-phase machine
Positive direction of armature and field currents indicated

In order to write equation 6 in terms of standard machine constants, 3 machine constants will be needed. The zero-phase-sequence current flows only in x_l and x_m, and if this current is applied,

$$i_a = i_b = i_c = \cos t \qquad I_d = I_q = 0$$

By equation 6

$$\psi_a = (x_l - 2x_m)\cos t$$

and

$$x_0 = (x_l - 2x_m) \tag{11}$$

Then from equations 9 and 10

$$x_d - x_q = p_1 - p_2 \tag{12}$$

$$x_d + x_q - 2(x_l + x_m) = p_1 + p_2 \tag{13}$$

Substituting in equation 6,

$$\begin{aligned}\psi_a &= I_d\cos\theta + I_q\sin\theta + i_a x_l - (i_b + i_c)x_m + \\ &\quad \frac{x_d + x_q}{3}\left(i_a - \frac{i_b + i_c}{2}\right) - \frac{2(x_l + x_m)i_a}{3} + \\ &\quad \frac{(x_l + x_m)(i_b + i_c)}{3} + \frac{x_d - x_q}{3}[i_a\cos 2\theta + \\ &\quad i_b \cos(2\theta - 120) + i_c\cos(2\theta + 120)] \\ &= I_d\cos\theta + I_q\sin\theta + (x_l - 2x_m)\frac{i_a}{3} + (x_l - 2x_m)\frac{i_b}{3} + \\ &\quad (x_l - 2x_m)\frac{i_c}{3} + \frac{x_d + x_q}{3}\left(i_a - \frac{i_b + i_c}{2}\right) + \\ &\quad \frac{x_d - x_q}{3}[i_a\cos 2\theta + i_b\cos(2\theta - 120) + i_c\cos(2\theta + 120)] \\ &= I_d\cos\theta + I_q\sin\theta + x_0\left(\frac{i_a + i_b + i_c}{3}\right) + \\ &\quad \frac{x_d + x_q}{3}\left(i_a - \frac{i_b + i_c}{2}\right) + \frac{x_d - x_q}{3}[i_a\cos 2\theta + \\ &\quad i_b\cos(2\theta - 120) + i_c\cos(2\theta + 120)]\end{aligned} \tag{14}$$

Corresponding equations can be written for phases b and c.

Equations 1 and 2 involve currents which produce magnetomotive forces in the so-called direct and quadrature axes of the machine. Thus, the unit of current in the direct axis may be defined as

$$i_d = \frac{2}{3}[i_a\cos\theta + i_b\cos(\theta - 120) + i_c\cos(\theta + 120)] \tag{15}$$

and that in the quadrature axis as

$$i_q = \frac{2}{3}[i_a\sin\theta + i_b\sin(\theta - 120) + i_c\sin(\theta + 120)] \tag{16}$$

Corresponding flux linkages and voltages may be similarly defined. Thus, for the flux linkages in the direct and quadrature axes

$$\psi_d = \frac{2}{3}[\psi_a\cos\theta + \psi_b\cos(\theta - 120) + \psi_c\cos(\theta + 120)] \tag{17}$$

$$\psi_q = \frac{2}{3}[\psi_a\sin\theta + \psi_b\sin(\theta - 120) + \psi_c\cos(\theta + 120)] \tag{18}$$

and for the corresponding voltages

$$e_d = \frac{2}{3}[e_a\cos\theta + e_b\cos(\theta - 120) + e_c\cos(\theta + 120)] \tag{19}$$

$$e_q = \frac{2}{3}[e_a\sin\theta + e_b\sin(\theta - 120) + e_c\cos(\theta + 120)] \tag{20}$$

An interrelationship of voltages, currents, and flux linkages in the direct and quadrature axes may be obtained from the following fundamental equations for the voltages induced by currents in a positive direction in the machine as assumed in figure 1. These equations are:

$$e_a = -p\psi_a - i_aR; \quad e_b = -p\psi_b - i_bR; \quad e_c = -p\psi_c - i_cR \tag{21}$$

Differentiating equations 17 and 18,

$$\begin{aligned}p\psi_d &= \frac{2}{3}[p\psi_a\cos\theta + p\psi_b\cos(\theta - 120) + p\psi_c\cos(\theta + 120)] - \\ &\quad \frac{2}{3}[\psi_a\sin\theta + \psi_b\sin(\theta - 120) + \psi_c\sin(\theta + 120)]p\theta\end{aligned}$$

$$\begin{aligned}p\psi_q &= \frac{2}{3}[p\psi_a\sin\theta + p\psi_b\sin(\theta - 120) + p\psi_c\sin(\theta + 120)] + \\ &\quad \frac{2}{3}[\psi_a\cos\theta + \psi_b\cos(\theta - 120) + \psi_c\cos(\theta + 120)]p\theta\end{aligned}$$

From equation 21 $p\psi_a = -e_a - i_aR$, etc. Substituting this in the preceding,

$$p\psi_d = -e_d - i_dR - \psi_q p\theta \qquad p\psi_q = -e_q - i_qR + \psi_d p\theta$$

or

$$e_d = -p\psi_d - \psi_q(p\theta) - i_dR \tag{22}$$

$$e_q = -p\psi_q + \psi_d(p\theta) - i_qR \tag{23}$$

These equations are complete and general for the direct and quadrature axes, if the I_d and I_q terms are generalized (in equation 14) to represent the total flux linkages of the rotor circuits in their respective axes with the armature. The first term is an induced voltage caused by a changing axis flux, the second a voltage caused by rotation, and the third a voltage caused by the resistance drop.

In these equations all parameters are constants and operational methods of solution prevail.

A finite number of rotor circuits may be assumed, these rotor circuits being included in the flux-linkage expressions for their respective axes. In order to simplify the solutions, a field circuit will be assumed in the direct axis only.

The circuital equation for the field circuit is then

$$E_{fd} = p\psi_{fd} + I_{fd}R_{fd} \tag{24}$$

Here

$$\psi_{fd} = X_{ffd}I_{fd} + X_{fad}i_d \tag{25}$$

The direct-axis flux linkage, in terms of the field circuit and the direct axis, is

$$\psi_d = X_{afd}I_{fd} + x_d i_d \tag{26}$$

Since no rotor circuits in the quadrature axis are assumed,

$$\psi_q = x_q i_q \tag{27}$$

Equations 22, 23, and 24 are sufficient to find the currents in the armature and field circuits under any symmetrical condition of phase connections, regardless of the applied voltage. By the use of equations 25, 26, and 27, the general equations 22, 23, and 24

may be expressed in terms of the machine constants, for any arbitrarily assumed voltage applied.

In order to convert 3-phase current, voltage, and flux values into corresponding armature phase values, equations 15, 16, 17, 18, 19, and 20 must be solved simultaneously.

Letting

$$\left.\begin{aligned} i_0 &= \frac{1}{3}(i_a + i_b + i_c) \\ \psi_0 &= \frac{1}{3}(\psi_a + \psi_b + \psi_c) \\ e_0 &= \frac{1}{3}(e_a + e_b + e_c) \end{aligned}\right\} \quad (28)$$

This gives, for the currents,

$$\left.\begin{aligned} i_a &= i_d \cos\theta + i_q \sin\theta + i_0 \\ i_b &= i_d \cos(\theta - 120) + i_q \sin(\theta - 120) + i_0 \\ i_c &= i_d \cos(\theta + 120) + i_q \sin(\theta + 120) + i_0 \end{aligned}\right\} \quad (29)$$

for the flux linkages,

$$\left.\begin{aligned} \psi_a &= \psi_d \cos\theta + \psi_q \sin\theta + \psi_0 \\ \psi_b &= \psi_d \cos(\theta - 120) + \psi_q \sin(\theta - 120) + \psi_0 \\ \psi_c &= \psi_d \cos(\theta + 120) + \psi_q \sin(\theta + 120) + \psi_0 \end{aligned}\right\} \quad (30)$$

and for the voltages

$$\left.\begin{aligned} e_a &= e_d \cos\theta + e_q \sin\theta + e_0 \\ e_b &= e_d \cos(\theta - 120) + e_q \sin(\theta - 120) + e_0 \\ e_c &= e_d \cos(\theta + 120) + e_q \sin(\theta + 120) + e_0 \end{aligned}\right\} \quad (31)$$

Constants and Methods of Measurement

Several of the machine constants that are used in these equations are different from those ordinarily used. For this reason, it is advisable to explain just how these constants are measured.

The 7 constants used are R, R_{fd}, X_{ffd}, x_d, x_q, X_{afd}, and X_{fad}. The constant R is simply the armature resistance per phase, and nothing further need be said about its measurement. The total resistance of the field circuit at the particular time under consideration is R_{fd}, and X_{ffd} is the reactance of the field circuit. The impedance Z_{ffd} can be measured by impressing rated frequency on the field, and from this the value of X_{ffd} can be obtained. For most practical cases it is equal to Z_{ffd}.

There are various methods used to measure x_d and x_q, probably the simplest being the slip method. With the field circuit open, balanced 3-phase voltage of rated frequency is applied to the machine with the rotor speed adjusted so as to give a small value of slip. The voltage across the field will be of a low frequency and when the field voltage passes through zero, the rotor is magnetized in the direct axis. The ratio of applied voltage to armature current at this instant is x_d. When the field voltage is maximum, the rotor is magnetized in the quadrature axis and the ratio of applied voltage to current is x_q. If the slip is low enough, measurements can be taken by observing the swings of the meters in the armature circuit, or, if necessary, the test values can be obtained from an oscillogram.

The constant X_{afd} is the mutual reactance between the direct-axis armature circuit and the main field circuit. It is measured by rotating the machine at normal speed and supplying direct current to the field. The ratio of the maximum value of the phase voltage to the direct current in the field is the ohmic value of X_{afd}. The maximum value of armature voltage is used in order that the per unit value of X_{afd} is equal to unity, as will be seen later.

The constant X_{fad} is the mutual reactance between the main field circuit and the direct axis of the armature circuit. It is measured with the machine stationary and balanced 3-phase currents supplied to the armature. The ratio of effective voltage across the field to effective phase current in the armature is the ohmic value of X_{fad}.

To convert to per unit values, simplification will result if the normal values of field current and voltage are chosen so as to give per unit values of R_{fd} and X_{afd} equal to unity. Normal values for armature voltage and current will be taken as rated values.

The machine used was a wound-rotor induction motor whose field was excited by direct current. Table I shows the normal values, the ohmic values of the machine constants, the method of conversion to per unit values, and the per unit values.

Applications of Method

Some of the applications which follow, although of no very great practical use, are given to show the flexibility with which the equations may be handled, and also to show how to use the equations in preparation for calculating the case of a 3-phase short circuit. The measured data in these special cases were taken both by meters and by an oscillograph, in order to obtain both the magnitudes and the phase relationships. The constants used in the calculations are those given in table I. Per unit values have been used only in the calculations of the removal and occurrence of a 3-phase short circuit, as it proved more convenient to use ohmic values in the other cases.

Case 1

Three-phase balanced voltage applied to armature; rotor short-circuited and at a standstill.

$$E_{fd} = p\psi_{fd} + I_{fd}R_{fd} \quad (2\text{-}$$

where

$$\psi_{fd} = X_{ffd}I_{fd} + X_{fad}i_d \quad (2$$

or

$$E_{fd} = pX_{ffd}I_{fd} + pX_{fad}i_d + I_{fd}R_{fd}$$

from which

$$I_{fd} = \frac{E_{fd}}{R_{fd} + pX_{ffd}} - \frac{pX_{fad}i_d}{R_{fd} + pX_{ffd}}$$

Field circuit is short-circuited so that $E_{fd} = 0$. Then

$$I_{fd} = -\frac{pX_{fad}i_d}{R_{fd} + pX_{ffd}}$$

Neglecting R_{fd},

$$I_{fd} = -\frac{X_{fad}}{X_{ffd}} i_d$$

where

$$i_d = \frac{2}{3}\left[i_a \cos\theta + i_b \cos(\theta - 120) + i_c \cos(\theta + 120)\right] \quad (15)$$

Here, because rotor is at standstill,

$\theta = t + \theta_0 = \theta_0$

(a) *Phase a in quadrature axis, $\theta_0 = 90$ degrees*

Measured data:

$i_a = 0.98\sqrt{2}\sin(t + 0)$ $\quad i_c = 10.65\sqrt{2}\sin(t + 93)$
$i_b = 10.65\sqrt{2}\sin(t - 93)$ $\quad I_{fd} = 30.2\sqrt{2}\cos t$

Calculated data: From equation 15,

$$i_d = \frac{2}{3}[0.98\sqrt{2}\cos 90\sin(t+0) + 10.65\sqrt{2}\cos(-30)\sin(t-93) + 10.65\sqrt{2}\cos 210\sin(t+93)]$$

$$i_d = \frac{2}{3}[2 \times 10.65\sqrt{2} \times -0.866 \times 0.999\cos t]$$

$$= -12.3\sqrt{2}\cos t$$

$$I_{fd} = -\frac{X_{fad}}{X_{ffd}} i_d$$

$$I_{fd} = \frac{9.7}{3.79} \times 12.3\sqrt{2} = 31.5\sqrt{2}\cos t$$

(b) *Phase a in direct axis, $\theta_0 = 0$ degrees*

Measured data:

$i_a = 11.3\sqrt{2}\sin(t + 0)$ $\quad i_c = 5.65\sqrt{2}\sin(t + 180)$
$i_b = 5.65\sqrt{2}\sin(t - 180)$ $\quad I_{fd} = 27.6\sqrt{2}\sin(t + 180)$

Calculated data:

$$i_d = \frac{2}{3}[11.3\sqrt{2}\cos 0\sin t + 5.65\sqrt{2}\cos(-120)\sin(t-180) + 5.65\sqrt{2}\cos 120\sin(t+180)]$$

$$i_d = \frac{2}{3}\left[11.3\sqrt{2}\sin t + 2 \times \frac{5.65\sqrt{2}}{2}\sin t\right] = 11.3\sqrt{2}\sin t$$

$$I_{fd} = -\frac{9.7}{3.79} \times 11.3\sqrt{2} = -28.9\sqrt{2}\sin t$$

Table I—Constants for Machine Tested

Constant	Ohmic Value	Conversion	Per Unit Value
R	0.388	$0.388 \times \frac{10\sqrt{2}}{63.5\sqrt{2}}$	0.0611
R_{fd}	0.136	$0.136 \times \frac{13.95}{1.89}$	1.0
X_{ffd}	3.79	$3.79 \times \frac{13.95}{1.89}$	28.0
$x_d = x_q$	17.78	$17.78 \times \frac{10\sqrt{2}}{63.5\sqrt{2}}$	2.8
X_{afd}	6.43	$6.43 \times \frac{13.95}{63.5\sqrt{2}}$	1.0
X_{fad}	9.7	$9.7 \times \frac{10\sqrt{2}}{1.89}$	72.6

Normal $e_a = 63.5\sqrt{2}$ volts $\quad$ Normal E_{fd} = 1.89 volts
Normal $i_a = 10\sqrt{2}$ amperes $\quad$ Normal I_{fd} = 13.95 amperes

CASE 2

(a) *60 cycles impressed on field, and rotor at a standstill. Armature open-circuited and phase a in direct axis, $\theta_0 = 0$ degrees*

$$e_d = -p\psi_d - \psi_q(p\theta) - i_d R \quad (22)$$

$$e_q = -p\psi_q + \psi_d(p\theta) - i_q R \quad (23)$$

$$\psi_d = X_{afd}I_{fd} + x_d i_d \quad (26)$$

$$\psi_q = x_q i_q \quad (27)$$

Under the given conditions,

$i_d = i_q = p\theta = 0$

Therefore

$\psi_d = X_{afd}I_{fd} \quad \psi_q = 0$

$e_d = -p\psi_d \quad e_q = 0$

$e_d = -pX_{afd}I_{fd}$

Assuming

$I_{fd} = I_{fd}\sin t$

$e_d = -pX_{afd}I_{fd}\sin t = -X_{afd}I_{fd}\cos$

Now

$$e_a = e_d\cos\theta + e_q\sin\theta \quad (31)$$

$\theta = t + \theta_0 = \theta_0 = 0$

$e_a = e_d\cos 0 = -X_{afd}I_{fd}\cos t$

Measured data:

$I_{fd} = 7.65\sqrt{2}\sin t \quad e_a = 47\sqrt{2}\cos(t + 180)$

Calculated data:

$e_a = -6.43 \times 7.65\sqrt{2}\cos t = -49.2\sqrt{2}\cos t$

(b) *60 cycles impressed on field; machine running at normal speed; armature circuit open*

Now, in equations 22, 23, 26, and 27,

$i_d = i_q = 0$ and $p\theta = 1$

$e_d = -p\psi_d \quad e_q = \psi_d(p\theta) = \psi_d$

$e_d = -X_{afd}I_{fd}\cos t \quad e_q = X_{afd}I_{fd}\sin t$

$e_a = X_{afd}I_{fd}[-\cos t\cos\theta + \sin t\sin\theta]$

but

$\theta = t$

$e_a = -X_{afd}I_{fd}[\cos^2 t - \sin^2 t] = -X_{afd}I_{fd}\cos 2t$

Measured data:

$I_{fd} = 7.7\sqrt{2}\sin t \quad e_a = -47.5\sqrt{2}\cos 2t$

Calculated data:

$e_a = -6.43 \times 7.7\sqrt{2}\cos 2t = -49.5\sqrt{2}\cos 2t$

(c) *60 cycles impressed on field; machine running at normal speed; armature short-circuited*

In equations 22, 23, 26, and 27,

$e_d = e_q = 0$ and $p\theta = 1$

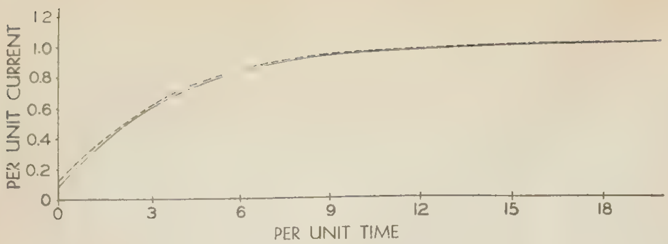

Fig. 2. Field current upon removal of 3-phase short circuit

Dashed curve from oscillogram; solid curve from calculation

Neglecting R,

$$0 = -p\psi_d - \psi_q = -pX_{afd}I_{fd} - px_d i_d - x_q i_q$$
$$0 = -p\psi_q + \psi_d = X_{afd}I_{fd} + x_d i_d + px_q i_q$$

Solving these 2 equations for i_d and i_q,

$$i_d = -\frac{X_{afd}}{x_d} I_{fd} \qquad \text{and} \qquad i_q = 0$$

$$i_a = i_d \cos\theta + i_q \sin\theta \tag{29}$$

$$i_a = i_d \cos t = -\frac{X_{afd}}{x_d} I_{fd} \sin t \cos t$$

$$= -\frac{1}{2}\frac{X_{afd}}{x_d} I_{fd} \sin 2t$$

Measured data:

$I_{fd} = 26.5\sqrt{2} \sin t$ $\qquad$ $i_a = -5.0\sqrt{2}\sin 2t$

Calculated data:

$$i_a = -\frac{1}{2}\frac{6.43}{17.78} \times 26.5\sqrt{2}\sin 2t = -4.8\sqrt{2}\sin 2t$$

Case 3

Negative-sequence currents in armature; machine running at normal speed; field circuit open

$$E_{fd} = p\psi_{fd} + I_{fd}R_{fd} \tag{24}$$

$$\psi_{fd} = X_{ffd}I_{fd} + X_{fad}i_d \tag{25}$$

or

$$E_{fd} = pX_{ffd}I_{fd} + pX_{fad}i_d + I_{fd}R_{fd}$$

but

$$I_{fd} = 0$$

Therefore

$$E_{fd} = pX_{fad}i_d$$

Now for negative-sequence currents,

$$i_a = I_a \sin t$$
$$i_b = I_a \sin(t + 120)$$
$$i_c = I_a \sin(t - 120)$$

$$i_d = \frac{2}{3}[i_a \cos\theta + i_b \cos(\theta - 120) + i_c \cos(\theta + 120)] \tag{15}$$

Substituting in the values for i_a, i_b, and i_c,

$$i_d = I_a \sin 2t$$

Therefore

$$E_{fd} = 2X_{fad} I_a \cos 2t$$

Measured data:

$i_a = 3.95\sqrt{2}\sin t$ $\qquad$ $E_{fd} = 77.5\sqrt{2}\cos 2t$

Calculated data:

$$E_{fd} = 2 \times 9.7 \times 3.95\sqrt{2}\cos 2t = 76.6\sqrt{2}\cos 2t$$

Case 4

Negative-sequence voltage impressed on armature; fiel short-circuited; machine running at normal speed

$$E_{fd} = p\psi_{fd} + I_{fd}R_{fd} \quad (24$$

$$\psi_{fd} = X_{ffd}I_{fd} + X_{fad}i_d \quad (2$$

$$E_{fd} = I_{fd}R_{fd} + pX_{ffd}I_{fd} + pX_{fad}i_d = 0$$

$$I_{fd} = -\frac{pX_{fad}}{R_{fd} + pX_{ffd}} i_d$$

$$\psi_d = X_{afd}I_{fd} + x_d i_d = -\frac{pX_{fad}X_{afd}i_d}{R_{fd} + pX_{ffd}} + x_d i_d$$

The initial value of ψ_d assuming no subtransient effects is

$$\psi_d = x_d' i_d$$

This initial value is also the value of ψ_d above where $p = \infty$

$$\psi_d = -\frac{X_{fad}X_{afd}}{X_{ffd}} i_d + x_d i_d$$

or

$$x_d' = x_d - \frac{X_{fad}X_{afd}}{X_{ffd}}$$

If $e_a = \sin t$, and e_b and e_c are negative-sequence voltages, then fror equations 19 and 20,

$e_d = \sin 2t$ $\qquad$ and $\qquad$ $e_q = -\cos 2t$

$$e_d = -p\psi_d - \psi_q(p\theta) - i_dR \quad (22$$

$$e_q = -p\psi_q + \psi_d(p\theta) - i_qR \quad (23$$

$$\psi_d = x_d' i_d$$

$$\psi_q = x_q' i_q = x_q i_q$$

With $p\theta = 1$ and neglecting R, equations 22 and 23 become

$$\sin 2t = -p\psi_d - \psi_q = -px_d' i_d - x_q i_q$$
$$-\cos 2t = -p\psi_q + \psi_d = -px_q i_q + x_d' i_d$$

Now i_d is of the form $\cos 2t$ and i_q is of the form $\sin 2t$, or

$i_d = A\cos 2t$ $\qquad$ $i_q = B\sin 2t$

$$\sin 2t = 2A\, x_d' \sin 2t - Bx_q \sin 2t$$

$$-\cos 2t = -2B\, x_q \cos 2t + A\, x_d' \cos 2t$$

$$1 = 2A\, x_d' - B\, x_q$$

$$-1 = -2B\, x_q + A\, x_d'$$

Solving these last 2 equations gives

$$A = \frac{1}{x_d'} \qquad B = \frac{1}{x_q}$$

or

$$i_d = \frac{\cos 2t}{x_d'} \qquad i_q = \frac{\sin 2t}{x_q}$$

$$_{} = i_d \cos\theta + i_q \sin\theta \qquad (29)$$

$$_{} = \frac{\cos 2t \cos\theta}{x_d'} + \frac{\sin 2t \sin\theta}{x_q}$$

eplacing θ by t,

$$_{} = \frac{1}{2x_d'}(\cos t + \cos 3t) + \frac{1}{2x_q}(\cos t - \cos 3t)$$

$$= \frac{1}{2}\left(\frac{1}{x_d'} + \frac{1}{x_q}\right)\cos t + \frac{1}{2}\left(\frac{1}{x_d'} - \frac{1}{x_q}\right)\cos 3t$$

hat is, the armature current is made up of a fundamental and a ird harmonic. If, from equation 29, the expressions for i_b and i_c ere to be written out, it would be observed that the fundamental urrent is negative sequence and the third harmonic current is ositive sequence. The ratio of these will be the ratio of the coeffi- ents.

$$\text{atio } \frac{\text{third harmonic}}{\text{fundamental}} = \frac{\frac{1}{x_d'} - \frac{1}{x_q}}{\frac{1}{x_d'} + \frac{1}{x_q}} = \frac{x_q - x_d'}{x_q + x_d'}$$

The negative-sequence reactance is defined as the ratio of negative- equence fundamental voltage to negative-sequence fundamental urrent. Impressed voltage has been assumed equal to unity.

$$_{2} = \frac{1}{\frac{1}{2}\left(\frac{1}{x_d'} + \frac{1}{x_q}\right)} = \frac{2x_q x_d'}{x_q + x_d'}$$

rom equations 24 and 25, and neglecting R_{fd}, as in case 1,

$$_{fd} = -\frac{X_{fad}}{X_{ffd}}\, i_d$$

In this case i_d is found from the measured values of ${}_1i_2$ and ${}_3i_1$, e fundamental negative-sequence current and the third-harmonic ositive-sequence current, respectively. If these are put in equation 5, the equation for i_d, a double-frequency current whose amplitude the sum of the amplitudes of ${}_1i_2$ and ${}_3i_1$ will be obtained.

ow

$$_{d}' = x_d - \frac{X_{fad}X_{afd}}{X_{ffd}}$$

om which

$$\frac{_{fad}}{_{ffd}} = \frac{x_d - x_d'}{X_{afd}}$$

herefore

$$_{d} = -\frac{x_d - x_d'}{X_{afd}}({}_1i_2 + {}_3i_1)$$

Since i_d is a double-frequency term, the field urrent will be a double-frequency current. The egative sign indicates the phase relationship be- ween I_{fd} and i_a.

ı) No external reactance in either armature or field ircuit. Harmonic ratios in the armature current easured by means of a harmonic analyzer

leasured data:

=	9.1 amperes	I_{fd}	= 31.7 amperes
=	16.2 volts	ratio	= 0.852
=	6.93 amperes	${}_3i_1$	= 5.9 amperes

$$= \frac{16.2}{6.93} = 2.335 \text{ ohms}$$

Calculated data:

$$x_d' = x_d - \frac{X_{afd}X_{fad}}{X_{ffd}}$$

$$= 17.78 - \frac{6.43 \times 9.7}{3.79} = 1.31$$

$$\text{ratio} = \frac{x_q - x_d'}{x_q + x_d'} = \frac{17.78 - 1.31}{17.78 + 1.31} = 0.863$$

$$x_2 = \frac{2x_q x_d'}{x_q + x_d'} = \frac{2 \times 17.78 \times 1.31}{19.09} = 2.44$$

$$I_{fd} = -\frac{(x_d - x_d')}{X_{afd}}({}_1i_2 + {}_3i_1) = \frac{16.47}{6.43} \times 12.83$$

$$= 32.9 \text{ amperes}$$

(b) Reactance of 12.4 ohms added externally in each phase

Measured data:

i_a	= 2.02 amperes	I_{fd}	= 6.32 amperes
e_a	= 35.0 volts	ratio	= 0.392
${}_1i_2$	= 1.89 amperes	${}_3i_1$	= 0.74 amperes

$$x_2 = \frac{35}{1.89} = 18.5 \text{ ohms}$$

Calculated data:

$$x_d' = 30.18 - \frac{6.43 \times 9.7}{3.79} = 13.71$$

$$\text{ratio} = \frac{30.18 - 13.71}{30.18 + 13.71} = 0.375$$

$$x_2 = \frac{2 \times 30.18 \times 13.71}{43.89} = 18.9$$

$$I_{fd} = \frac{16.47}{6.43} \times 2.63 = 6.62 \text{ amperes}$$

(c) Reactance of 4.13 ohms added externally to field circuit

Measured data:

i_a	= 3.0 amperes	I_{fd}	= 4.29 amperes
e_a	= 35.0 volts	ratio	= 0.289
${}_1i_2$	= 2.88 amperes	${}_3i_1$	= 0.86 amperes

$$x_2 = \frac{35}{2.88} = 12.15 \text{ ohms}$$

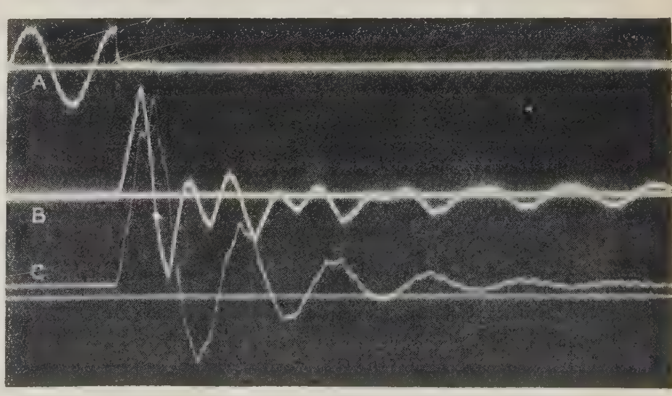

Fig. 3. Oscillogram of 3-phase short circuit

A—Phase voltage *B*—Armature current *C*—Field current

Calculated data:

$$x_d' = 17.78 - \frac{6.43 \times 9.7}{7.92} = 9.89$$

$$\text{ratio} = \frac{17.78 - 9.89}{17.78 + 9.89} = 0.285$$

$$x_2 = \frac{2 \times 17.78 \times 9.89}{27.67} = 12.74$$

$$I_{fd} = \frac{7.89}{6.43} \times 3.74 = 4.59 \text{ amperes}$$

(*d*) *Reactance of 6.22 ohms added externally to field circuit*

Measured data:

i_a = 2.7 amperes	I_{fd} = 3.02 amperes
e_a = 35.0 volts	ratio = 0.2045
${}_1i_2$ = 2.66 amperes	${}_3i_1$ = 0.54 amperes

$$x_2 = \frac{35}{2.66} = 13.17 \text{ ohms}$$

Calculated data:

$$x_d' = 17.78 - \frac{6.43 \times 9.7}{10.01} = 11.55$$

$$\text{ratio} = \frac{17.78 - 11.55}{17.78 + 11.55} = 0.213$$

$$x_2 = \frac{2 \times 17.78 \times 11.55}{29.33} = 14.0$$

$$I_{fd} = \frac{6.23}{6.43} \times 3.2 = 3.11 \text{ amperes}$$

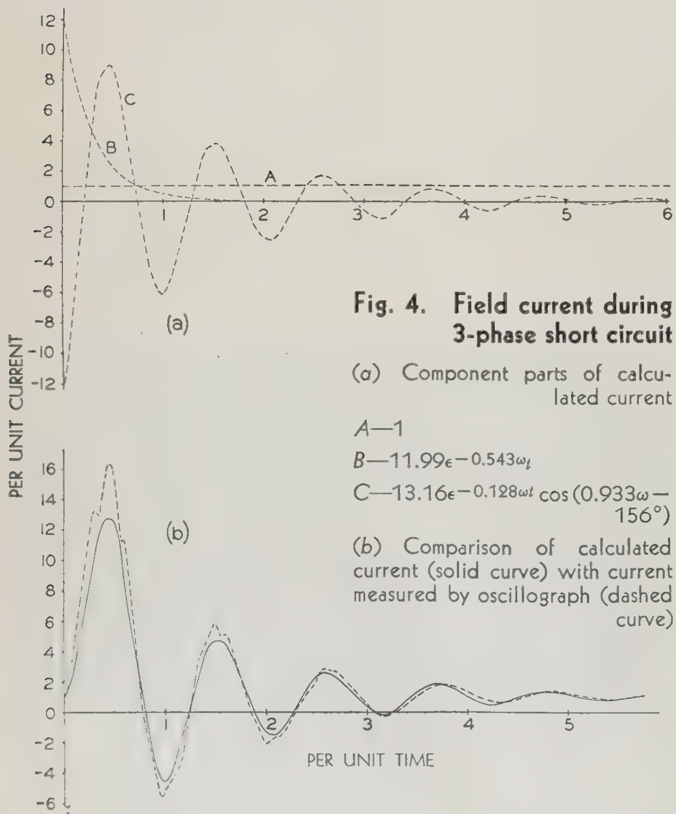

Fig. 4. Field current during 3-phase short circuit

(*a*) Component parts of calculated current

A—1

B—$11.99\epsilon^{-0.543\omega t}$

C—$13.16\epsilon^{-0.128\omega t}\cos(0.933\omega - 156°)$

(*b*) Comparison of calculated current (solid curve) with current measured by oscillograph (dashed curve)

(*e*) *Reactance of 6.22 ohms added to field circu 12.4 ohms reactance added to each armature phase*

Measured data:

i_a = 2.73 amperes	I_{fd} = 2.85 amperes
e_a = 69.0 volts	ratio = 0.1135
${}_1i_2$ = 2.71 amperes	${}_3i_1$ = 0.31 amperes

$$x_2 = \frac{69}{2.71} = 25.45 \text{ ohms}$$

Calculated data:

$$x_d' = 30.18 - \frac{6.43 \times 9.7}{10.01} = 23.95$$

$$\text{ratio} = \frac{30.18 - 23.95}{30.18 + 23.95} = 0.115$$

$$x_2 = \frac{2 \times 30.18 \times 23.95}{54.13} = 26.7$$

$$I_{fd} = \frac{6.23}{6.43} \times 3.02 = 2.93 \text{ amperes}$$

Case 5

Removal of a 3-phase short circuit

From equations 24 and 25,

$$E_{fd} = pX_{fad}i_d + (R_{fd} + pX_{ffd})I_{fd}$$

from which

$$I_{fd} = E_{fd} - \frac{p\,X_{fad}\,i_d}{R_{fd} + pX_{ffd}}$$

The effect of removing a short circuit is the same as sudde impressing the negative of the current that is flowing during short circuit. The value of i_d as calculated later in the 3-ph short circuit case is −0.357. There is no change made in when the short circuit is removed. In order to calculate the cha in field current, the negative of i_d should be applied and E_{fd} equal to zero in the equation just given. That is,

$$I_{fd} = -\frac{pX_{fad}(0.357)}{R_{fd} + pX_{ffd}}\,1$$

$$I_{fd} = -\frac{72.6 \times 0.357p}{1 + 28p}\,1 = -0.925\epsilon^{-0.0357\omega t}$$

This is only the change in field current resulting from the remo of the short circuit, and to this must be added the original fi current, taken to be equal to unit value. Therefore the actual fi current is

$$I_{fd} = 1 - 0.925\epsilon^{-0.0357\omega t}$$

A comparison of the calculated current and the field curr measured by an oscillograph is shown in figure 2.

Solution for 3-Phase Short Circuit

The fundamental equations necessary for the so tion of the case for a 3-phase short circuit are 23, 24, 25, 26, and 27. These may be recombin into the following 3 equations. Assuming rat speed, $p\theta = 1$.

$$\left.\begin{aligned} e_d &= -(R + px_d)i_d - x_q i_q - pX_{afd}I_{fd} \\ e_q &= x_d i_d - (R + px_q)i_q + X_{afd}I_{fd} \\ E_{fd} &= pX_{fad}i_d + (R_{fd} + pX_{ffd})I_{fd} \end{aligned}\right\} \quad (\ldots$$

These equations solved simultaneously by means of determinants give the following equations for the 3 unknown currents:

$$i_d = [e_q(x_qR_{fd} + px_qX_{ffd}) - e_d(p^2x_qX_{ffd} + px_qR_{fd} + pRX_{ffd} + RR_{fd}) - E_{fd}X_{afd}(p^2x_q + pR + x_q)]/D(p)$$

$$i_q = [e_d(pX_{afd}X_{fad} - px_dX_{ffd} - x_dR_{fd}) + E_{fd}(RX_{afd}) - e_q(p^2x_dX_{ffd} - p^2X_{afd}X_{fad} + pRX_{ffd} + pR_{fd}x_d + RR_{fd})]/D(p)$$

$$I_{fd} = [E_{fd}(p^2x_dx_q + pRx_q + pRx_d + R^2 + x_dx_q) - e_q(pX_{fad}x_q) + e_d(p^2x_qX_{fad} + pRX_{fad})]/D(p)$$

where

$$D(p) = p^3(x_dx_qX_{ffd} - X_{afd}X_{fad}x_q) + p^2(x_qx_dR_{fd} + Rx_dX_{ffd} + Rx_qX_{ffd} - X_{afd}X_{fad}R) + p(RR_{fd}x_d + RR_{fd}x_q + R^2X_{ffd} + x_dx_qX_{ffd} - X_{afd}X_{fad}x_q) + (R^2R_{fd} + x_dx_qR_{fd})$$

The current in phase a is obtained from equation 29 as

$$i_a = i_d \cos\theta + i_q \sin\theta$$

where $\theta = (t + \theta_0)$.

By using the per unit constants of the machine as given in table I, the following numerical equations are obtained. The 3 roots of $D(p)$ were solved for and are given in factorial form for illustrative purposes.

$$I_{fd} = \frac{(7.84p^2+0.34p+7.84)E_{fd}-(203.3p)e_q+(203.3p^2+4.43p)e_d}{16.24(p+0.5433)(p+0.1278+j0.933)(p+0.1278-j0.933)}$$

$$i_d = \frac{(78.4p+2.8)e_q-(78.4p^2+4.51p+0.0611)e_d-(2.8p^2+0.0611p+2.8)E_{fd}}{16.24(p+0.5433)(p+0.1278+j0.933)(p+0.1278-j0.933)}$$

$$i_q = \frac{(0.0611)E_{fd}-(5.8p+2.8)e_d-(5.8p^2+4.51p+0.0611)e_q}{16.24(p+0.5433)(p+0.1278+j0.933)(p+0.1278-j0.933)}$$

These equations are general in that they give the currents resulting from any change whatsoever that might be made in E_{fd}, e_d, or e_q. Before the short circuit, the 3 phase voltages are assumed to be balanced. From equations 19 and 20, and with rated field current assumed, this gives $e_d = 0$ and $e_q = 1$. When a fault occurs, the 3 phase voltages become zero as does also e_q. This is the same as though $-e_q$ were suddenly applied to the machine. Then, in the above equations, the values $E_{fd} = e_d = 0$ are inserted because there is no change in either of them. The term e_q is put equal to -1, and by solving the equations by use of the expansion theorem the following results are obtained:

$$I_{fd} = 1 + 11.99\epsilon^{-0.543\omega t} + 13.16\epsilon^{-0.128\omega t}\cos(0.933\omega t - 156°)$$

$$i_a = 0.357\sin(\omega t + 8°\,15') + 4.32\epsilon^{-0.543\omega t}\sin(\omega t + 8°) + 2.36\,\epsilon^{-0.128\omega t}\sin(1.933\omega t + 210°\,45') + 2.71\,\epsilon^{-0.128\omega t}\sin(0.067\omega t + 168°\,18')$$

Figure 3 shows an oscillogram of field current and armature current when a 3-phase short circuit occurs at the alternator terminals.

Figure 4*a* shows the component parts of the calculated field current with each part plotted separately. Figure 4*b* shows the comparison between the measured and calculated field currents.

Figure 5*a* shows the component parts of the calculated armature current with each part plotted separately. Figure 5*b* shows the comparison between the measured and calculated armature currents.

Comments on Results

The calculated and test data check with an average error of 3.7 per cent. The machine used did not fulfill the requirements to a high degree of accuracy and measurements were made by standard laboratory instruments. In view of all the possible sources of error, the results check theoretically to a very high degree. The discrepancies between the oscillographic and calculated curves are very largely the result of tooth ripple, which is amplified at the larger values of current.

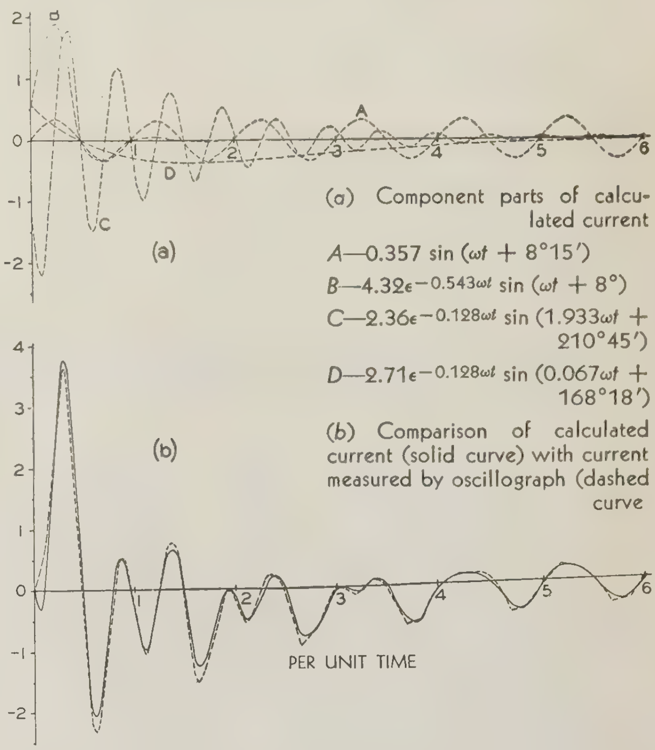

Fig. 5. Armature current during 3-phase short circuit

The tests described were made with symmetrical connections to the armature circuits, as mentioned before. Unsymmetrical conditions require the addition of negative- and zero-sequence currents, and the consequent introduction of corresponding constants. These currents will in turn add harmonics to the armature and field currents. It is proposed to show the application of the operational solution to these cases at a future date.

Appendix I—Extension to Additional Rotor Circuits

Equations 32 are the simplest that can be obtained for a general solution, wherein only a direct axis, quadrature axis, and field circuit currents are involved.

In practice, the rotor may have additional windings in either axis.

This condition would modify equations 25, 26, and 27, respectively, as follows, for a single additional circuit in each axis.

$$\psi_{fd} = X_{ffd}I_{fd} + X_{fad}i_d + X_{f1d}I_{1d} \tag{25a}$$

$$\psi_d = x_d i_d + X_{afd}I_{fd} + X_{a1d}I_{1d} \tag{26a}$$

$$\psi_q = x_q i_q + X_{a1q}I_{1q} \tag{27a}$$

Also, 2 additional voltage equations would appear for these circuits, as

$$0 = -p\psi_{1d} - I_{1d}R_{1d} \tag{33}$$

$$0 = -p\psi_{1q} - I_{1q}R_{1q} \tag{34}$$

where

$$\psi_{1d} = X_{11d}I_{1d} + X_{1fd}I_{fd} + X_{1ad}i_d \tag{35}$$

and

$$\psi_{1q} = X_{11q}I_{1q} + X_{1aq}i_q$$ (

Substituting equations 25*a*, 26*a*, and 27*a* into equations 22, and 24, and also substituting equations 35 and 36 into equati 33 and 34, the following 5 equations are obtained, when $(p\theta)$ =

$$\left.\begin{aligned} e_d &= -(R + px_d)i_d - x_q i_q - pX_{afd}I_{fd} - pX_{a1d}I_{1d} - X_{a1q}I_{1q} \\ e_q &= x_d i_d - (R + px_q)i_q + X_{afd}I_{fd} + X_{a1d}I_{1d} - pX_{a1q}I_{1q} \\ E_{fd} &= pX_{fad}i_d + (R_{fd} + pX_{ffd})I_{fd} + pX_{f1d}I_{1d} \\ 0 &= -pX_{1ad}i_d - pX_{1fd}I_{fd} - (R_{1d} + pX_{11d})I_{1d} \\ 0 &= -pX_{1aq}i_q - (R_{1q} + pX_{11q})I_{1q} \end{aligned}\right\}$$ (

These equations may be solved simultaneously for the 5 unkno currents. The extension to any number of rotor circuits may made in a similar manner.

Self-Regulated Compounded Rectifiers

A mercury-vapor rectifier can function as its own automatic regulator with the aid of the grid-control circuit described in this paper. The control equipment is practically inertialess and, therefore, is almost instantaneous in its operation. The circuit may be compounded to have a wide variety of characteristics.

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By means of the grid-control circuit described in this paper, a power rectifier can act as *its own automatic regulator* without the assistance of a vibrating contact. At the same time, the circuit may be compounded with respect to load current and input voltage, giving a wide variety of characteristics, ranging from cumulative compound to constant current. These characteristics cover wider range than do those easily obtained from d generators. Furthermore, the automatic control nearly instantaneous, which is certainly not true d-c generator equipment or vibrating and therm types of regulators. Compared with electronic regu lators, the simplest of the circuits has the advanta of not employing auxiliary tubes, which will great lower the cost of small installations. For example, 3-tube rectifier employing grid-controlled gas-fill tubes, with electronic regulator, might employ tubes; but in the new circuit, only the 3 main pow tubes themselves are necessary.

In order that the new circuit may be readily unde stood, several steps in its evolution are describe In the first step, the usual alternating voltage is n applied to the grid, because the definite phase ang introduced by such an a-c wave would destroy t sensitivity of the new self-regulating circuit. I stead, the various grids, each with its own seri resistance, are connected together and to a comm grid-voltage source of such a nature as to adju automatically the phase angle at which the gri operate to fulfill the needs of the load. In this d cussion, the use of a common cathode or catho connection is assumed.

The circuit for step 1 is shown in figure 1*a*. T power rectifier shown is a 3-anode polyphase rec fier, but a 2-anode single-phase rectifier also has be used. The grid resistors r consisted of small tungst 110-volt lamps, which are convenient when the be value of resistance is to be determined by trial. T common grid source consists of a potentiometer across the d-c load, together with a source of *posit* d-c grid bias, represented on the diagram by t battery E and usually about 90 volts.

The smoothing inductance L, *must* be connect to the anode transformer neutral. If accidenta connected in the cathode lead, the alternating vo age across the inductance would appear in the g circuit with serious consequences. If connection is moved to point b to include the inductance the control potentiometer S would not receive t benefit of the inductance, with poor results in the g circuit.

A paper recommended for publication by the AIEE committee on electrical machinery, and scheduled for discussion at the AIEE winter convention, New York, N. Y., Jan. 25–29, 1937. Manuscript submitted June 5, 1936; released for publication Sept. 8, 1936.

The authors wish to thank Dean H. E. Clifford and Prof. C. L. Dawes, both of the graduate school of engineering, Harvard University, Cambridge, Mass., for reading and making suggestions concerning the manuscript.

A simplified picture of operation of the common rid source may now be given, assuming that the nductance L so smooths the output that no com-ensation for ripple in the load voltage across S eed be made. It will be assumed that an anode has ust "fired," as a result of the grid reaching its ritical voltage e_g, which is small in comparison vith the various sources of grid voltage. The result-ng power impulse causes the current in L and conse-uently the load voltage V to rise slightly. The volt-ge V, acting through the potentiometer S, therefore lecreases the total grid voltage. The relation may be xpressed thus:

$$\text{otal grid voltage} = E - \alpha V \quad (1)$$

vhere

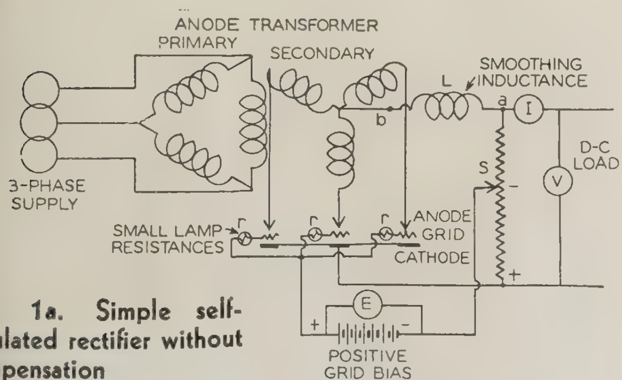
E is the *positive* grid bias
is a fraction determined by the setting of the potentiometer S

The decrease in grid voltage, below the initial r critical value, will prevent the next anode from 'firing" until the load voltage V has fallen to the nitial value. Hence, the next anode *automatically* elects a phase angle of "firing" that will maintain his initial voltage V at which the grid voltage be-omes equal to the critical voltage. This is clearly a egulator action, with variable phase angle. Hence, pproximately,

$$V = \frac{E - e_g}{\alpha} \quad (2)$$

vhere

e_g is the critical grid voltage
is the setting of potentiometer S



ig. 1a. Simple self-egulated rectifier without ompensation

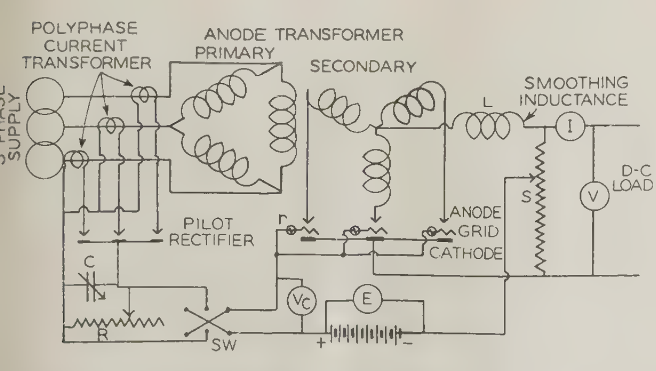


Fig. 1b. Self-regulated rectifier with current compounding

Therefore, if E be large enough to mask any variations in e_g, the output voltage V will be approximately constant, even if the voltage of the power source should vary. This result is shown in figure 2 which is a curve, determined experimentally under load conditions, of output voltage as a function of

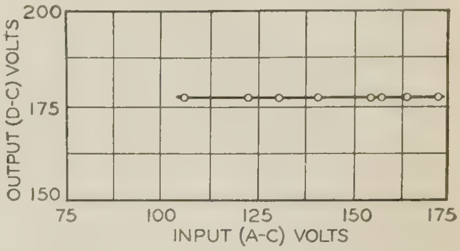


Fig. 2. Constancy of output voltage with variation of input voltage

Rectifier completely compensated against load and input voltages

input voltage. The variation in output voltage is actually too small to read on an accurate portable voltmeter, even for very large variations in input voltage. The equipment employed was compensated by the compounding method described later in the paper. Without compensation, a variation of output from 184 to 185 volts occurred for a variation of input from 127 to 163 volts.

If the variation of input voltage occurs suddenly the results are even more startling. With an alternator for power source, a pure resistance load was connected across the alternator terminals, in parallel with the rectifier input, by closing a switch. The rectifier output voltmeter showed no flicker of its pointer or change of reading either *at the start, during the transient, or after the transient.* In spite of this, a voltmeter across the input showed that the sudden impulse was followed by a steady drop in input voltage of 20 or 30 per cent. A mechanical regulator cannot thus exceed the response speed of voltmeters because of the inertia of the mechanism.

The discussion of such transients involves many factors, such as the ratio of resistance to inductance of the d-c load circuit, alternator damper windings, and others. It is rather doubtful if the smoothing inductance alone can account for such perfect performance during a transient, since the resistance-to-inductance ratio of the smoothing inductance circuit is high when a resistance load is employed. Unquestionably the 180-cycle impulse speed of 3 anodes is involved.

When the load current is varied, changes occur which prevent the output voltage from being perfectly constant. The proportion of ripple in the load current is changed because the absolute magnitude of the ripple itself does not change much. This is because the a-c impedance of the load circuit is chiefly that of the smoothing inductance. It should be realized that the grid does not regulate the d-c value of the load voltage, but rather the "minimum instantaneous value." A large proportion of ripple at light load should cause an increase in the average load voltage when approaching light load. The zero-compounding curve of figure 3 shows such a trend toward higher voltages at light load.

Another change is the decrease in tube temperature at light load when the tubes are operated under

practical conditions, without oil bath, as they were in these experiments. A decrease in temperature makes the critical voltage, e_g, *more positive*, which, according to equation 2, should decrease the output voltage at light load.

A third change is the effect of instantaneous anode voltage on e_g, which enters because the regulator neutralizes the natural regulation of the circuit by altering the phase angle, or point on the anode voltage wave at which the tube "fires." At light load, the phase angle of the grid firing should lag more to reduce natural regulation. This increases the forward voltage on the tube, just before firing, and therefore makes e_g *more negative*, a change tending again to give higher voltages at light load.

If, as in figure 2, the input voltage is increased, the anode, and hence forward voltage, is increased directly by the power transformer as well as indirectly by the lagging of the grid phase angle. As before this makes e_g more negative and increases the output. Both of these effects of anode voltage are undesirable and may be reduced by using grids that almost have to strike a grid arc, thus reducing the effect of the anode. "Positive-grid" gas-filled tubes were used for the investigation reported in this paper. These effects, however, can be compensated partially by compounding.

Both the question of compensating these changes, and the more general application of the circuit, suggest compounding, which is the second step in the development of the regulator.

Compounding

Compounding is accomplished by introducing other d-c automatic control voltages into the common grid circuit. It will be desired, therefore, to have a d-c automatic-control voltage that is proportional to the load current.

For the purpose of discussion, the anode power

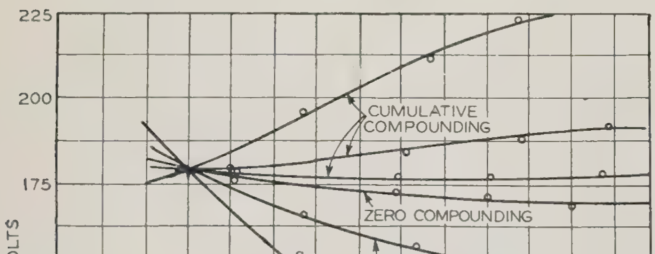


Fig. 3. Effect of current compounding, with different settings of grid-control rheostats; input voltage constant

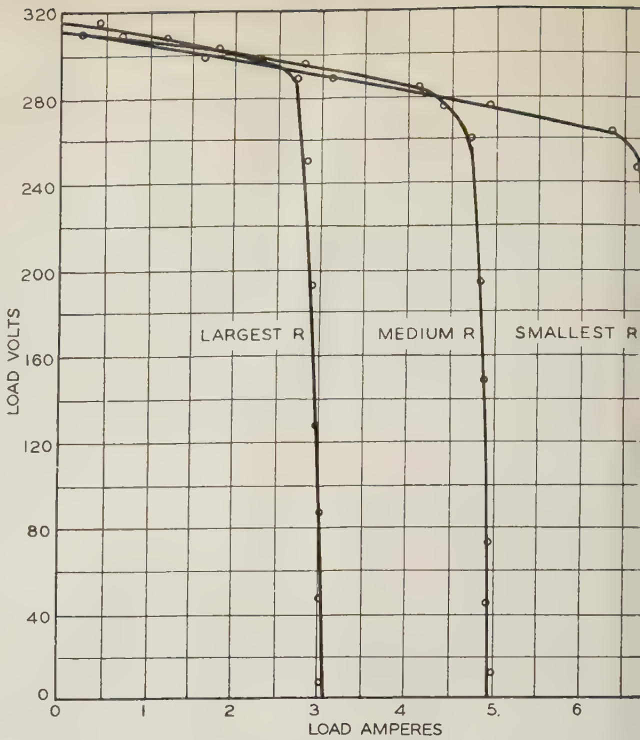


Fig. 4. Constant-current output with varying load voltage, for 3 settings of rheostat in grid circuit; load voltage potentiometer set at zero

transformer will be assumed to be unsaturated. Tl assumption is probably untrue, but serves as a co venient introduction. It follows, from the laws transformers, that the input (alternating) curre is nearly proportional to the load (direct) curren although its phase angle is variable and its wa shape slightly variable. The problem reduces that of obtaining a d-c voltage proportional the input current. For this purpose a polypha *current transformer* is inserted, as shown in figu 1*b* operating a pilot rectifier, which in turn is load by the rheostat *R*. The voltage output across tl rheostat is proportional to the load current and t resistance of the rheostat, and inversely proportion to the ratio of the current transformer. It is co nected into the common grid circuit through t polarity reverse switch *Sw*, which makes possib both positive and negative compounding, the latt having as many applications (see constant-curre operation discussed later in the paper) as the form Polarity reversal may be obtained smoothly using a potentiometer instead of rheostat *R*, a connecting the grid circuit to contact and to cent of potentiometer. The capacitance *C*, determin by trial, is sometimes unnecessary. The maximu value required in these experiments was 13 mici farads, which should be variable in 1-microfar steps. The tubes employed must be reliable, le erratic results be obtained. About 10 per cent ov compounding was found to be sensitive to th Knowledge of reliability is obtained only by su stituting tubes.

Results of the compounding in figure 1*b* are sho in figure 3, for different settings of the rheostat

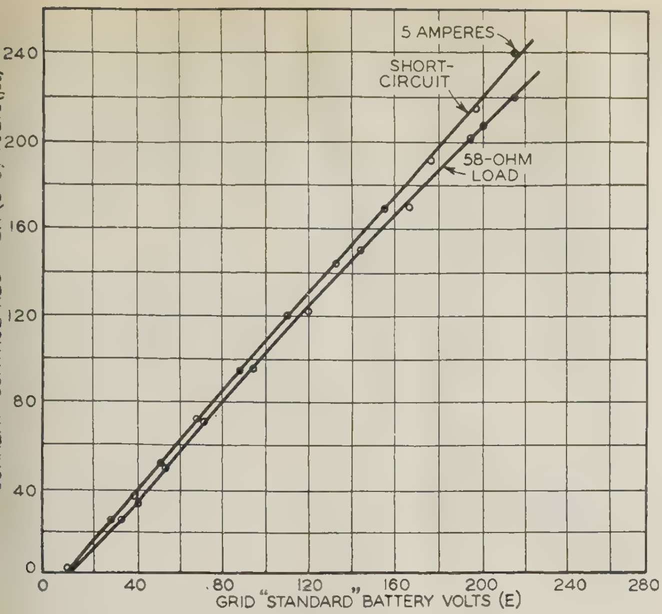


Fig. 5. Voltages in grid circuit with current regulation

vhich controls the slope of the curve. The height f the curve as a whole is determined as before by otentiometer S. In equation form:

$$_g = \text{total grid voltage} = E - \alpha V \pm \beta I \tag{3}$$

vhere

is proportional to the setting of potentiometer S
is approximately proportional to the setting of rheostat R

r

$$V = \frac{E \pm \beta I - e_g}{\alpha} \tag{4}$$

The departure of the curves from the straight lines f equation 4 may be seen to follow almost exactly he departure of the zero-compounding curve (β = 0) which already has been discussed. Cumulative ompounding of slope at any point greater than that f a straight line drawn through the origin to the ame point is not to be desired. This is because ny device with such slope would not have stable peration at that point, even with pure resistance oad, for the output voltage would rise faster (greater lope) than the voltage of the pure resistance load. Jsing pure resistance load, it was found that as the lope approached that of a resistance line, operation oon became unstable. Probably a special laboratory oad, connected in series with a d-c constant-current ircuit, could permit measurements in such unusual egions, but such characteristics do not seem to have ny practical applications. It might be mentioned hat because of saturation the ordinary series-vound d-c generator, which represents the limit f compounding of a compound-wound generator, as less slope than any resistance line. As is well nown, d-c generators having straight-line saturation urves are unstable unless separately excited, or else egulated.

Thus, it may be concluded that the compounding f a self-regulated rectifier is limited chiefly by those circuit considerations that apply to all d-c sources. It might be interesting to test the limit of differential compounding. For this limit, $\alpha = 0$, in order to raise the height of the hypothetical open-circuit voltage to infinity. Then in equation 4,

$$V = \frac{E \pm \beta I - e_g}{0} \tag{5}$$

where V is limited to finite voltages, and particularly to the maximum voltage the rectifier can generate. Therefore, the numerator of equation 5 must be zero and

$$\pm \beta I = e_g - E \tag{6}$$

But I is positive only, in a properly designed rectifier, so that operation is possible only with the minus sign, or in the differential-compounding direction. Finally,

$$\beta I = E - e_g \tag{7}$$

The value of I is dependent on e_g which is negligible, E which is constant, and β which depends on the setting of resistance R of figure 1*b*. Therefore, I is nearly a constant current, independent of load voltage or input voltage.

The experimental results obtained with these adjustments are shown in figure 4. The 3 curves are for 3 settings of the rheostat R which controls the current. Up to the working voltage of 180 volts, or even up to 220 volts, the output current may be seen to be nearly constant. The slight decrease in the current shown in each characteristic, as load voltage is increased to 180 volts, corresponds to the high light-load voltages of figure 3. For example, in equation 7, an increase of load voltage makes the phase angle of grid firing less lagging, giving less forward voltage, and more positive e_g. Because of the minus sign of e_g, I is reduced.

The top portions in figure 4 are interesting because when the supply voltages to the anodes are insufficient for the load voltage, regulation must cease

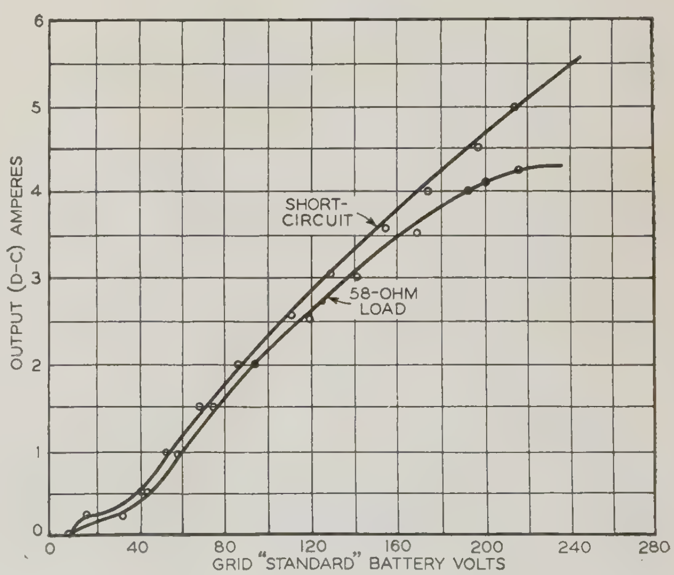


Fig. 6. Linear current control by varying battery voltage in grid circuit

and the curves flatten out as shown. Each curve then consists of a constant-current portion and a nearly constant-potential portion, smoothly joined by a rounded knee, as in magnetic saturation curves. The practical applications of such a characteristic are, therefore, multifold. Not only may constant direct current be obtained from alternating current at constant potential, but *both* parts of the characteristic might be used in the *same* application. For example, if the armature of a separately excited d-c motor be connected as load, then during the starting period the current and hence torque will be constant, which is desirable economically and mechanically. When the motor reaches the operating speed, its back electromotive force (the load voltage) reaches the constant-potential portion of the characteristic and the speed becomes nearly constant. It may be noted that there are no steps in the starting characteristic, nor is there any starting resistance loss. Starting equipment is eliminated, and the torque may be high because normal d-c motors may be used at voltages to which they are best adapted. Speed is controlled as usual by the motor field, which can be supplied from the same a-c source by a small grid-controlled rectifier, eliminating field rheostats, and giving "vest-pocket" or even photoelectric-cell control. The maximum permissible transient-load torque is equal to the starting torque which may be set by the operator by means of the rheostat R, as is usual, at some value between 100 and 300 per cent, for example. At any time, the starting current can be reduced below the maximum permissible value by inserting a resistance in the grid-control rheostat R during starting only. As a machine that has been started can be operated with entirely different characteristics (as flat, drooping, or rising speed-torque characteristics), the possibilities are rather numerous and attractive.

An experimental study of the quantities directly in equation 7 was made on a 2-tube setup. In figure

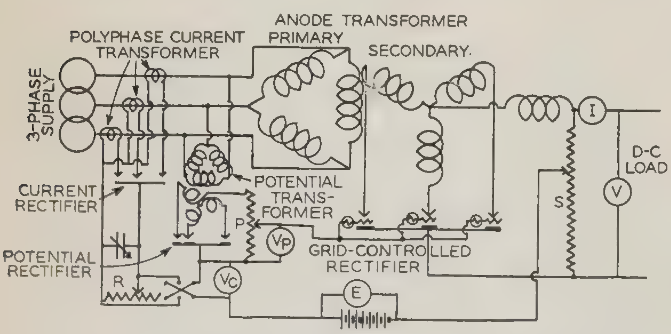


Fig. 7a. Rectifier circuit with current and potential compounding

5, the ordinates are the voltages of the current-control pilot rectifier; this is the voltage βI of equation 7. The abscissas are the positive grid-bias or "standard" voltages E of equation 7. The approximate linear relationship, which equation 7 postulates, is shown by 2 curves, one for short circuit of the load and the other for a 58-ohm resistance load. It may be noted that both intercept the axes at $\beta I = 0$ and $E = +12$, which accordi[ng] to equation 7 demands a value of e_g of $+12$ vol[ts]. The tubes were "positive-grid" gas-filled units who[se] critical grid voltage is ordinarily 10–13 volts accor[d]ing to temperature. The 58-ohm curve shows [a] value of e_g, or $E - \beta I$, which decreases to [–] volts. The short-circuit curve decreases e_g by [a] larger amount to -25 volts. These checks, repr[e]senting difference quantities, measure the deviatio[n] resulting from causes that already have been di[s]cussed, and represent increased demand (βI) on t[he] pilot rectifier as compared with the approxima[te] theory.

Curves of output current versus E for the sa[me] apparatus are shown in figure 6. The hooks at t[he] bottom probably represent a variation in β of [...] at low currents caused by exciting currents of pow[er] and current transformers, load current ripple affec[t]ing the input-current wave forms, etc. But t[he] smooth control by the operator of a short circu[it] with a constant-potential source and without limi[t]ing resistance or reactance is shown clearly by t[he] slow noncritical characteristic practically starti[ng] at the origin. *This has not been accomplished by oth[er] power circuits.* For example, an ordinary phas[e]-shift-controlled rectifier when short-circuited [is] very critical when the current is adjusted by t[he]

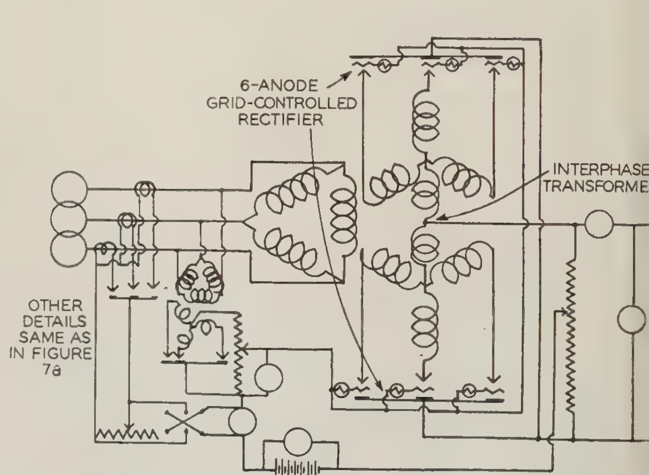


Fig. 7b. Double-star self-regulated compound circuit

phase-shifting device. This is because 2 or 3 d[e]grees of grid phase-shift will give sufficient volta[ge] to vary the short-circuit current from low to fu[ll] value. These ordinary circuits do not possess aut[o]matic control or regulator action. One might thi[nk] that a mechanical regulator may be arranged to [do] this, but the authors know of no mechanical regul[a]tor which is sufficiently fast to handle sudden sho[rt] circuits. In rotating d-c generators, whether serie[s], shunt, or compound, a sudden short circuit acro[ss] the terminals is not permissible.

Potential Compounding

Analogous to current compounding is potenti[al] compounding, which makes any variation of t[he]

lternating input voltage affect the output voltage n some desired manner. The purpose is to com- ensate for any failure of the regulator action to eutralize exactly variations in input potential. lso with potential compounding, current com- ounding may become smooth over a wider range of nput potentials. In order to accomplish these vari- us purposes, a potential transformer, figure 7*a*, s connected to the 3-phase input. The secondary of he potential transformer operates another pilot ectifier, this time loaded with a potentiometer (P) nstead of a rheostat (as R), because potential rather han current is impressed on the resistance. The

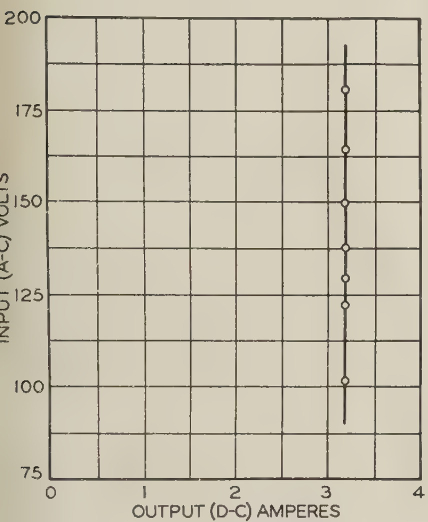


Fig. 8. Constancy of load current during input voltage variations

Characteristic obtained by varying only the field resistance of the input alternator

ariable voltage output of this potentiometer is in- erted, in series with the grid circuit, in a negative irection, opposing firing of the main grids. Thus ny increase of the alternating input voltage will be pposed by the grid control. With this connection, t should be remembered that P must carry any ositive grid current of the circuit and that the oad on the potential pilot rectifier, therefore, is educed. Hence, if the resistance of P is too high, he current tends to flow backward in the pilot ectifier, which therefore cuts off, making the current ero, and then the potential pilot rectifier does not unction. The opposite connection, potential recti- ier aiding the grids, is of course possible, but would e needed only if the main output already decreased vith increasing input.

The experimental results are shown in figure 8 for ariation of input and figure 9 for variation of load. he input source for figure 9 was an unregulated lternator the terminal voltage of which varied videly because of the load variation. Note that the haracteristic of figure 8 is vertical, showing perfect egulation. In figure 9, the curve is flat over a wide ange of 3-to-1 variation in load; at very light oad the usual rise resulting from ripple and other auses occurs. No capacitor was required in these ircuits.

Potential compounding is necessary only if flat egulation or perfect compensation for input fluctua- ion is required. For obtaining motor-starting characteristics, rising and drooping volt-ampere characteristics, and other practical purposes, it may be omitted. There might, however, be some application for a rectifier whose output voltage rose on decrease of the input voltage.

The foregoing results have been obtained with 3-anode circuits, and it may be asked if more than 3 anodes can be used. In the double-star interphase-

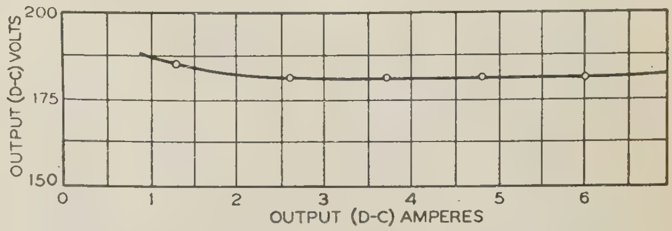


Fig. 9. Characteristic of regulator adjusted for both load and input variation, showing constant-potential output over wide current range

transformer circuit of figure 7*b*, each star circuit is an independent self-regulated rectifier. It would not be desirable to regulate only one of 2 or 4 star circuits, since the regulation load range would be halved or quartered, and, because of unbalance, the ripple would be increased. Therefore, 2 or more potential-regulated power circuits in parallel are used. This tends to be unstable, since both potentials are forced to be identical. If the regulators have rising characteristics, the situation is analagous to 2 compound d-c generators in parallel without an equalizer bus. In instantaneous action, the firing of an anode in one star circuit tends to prevent (by decreasing grid potential) the firing in the other, one anode of the latter being next in sequence. Also, there is not sufficient time, and hence voltage, for the power impulse to be reliable; this criticism applies also to 6-anode star circuits (not illustrated). The interphase transformer, to the utmost, attempts to force the circuit to operate, and considerable voltage appears on the interphase transformer.

In spite of these factors, the 6-anode circuit of figure 7*b* has operated in the authors' laboratory. Much research remains to be done in developing a satisfactory multianode technique. One method, now on trial, is the insertion of alternating voltage in the individual grid circuits, using a grid-transformer, and retaining in the common grid return the grid-circuit features of this paper. Time delay, or damping of the grid circuit, also may be necessary.

In conclusion, it may be said that a new field of power-rectifier technique and application has been opened. The principles of self-regulation and compounding may be applied to almost any variable. Self-regulation becomes automatic control and thus enters an infinity of control applications; the conversion of constant-potential input into constant-current d-c output, without the use of capacitors or a-c reactors, may be applied either for power transmission or power utilization. The surface scarcely has been scratched, concerning applications outside the original compound-wound rectifier.

Induction Motors on Unbalanced Voltages

Characteristics of polyphase induction motors are analyzed in this paper by the method of symmetrical components in a manner different from other recently published articles. The analysis is neither tedious nor highly involved, and curves of motor characteristics obtained by tests under unbalanced conditions agree closely with curves obtained by calculation.

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THE METHOD of symmetrical components as herein applied in the analysis of the starting and performance characteristics of polyphase induction motors operating on unbalanced voltages is quite different from those which have appeared in the recent literature.[1,2] The validity of the method is proved by the exactness with which test curves obtained by operating polyphase motors under several unbalanced conditions compare with the calculated curves. Lunn,[1] in his paper, outlines a method that is quite general but states ". . . its utility being limited only by the tedious algebra involved." As herein presented the analysis is neither tedious nor highly involved, as it uses only the exact equivalent circuit and the analytical determination of the characteristics which are familiar to all designers of induction motors.

Any unbalanced 3-phase voltage can be represented by 3 balanced systems, the positive-, negative-, and zero-sequence systems. In the case of the 3-phase motor without neutral connection, only the first 2 of these systems need be considered. Likewise 3-wire 2-phase systems can be represented by positive- and negative-sequence systems of vectors each having 2 vectors at right angles.

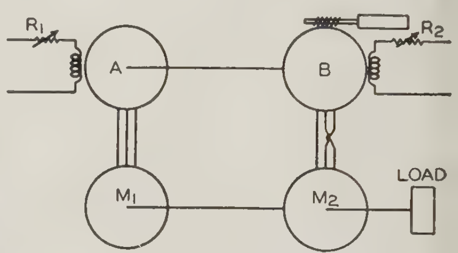
The positive-sequence voltages produce torque in the direction of rotation of the motor in question, and the negative-sequence voltages produce torque opposite to the direction of rotation. Motor cur rents are the vector sums of the positive- and nega tive-sequence currents.

Thus, assuming that saturation effects may b neglected, a polyphase induction motor on unbal anced voltages may be considered as equivalent t 2 identical motors (equal in every respect to th original) mounted on the same shaft. One of thes motors will be considered as being supplied wit balanced polyphase voltage of the same sequenc and magnitude as the positive-sequence componen of the unbalanced voltage, and the other as bein supplied with balanced voltage of the same sequenc and magnitude as the negative-sequence componen of the unbalanced voltage.

This concept is pictorially illustrated in figure for the 3-phase case. Alternator *A* supplies voltag of positive-phase sequence and equal in magnitud to this component of the unbalanced voltage. Al ternator *B* supplies the negative-sequence voltage The stator-shifting screw on alternator *B* is to indi cate that the proper phase relationship must exis between the alternators before the line currents ca be superposed to produce the actual line currents o the 3-phase motor operating on unbalanced voltages Except for starting, when the slip for both motors i 100 per cent, motor M_1 always runs at less than 10 per cent slip and motor M_2 always runs at mor than 100 per cent slip.

Inasmuch as balanced polyphase motors operatin either single-phase, or single-phase as capacito motors, are special cases of unbalance they also ma be analyzed by the above method. The analysis wil be applied to a 5-horsepower 3-phase 220-vol

Fig. 1. Illustration of concept of assuming a polyphase induction motor operating on unbalanced voltage as equivalent to 2 identical motors mounted on the same shaft



Resistors R_1 and R_2 are adjusted to give voltages equal to the positive and negative-sequence voltages, respectively

60-cycle 6-pole induction motor under the followin conditions:

1. Operating single phase
2. Operating single phase as capacitor motor, with fixed capacitanc
3. Operating 3 phase with 10 per cent degree of unbalance

The analysis also will be applied to a 3/4-horsepowe 3-phase 220-volt 60-cycle 4-pole induction moto under the following condition:

1. Starting single phase as capacitor motor, with variable capac tance

Finally, it will be applied to a 2-horsepower 2-phas 220-volt 60-cycle 6-pole induction motor under th

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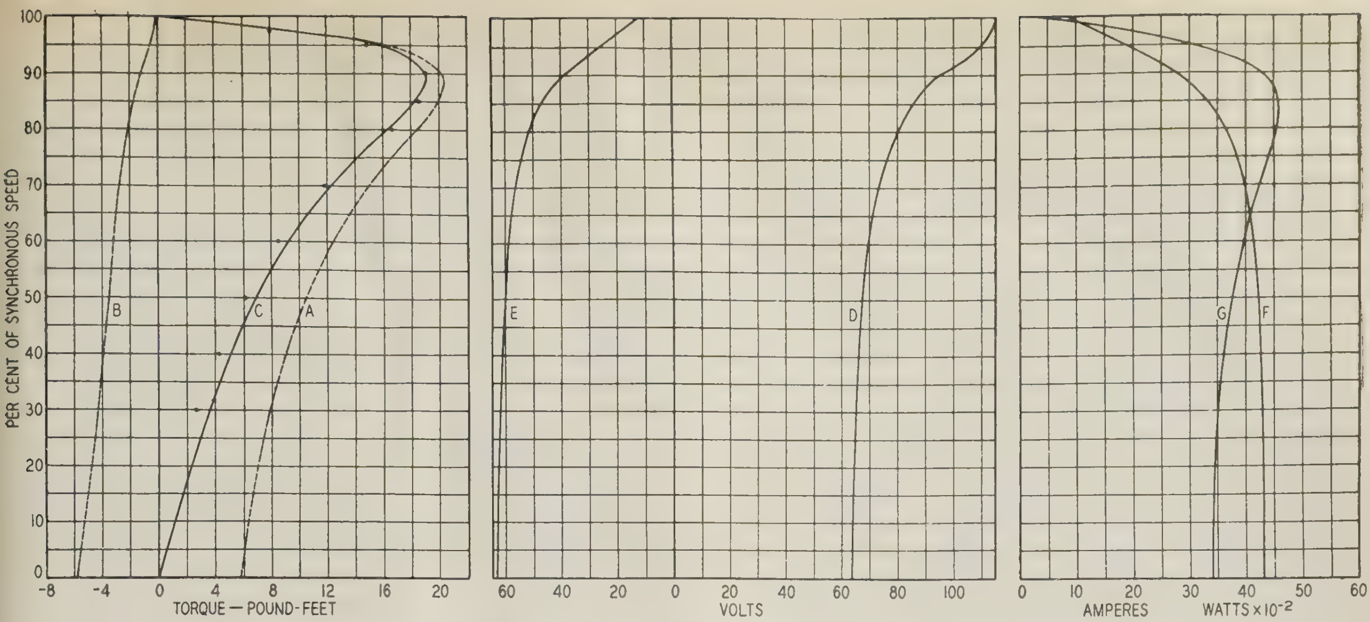
1. For all numbered references see list at end of paper.



Figs. 2–4. Characteristics for 5-horsepower 3-phase motor operating on single-phase supply

A—Positive-sequence torque; *B*—Negative-sequence torque; *C*—Net torque; *D*—Positive-sequence voltage V_{cn1}; *E*—Negative-sequence voltage V_{cn2}; *F*—Current; *G*—Power input

following conditions:

1. Operating single phase as capacitor motor, with fixed capacitance
2. Starting single phase as capacitor motor, with variable capacitance

Three-Phase Motor Operating Single Phase

The single-phase voltage V_{ca} may be assumed to be impressed across 2 phases of the 3-phase machine, as shown in sketch *A*.
Then

$$V_{ca} = V_{cn} - V_{an} \qquad (1)$$

$$V_{cn} = V_{cn1} + V_{cn2} \qquad V_{an} = V_{an1} + V_{an2}$$

but

$$V_{an1} = \alpha^2 V_{cn1} \qquad V_{an2} = \alpha V_{cn2}$$

where

$$\alpha = -0.5 + j0.866 \qquad \alpha^2 = -0.5 - j0.866$$

and so

$$V_{ca} = (1 - \alpha^2)V_{cn1} + (1 - \alpha)V_{cn2} \qquad (2)$$

$$I_{c1} = V_{cn1}/Z_1 \qquad I_{c2} = V_{cn2}/Z_2$$

where Z_1 = positive-sequence impedance of equivalent circuit, and Z_2 = negative-sequence impedance of equivalent circuit.
Since

$$I_b = I_{b1} + I_{b2} = 0$$

$$I_{b1} = \alpha I_{c1} \qquad I_{b2} = \alpha^2 I_{c2}$$

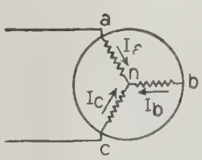
$$\alpha I_{c1} = -\alpha^2 I_{c2}$$

and

$$\alpha V_{cn1}/Z_1 = -\alpha^2 V_{cn2}/Z_2$$

then

$$V_{cn1} = -\alpha \frac{Z_1}{Z_2} V_{cn2}$$



Sketch A

and

$$V_{cn2} = -\alpha^2 \frac{Z_2}{Z_1} V_{cn1}$$

Substituting these values in equation 2 and simplifying

$$V_{ca} = (1 - \alpha^2)(1 + Z_2/Z_1)V_{cn1}$$

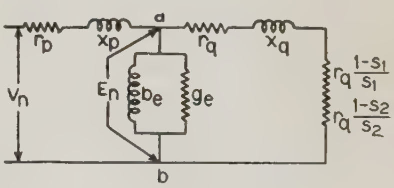
from which

$$V_{cn1} = \frac{V_{ca}}{(1 - \alpha^2)(1 + Z_2/Z_1)} \qquad (3)$$

Similarly,

$$V_{cn2} = \frac{V_{ca}}{(1 - \alpha)(1 + Z_1/Z_2)} \qquad (4)$$

To determine Z_1 and Z_2, the equivalent circuit per phase is as shown in sketch *B*. Meanings of the symbols are: V_n = impressed voltage to neutral, s_1 = slip for positive-sequence voltage, and $s_2 = 2 - s_1$ = slip for negative-sequence voltage.



Sketch B

The apparent conductance of the rotor and load is $g_q = r_q s/(r_q^2 + s^2 x_q^2)$ and the apparent susceptance of the rotor and load is $b_q = x_q s^2/(r_q^2 + s^2 x_q^2)$. The conductance and susceptance of the parallel branch *ab* is $g_{ab} = g_e + g_q$ and $b_{ab} = b_e + b_q$.

The positive-sequence impedance of the entire circuit $Z_1 = (r_p + r_{ab}) + j(x_p + x_{ab})$ is obtained by substituting s_1 in the foregoing equations. The negative-sequence impedance Z_2 of the entire circuit

is obtained by substituting s_2 in the equations. The equivalent resistance of the branch ab is r_{ab} and the equivalent reactance is x_{ab}. These impedances are used to determine the positive- and negative-sequence voltages V_{cn1} and V_{cn2}.

The internal torque developed by the positive-sequence voltage is

$$T_{d1} = \frac{550}{746} \cdot \frac{p}{4\pi f} \cdot |E^2_{cn1}| \cdot g_q \text{ pound-feet per phase}$$

in which

p = number of poles
f = frequency of applied voltage
$|E_{cn1}| = |V_{cn1}| / \sqrt{M^2 + N^2}$

where

$$M = 1 + g_{ab}r_p + b_{ab}x_p \qquad N = b_{ab}r_p - g_{ab}x_p$$

Similarly, the torque T_{d2} developed by the negative-sequence voltage may be found.

The net torque $T_d = T_{d1} - T_{d2}$ pound-feet per phase.

$$I_c = I_{c1} + I_{c2} = V_{cn1}/Z_1 + V_{cn2}/Z_2 = V_{ca}/(Z_1 + Z_2)$$

and power in kilowatts is $|V_{ca}| I_{cr} \times 10^{-3}$ where I_{cr} is the component of I_c in phase with V_{ca}. An illustrative calculation is given in appendix I.

Figures 2, 3, and 4 are for the 5-horsepower 3-phase 220-volt motor. Figure 2 shows the relationship between positive- and negative-sequence torques and net torque and per cent synchronous speed. The circles represent test points which check the calculated curve very closely, especially within the operating range. It may be noted that the negative-sequence torque does not reach zero at synchronous speed as does the positive-sequence torque. Hence a single-phase motor, even though it could be manufactured to have no mechanical losses, could not run at synchronous speed at no load, but would run at that speed at which the positive- and negative-sequence torques were equal.

The relationship between positive- and negative-sequence voltages and per cent synchronous speed is shown in figure 3. Since the negative-sequence voltage is never zero, a 3-phase induction motor cannot be a perfect phase converter in changing from single phase to 3 phase, as the presence of negative-sequence voltage indicates a terminal voltage unbalance.

In figure 4 is shown the relationship between the line current and watts input and per cent synchronous speed.

Three-Phase Motor Operating Single Phase as Capacitor Motor[4]

A diagram for connections as a capacitor motor is shown in sketch C. In the bnc loop

$$V_{bn} - V_{cn} + V_{\text{capacitor}} = 0 \quad V_{bc} = V_{bn} - V_{cn}$$

$$V_{\text{capacitor}} = V_{cb} = -jI_bX_c \tag{5}$$

$$V_{ca} = V_{cn} - V_{an}, \; V_{bn} - V_{cn} - jI_bX_c = 0 \; V_{ab} = V_{an} - V_{bn}$$

Referring to sketch D,

$$V_{cn1} = V_{ca1}j\alpha^2/\sqrt{3} \tag{6}$$

$$V_{cn2} = -V_{ca2}j\alpha/\sqrt{3} \tag{7}$$

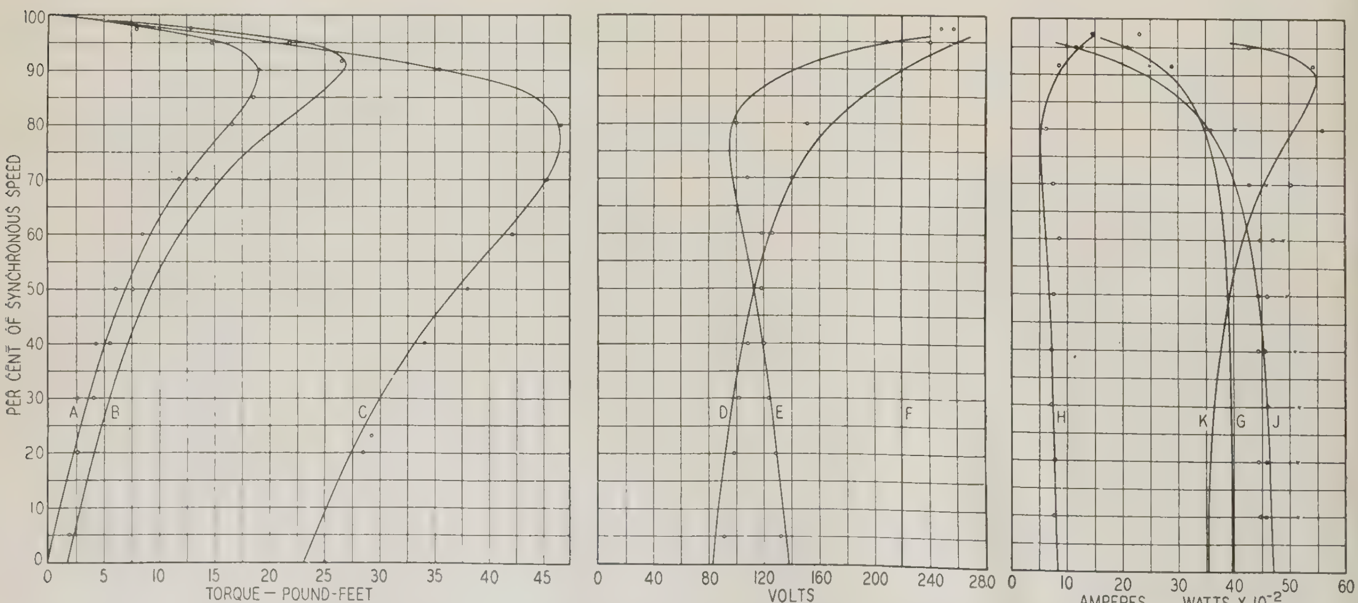
but

$$V_{ca1} = \frac{1}{3}(V_{ca} + \alpha^2 V_{bc} + \alpha V_{ab}) \tag{8}$$

and

$$V_{ca2} = \frac{1}{3}(V_{ca} + \alpha V_{bc} + \alpha^2 V_{ab}) \tag{9}$$

$$V_{ab} = -V_{bc} - V_{ca} = -V_{ca} - jI_bX_c \tag{10}$$



Figs. 5–7. Characteristics for 5-horsepower 3-phase motor operating on single-phase supply as capacitor motor

A—Torque, single phase; B—Torque, single phase with capacitor; C—Torque, 3 phase; D—Terminal voltage V_{ab}; E—Terminal voltage V_{bc}; F—Terminal voltage V_{ca}; G—Current I_a; H—Current I_b; J—Current I_c; K—Power input

By substituting equations 5 and 10 in equation 8, then in equation 6,

$$V_{cn1} = \frac{j\alpha^2}{3\sqrt{3}}(V_{ca} + j\alpha^2 I_b X_c - \alpha V_{ca} - j\alpha I_b X_c)$$

$$= \frac{1}{3}(-\alpha V_{ca} + j\alpha^2 I_b X_c) \qquad (11)$$

Similarly

$$V_{cn2} = \frac{-j\alpha}{3\sqrt{3}}(V_{ca} + j\alpha I_b X_c - \alpha^2 V_{ca} - j\alpha^2 I_b X_c)$$

$$= \frac{1}{3}(-\alpha^2 V_{ca} + j\alpha I_b X_c) \qquad (12)$$

$$I_{c1} = V_{cn1}/Z_1 = V_{cn1}Y_1 = V_{cn1}(G_1 - jB_1)$$
$$I_{c2} = V_{cn2}/Z_2 = V_{cn2}Y_2 = V_{cn2}(G_2 - jB_2)$$
$$I_{b1} = \alpha I_{c1} = \alpha V_{cn1}(G_1 - jB_1) \quad I_{b2} = \alpha^2 I_{c2} = \alpha^2 V_{cn2}(G_2 - jB_2)$$
$$I_b = I_{b1} + I_{b2} = \alpha V_{cn1}(G_1 - jB_1) + \alpha^2 V_{cn2}(G_2 - jB_2)$$
$$= \frac{\alpha}{3}(-\alpha V_{ca} + j\alpha^2 I_b X_c)(G_1 - jB_1) + \frac{\alpha}{3}(-\alpha^2 V_{ca} + j\alpha I_b X_c)(G_2 - jB_2)$$

from which

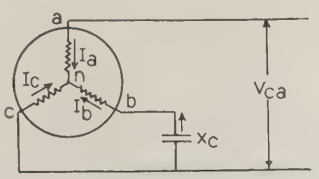
$$I_b = \frac{-V_{ca}[\alpha^2(G_1 - jB_1) + \alpha(G_2 - jB_2)]}{3 - jX_c[(G_1 + G_2) - j(B_1 + B_2)]} \qquad (13)$$

By substituting equation 13 into equations 11 and 12 V_{cn1} and V_{cn2} are obtained. These voltages are applied to the equivalent circuit and the torque calculated. An illustrative calculation is given in appendix II.

$$I_c = I_{c1} + I_{c2} = V_{cn1}(G_1 - jB_1) + V_{cn2}(G_2 - jB_2)$$
$$I_a = -I_b - I_c$$
$$V_{bc} = jI_b X_c \quad \text{and} \quad V_{ab} = -V_{bc} - V_{ca}$$
$$\text{Watts input} = |V_{ca}|\, I_{ar}$$

where I_{ar} is the component of I_a in phase with V_{ca}.

Figure 5 is a comparison of the speed-torque curves of the 5-horsepower 3-phase 220-volt motor operating 3 phase, single phase, and single phase as a capacitor motor with a 160-microfarad capacitor. The test points closely check the calculated curves especially above the breakdown points. The reason for the curve for operation as a capacitor motor rising above the curve for 3-phase operation is that the terminal voltages are higher for the former as may be seen from figure 6, which shows the relationship between terminal voltages and per cent synchronous speed for the 3-phase motor operating as a capacitor motor with 160 microfarads of capacitance.



Sketch C

The relationship between phase currents and watts input and per cent synchronous speed is shown in figure 7. The test points follow the calculated points for the capacitor current, but, although following the general trend for the other 2 phase currents, show some deviation. This is likely due to decreased reactance because of saturation effects which the method does not take into account.

Three-Phase Motor Starting Single Phase as Capacitor Motor

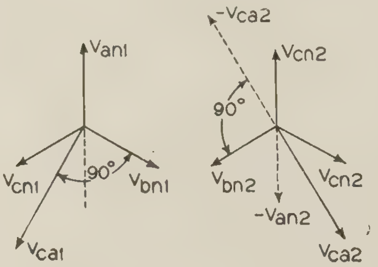
At standstill, $Z_1 = Z_2$ and $Y_1 = Y_2$, in which case equation 13 for I_b reduces to

$$I_b = \frac{V_{ca}(G_1 - jB_1)}{3 - 2X_c(B_1 + jG_1)}$$

and

$$I_c = (V_{cn1} + V_{cn2})(G_1 - jB_1)$$

By obtaining the starting torque with balanced normal 3-phase voltage applied, the starting torque resulting from the positive-sequence voltage may be obtained from the ratio of the squares of the 2 voltages. The torque resulting from the negative-sequence voltage is also obtained from the ratio of the squares of the 2 voltages. The net starting torque is the difference between the 2 torques:



Sketch D

$$T_s = \frac{|V_{cn1}^2| - |V_{cn2}^2|}{V_n^2} T_3 \text{ pound-feet}$$

where T_3 is the torque obtained with the application of balanced 3-phase voltage V_n to neutral.

Figures 8, 9, and 10 show starting torque and watts input, phase currents, and terminal voltages plotted against capacity in microfarads for the $^3/_4$-horsepower 3-phase motor starting on 224 volts single phase as a capacitor motor. The maximum starting torque is 3.82 pound-feet with 240 microfarads of capacitance and with an average terminal voltage of 214 volts, which is practically the rated value. At this capacitance the average phase current is 17.2 amperes and the power input is 3,800 watts. Balanced 3-phase starting torque is 4.27 pound-feet, current is 17.9 amperes, power input is 4,080 watts, and the power factor is 58.7 per cent. Were the power factor 50 per cent, 3-phase starting conditions could be duplicated exactly. The circles in the figures represent test points.

Two-Phase Motor Operating Single Phase as Capacitor Motor

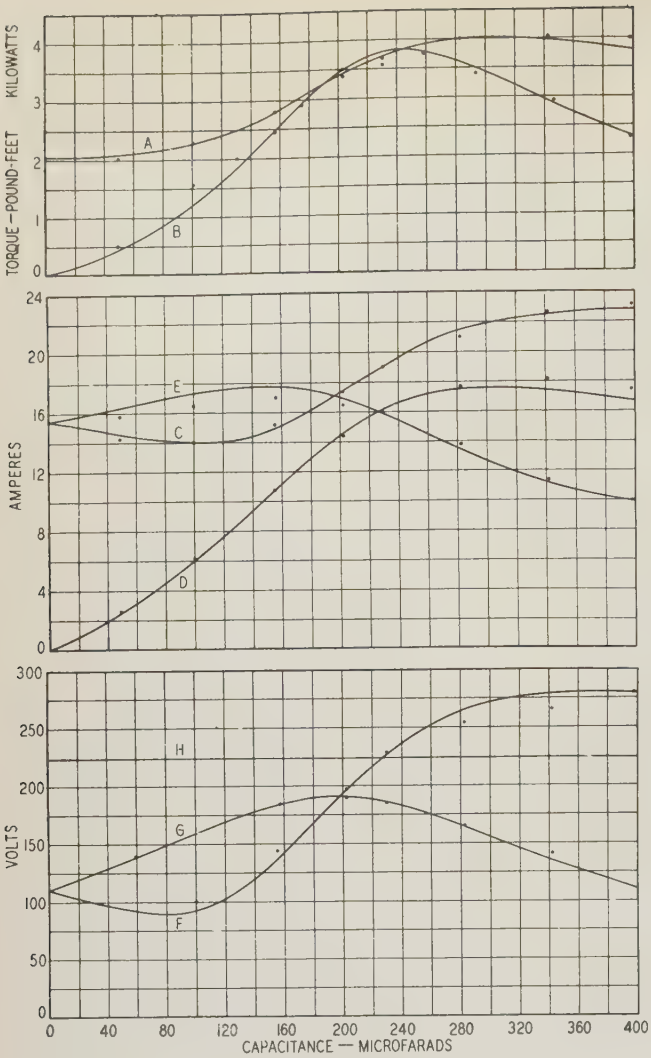
From the diagram shown in sketch E it may be seen that

$$V_{bn} = -jI_a X_c + Z_1 I_{a1} + Z_2 I_{a2} = I_{b1} + I_{b2} Z_2 \qquad (14)$$

where Z_1 = positive-sequence impedance of equivalent circuit, and Z_2 = negative-sequence impedance of equivalent circuit.

Since

$$I_a = I_{a1} + I_{a2} \qquad I_{a1} = jI_{b1} \qquad I_{a2} = -jI_{b2} \qquad (15)$$



Figs. 8–10. Characteristics for 3/4-horsepower 3-phase motor starting on single-phase supply as capacitor motor

A—Watts input
B—Torque
C—Current I_a
D—Current I_b
E—Current I_c
F—Terminal voltage V_{ab}
G—Terminal voltage V_{bc}
H—Terminal voltage V_{ca}

substitution of equation 15 in equation 14 gives

$$I_{b1}[X_c + (j-1)Z_1] = I_{b2}[X_c + (j+1)Z_2]$$

and

$$V_{bn} = I_{b1}Z_1 + I_{b1}Z_2\frac{X_c + (j-1)Z_1}{X_c + (j+1)Z_2}$$

Therefore

$$I_{b1} = \frac{V_{bn}[X_c + (j+1)Z_2]}{X_c(Z_1 + Z_2) + 2jZ_1Z_2} \qquad V_{bn1} = \frac{V_{bn}Z_1[X_c + (j+1)Z_2]}{X_c(Z_1 + Z_2) + 2jZ_1Z_2}$$

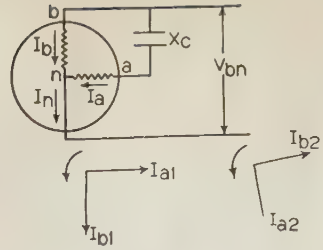
Similarly

$$I_{b2} = \frac{V_{bn}[X_c + (j-1)Z_1]}{X_c(Z_1 + Z_2) + 2jZ_1Z_2} \qquad V_{bn2} = \frac{V_{bn}Z_2[X_c + (j-1)Z_1]}{X_c(Z_1 + Z_2) + 2jZ_1Z_2}$$

By applying the voltages V_{bn1} and V_{bn2} to the equivalent circuit the torque is calculated.

$$I_b = I_{b1} + I_{b2} \qquad I_a = I_{a1} + I_{a2} = j(I_{b1} - I_{b2}) \qquad I_n = I_a + I_b$$

$$V_{ab} = jI_aX_c \qquad V_{an} = V_{bn} + jI_aX_c$$



Sketch E

An illustrative calculation is given in appendix III.

The speed-torque curves of the 2-horsepower 2-phase 220 volt motor operating 2 phase and single phase as a capacitor motor with a 160-microfarad capacitor are shown in figure 11. The dotted curve, shown with the test points, closely checks the calculated curve. The curve for capacitor operation rises above the curve for 2-phase operation within the operating range because of the high phase

Fig. 11. Speed-torque characteristics for 2-horsepower 2-phase motor

A—Operating as capacitor motor on single-phase supply

B—Operating on 2-phase supply

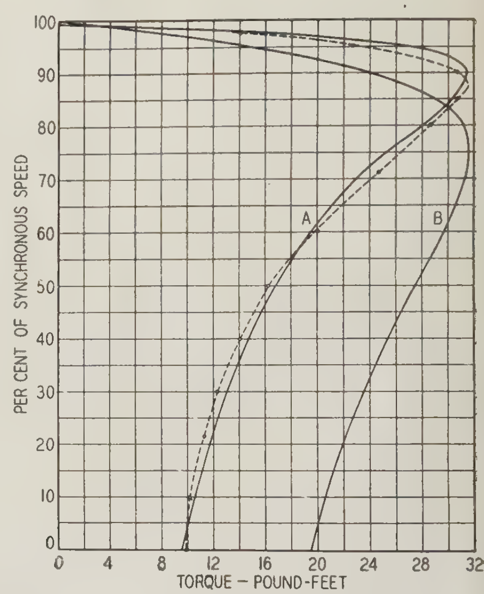
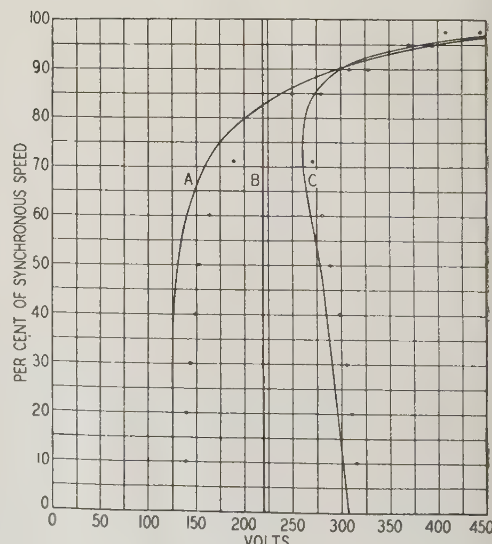


Fig. 12. Terminal voltages for 2-horsepower 2-phase motor operating on single-phase supply as capacitor motor

A—V_{an}
B—V_{bn}
C—V_{ab}



voltages within this range, as may be seen from figure 12. This figure shows the relationship between phase and terminal voltages and per cent synchronous speed for the 2-phase motor operating as a

capacitor motor with 160 microfarads of capacitance. The circles in the figure represent test points.

Two-Phase Motor
Starting Single Phase as Capacitor Motor

The method as given in the preceding is applicable to the starting conditions but the following is a much simpler solution for the starting case:

$$V_{an} = V_{an1} + V_{an2} \tag{16}$$

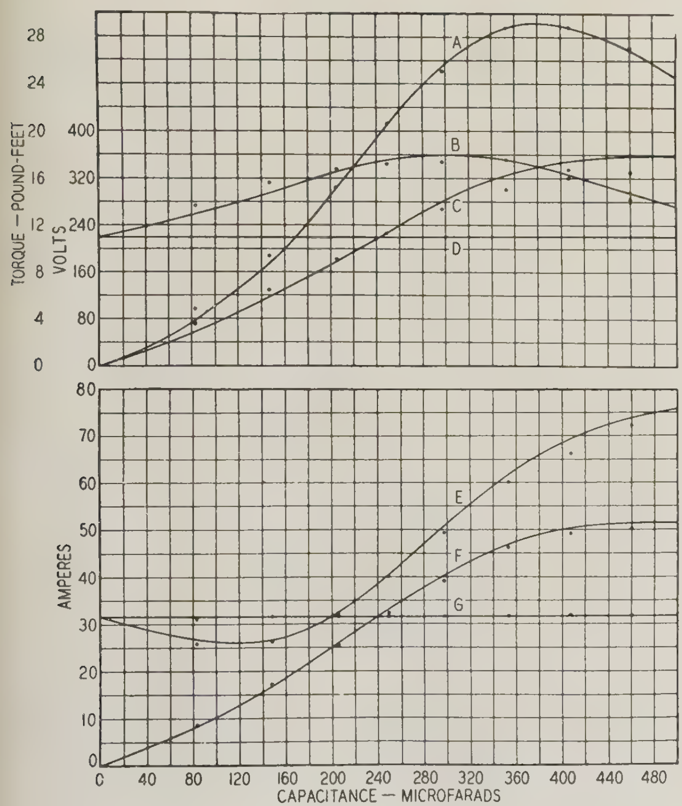
$$V_{bn} = V_{bn1} + V_{bn2} \tag{17}$$

$$= -jV_{an1} + jV_{an2} \tag{18}$$

Multiplying equation 16 by $-j$ and adding to equation 18,

$$-jV_{an} + V_{bn} = -jV_{an1} - jV_{an2} - jV_{an1} + jV_{an2}$$

$$V_{an1} = \frac{1}{2}(V_{an} + jV_{bn}) \tag{19}$$



Figs. 13 and 14. Characteristics for 2-horsepower 2-phase motor starting on single-phase supply as capacitor motor

A—Torque
B—Terminal voltage V_{ab}
C—Terminal voltage V_{an}
D—Terminal voltage V_{bn}
E—Current I_n
F—Current I_a
G—Current I_b

Multiplying equation 16 by j and adding to equation 18,

$$jV_{an} + V_{bn} = jV_{an1} + jV_{an2} - jV_{an1} + jV_{an2}$$

$$V_{an2} = \frac{1}{2}(V_{an} - jV_{bn}) \tag{20}$$

$$I_a = V_{bn}/(Z_a - jX_c)$$

where Z_a is the phase impedance of the windings.

$$V_{an} = I_aZ_a = Z_aV_{bn}/(Z_a' - jX_c)$$

$$I_b = V_{bn}/Z_b \qquad \text{and} \qquad I_n = I_a + I_b$$

By obtaining the starting torque with balanced 2-phase voltage applied, the starting torque resulting

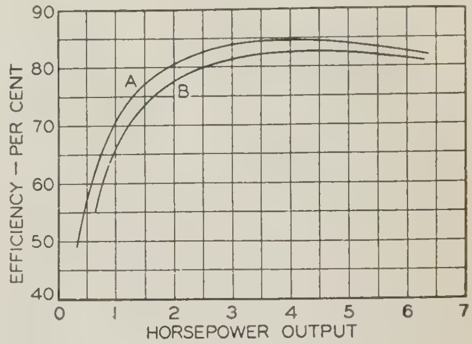


Fig. 15. Efficiency curves for 5-horsepower 3-phase motor operating on 3-phase supply

A—Balanced

B—With 10 per cent degree of unbalance

from the positive-sequence voltage may be obtained from the ratio of the squares of the 2 voltages. The torque resulting from the negative-sequence voltage is also obtained from the ratio of the squares of the 2 voltages. The net starting torque is the difference between the 2 torques.

$$T_s = \frac{|V_{an1}^2| - |V_{an2}^2|}{V_{bn}^2} T_2 \text{ pound-feet}$$

where T_2 is the torque obtained with the application of balanced 2-phase voltage V_{bn} to neutral. An illustrative calculation is given in appendix IV.

Figures 13 and 14 show starting torque, voltages, and currents plotted against capacitance in microfarads for the 2-horsepower 2-phase motor starting on 220 volts single phase as a capacitor motor. The maximum starting torque is 29.2 pound-feet at 380 microfarads of capacitance with an average neutral voltage of 279, which accounts for the starting torque being higher than the 2-phase starting torque of 20.0 pound-feet. At 244 microfarads the average neutral voltage is the rated value of 220, the starting torque is 19.8 pound-feet, and the average phase current is 31.8 amperes. Balanced 2-phase starting current is 31.5 amperes, and the power factor is 61.3 per cent. Were the power factor 70.7 per cent, 2-phase starting conditions could be duplicated exactly. The circles in the figures represent test points.

Three-Phase Motor
Operating on Unbalanced 3-Phase Supply

The positive- and negative-sequence voltages to neutral are determined and impressed across the equivalent circuit. The torques resulting from these sequence voltages are determined as previously outlined. An illustrative calculation is given in appendix V.

Since the net torque varies but slightly from the torque obtained with balanced 3-phase voltage ap-

plied, the results of the series of calculations outlined in appendix V are given in the following tabulation. Figure 5 shows the torque versus per cent synchronous speed; the circles represent test points.

Per Cent of Synchronous Speed	Three-Phase Torque, Pound-Feet	Unbalanced 3-Phase Torque, Pound-Feet
100	0.00	−0.13
90	36.00	35.62
80	46.41	45.97
70	45.34	44.97
60	41.33	40.88
50	36.97	36.60
40	33.23	32.85
30	30.05	29.70
20	27.42	27.03
10	25.12	24.73
0	23.22	22.82

Because there is such a small difference in the torques developed under the 2 conditions of operation a method for determining the efficiencies within the operating range will be given.

Inasmuch as the torque delivered to the load is the difference between the torques developed by the positive- and the negative-sequence voltages while the power input is the sum of the powers resulting from the positive- and the negative-sequence voltages it follows that the efficiency of the motor is much lower when operating on unbalanced voltages than when operating on balanced 3-phase voltages.

The power developed at the pulley by the positive-sequence voltage is

$$P_{p1} = |E_{n1}^2|\, g_q(1 - s_1) - \text{(windage and friction per phase)} \quad \text{watts per phase}$$

that developed by the negative-sequence voltage is

$$P_{p2} = |E_{n2}^2|\, g_q(1 - s_2) \text{ watts per phase}$$

and

$$P_p = P_{p1} + P_{p2} \text{ watts per phase}$$

The power input for the positive-sequence voltage is

$$P_{i1} = |E_{n1}^2|\,(Mg_{ab} + Nb_{ab}) \text{ watts per phase}$$

and that for the negative-sequence voltage is

$$P_{i2} = |E_{n2}^2|\,(Mg_{ab} + Nb_{ab}) \text{ watts per phase}$$

where g_q, M, g_{ab}, N, and b_{ab} are the respective values for the positive and negative sequences.

$$P_i = P_{i1} + P_{i2}$$

The efficiency is the ratio of P_p to P_i. The former in terms of the torques developed is given by

$$P_p = 2\pi n(1 - s_1)(T_1 - T_2) \times 746/33{,}000 \text{ watts}$$

where T_1 and T_2 are the torques developed by the positive and negative sequences, respectively

n = synchronous rpm

$$T_1 = \frac{K}{1 - s_1}\left[|E_{n1}^2|\, g_q(1 - s_1) - \text{(windage and friction per phase)}\right] \quad \text{pound-feet per phase}$$

$$T_2 = \frac{K}{1 - s_2}\left[|E_{n2}^2|\, g_q(1 - s_2)\right] \text{ pound-feet per phase}$$

and

$$K = \frac{550}{746} \times \frac{p}{4\pi f}$$

The relationship between efficiency and horsepower output for the 5-horsepower 3-phase motor is shown in figure 15.

Appendix I—Three-Phase Motor Operating Single Phase

Five-horsepower 3-phase 220-volt 60-cycle 6-pole induction motor

r_p = 0.383 ohm $\quad r_q$ = 0.530 ohm (ohmic) $\quad g_e$ = 0.0026 mh

$x_p = x_q$ = 1.207 ohm $\quad r_q$ = 0.585 ohm (effective)

$V_{ca} = 220 + j0$ $\quad b_e$ = 0.0436 mh

	Positive Sequence	Negative Sequence		Positive Sequence	Negative Sequence
s_1	0.10		Z_1	$4.94 + j3.32$	
s_2		1.90	Z_2		$0.64 + j2.3$
r_q	0.535	0.535	$\lvert V_{cn1}\rvert$	95.06	
g_q	0.1779	0.1833	$\lvert V_{cn2}\rvert$		38.86
b_q	0.0401	0.786	M	1.170	2.073
g_{ab}	0.1805	0.186	N	−0.186	0.0934
b_{ab}	0.0837	0.830	$\lvert E_{n1}\rvert$	80.22	
r_{ab}	4.560	0.257	$\lvert E_{n2}\rvert$		18.73
x_{ab}	2.114	1.147	$3 \times T_{d1}$	20.16	
			$3 \times T_{d2}$		1.13

$$3 \times T_d = 19.03 \text{ pound-feet}$$
$$I_c = 220/(5.58 + j5.67) = 19.4 - j19.7$$
$$\text{Watts} = 220 \times 19.4 = 4{,}270$$

Appendix II—Three-Phase Motor Operating Single Phase as Capacitor Motor

Same motor as in appendix I, C = 160 microfarads, $s_1 = 0.10$, $V_{ca} = 220 + j0$

From appendix I: $Z_1 = 4.94 + j3.32$ and $Z_2 = 0.64 + j2.35$

Then $Y_1 = 0.1392 - j0.0935$ and $Y_2 = 0.108 - j0.396$

$$I_b = \frac{-220[\alpha^2(0.1392 - j0.0935) + \alpha(0.108 - j0.396)]}{3 - j16.6(0.2472 - j0.4895)} = 8.17 + j2.79$$

$|I_b|$ = 8.64 amperes

$$V_{cn1} = \frac{1}{3}[-\alpha \cdot 220 + j\alpha^2(8.17 + j2.79) \cdot 16.6] = 83.6 - j72.75$$

$|V_{cn1}|$ = 110.8 volts

$$V_{cn2} = \frac{1}{3}[-\alpha^2 \cdot 220 + j\alpha(8.17 + j2.79) \cdot 16.6] = 5.25 + j27.5$$

$|V_{cn2}|$ = 28.0 volts

By applying these voltages to the equivalent circuit and calculating the torque,

T_{d1} = 27.4 pound-feet

T_{d2} = 0.59 pound-feet

T_d = 26.81 pound-feet

$I_{c1} = (83.6 - j72.75)(0.1392 - j0.0935) = 4.86 - j17.96$

$I_{c2} = (5.25 + j27.5)(0.108 - j0.396) = 11.47 + j0.89$

$I_c = 16.33 - j17.07$ $\quad |I_c|$ = 23.6 amperes

$I_a = -24.50 + j14.28$ $\quad |I_a|$ = 28.35 amperes

$V_{bc} = -46.3 + j135.8$ $\quad |V_{bc}|$ = 143.3 volts

$V_{ab} = -173.7 - j135.8$ $\quad |V_{ab}|$ = 220.2 volts

Watts = 220 × 24.50 = 5,400

Appendix III—Two-Phase Motor Operating Single Phase as Capacitor Motor

Two-horsepower 2-phase 220-volt 60-cycle 6-pole induction motor

r_p = 2.56 ohms $\quad x_p$ = 2.75 ohms $\quad g_e$ = 0.00079 mho
r_q = 1.45 ohms (ohmic) $\quad x_q$ = 2.76 ohms $\quad b_e$ = 0.0122 mho
r_q = 1.72 ohms (effective) $\quad C$ = 160 microfarad $\quad V_{bn} = 220 + j0$

	Positive Sequence	Negative Sequence		Positive Sequence	Negative Sequence
s_1	0.10		M	1.237	2.199
s_2		1.90	N	−0.120	0.627
r_q	1.477	1.477	I_{b1}	$21.58 - j8.72$	
g_q	0.0655	0.0944	I_{b2}		$4.34 + j8.05$
b_q	0.0122	0.335	$\lvert V_{bn1} \rvert$	254.2	
g_{ab}	0.0663	0.0952	$\lvert V_{bn2} \rvert$		58.2
b_{ab}	0.0244	0.347	$\lvert E_{bn1} \rvert$	205.0	
r_{ab}	13.25	0.735	$\lvert E_{bn2} \rvert$		25.5
x_{ab}	4.88	2.68	$2 \times T_{d1}$	32.2	
Z_1	$15.81 + j7.63$		$2 \times T_{d2}$		0.72
Z_2		$3.30 + j5.43$			

$$2 \times T_d = 31.48$$

$I_b = 15.92 - j0.67 \qquad |I_b| = 15.93$ amperes
$I_a = 16.77 + j7.24 \qquad |I_a| = 18.24$ amperes
$I_n = 32.69 + j6.57 \qquad |I_n| = 33.35$ amperes
$V_{ab} = j(16.77 + j7.24) \times 16.6 = -120 + j278$
$|V_{ab}| = 303$ volts

$V_{an} = 220 + j(16.77 + j7.24) \times 16.6 = 100 + j278$
$|V_{an}| = 295$ volts

Appendix IV—Two-Phase Motor Starting Single-Phase as Capacitor Motor

Same motor as in appendix III, C = 40 microfarads, $Z_a = 4.28 + j5.51$, $V_{bn} = 220 + j0$

$$V_{an} = \frac{(4.28 + j5.51) \times 220}{4.28 + j5.51 - j66.3} = -18.75 + j16.8$$

$$V_{an1} = \frac{1}{2}(-18.75 + j16.8 + j220) = -9.38 + j118.4$$

$|V_{an1}| = 118.9$ volts

$$V_{an2} = \frac{1}{2}(-18.75 + j16.8 - j220) = -9.38 - j101.6$$

$|V_{an2}| = 102.0$ volts

$$T_s = \frac{118.9^2 - 102.0^2}{218.5^2} \times 19.75 = 1.54 \text{ pound-feet}$$

where T_2 = 19.75 pound-feet with 2-phase voltage of 218.5 volts applied.

$I_a = 220/(4.28 - j60.8) = 0.253 + j3.60 \qquad |I_a| = 3.605$ amperes
$I_b = 220/(4.28 + j5.51) = 19.31 - j24.9 \qquad |I_b| = 31.5$ amperes
$I_n = 19.56 - j21.3 \qquad |I_n| = 28.9$ amperes
$|V_{ab}| = |I_a| X_c = 3.605 \times 66.3 = 239.0$ volts

Appendix V—Three-Phase Motor Operating 3 Phase With 10 Per Cent Degree of Unbalance

Same motor as in appendix I. The terminal voltages are

$|V_{ac}| = 200$ volts $\qquad |V_{cb}| = 220$ volts $\qquad |V_{ba}| = 240$ volts

With V_{cb} as reference the phase voltages are
$V_{na} = -26.7 + j124.8$
$V_{nb} = 123.3 - j62.4$
$V_{nc} = -96.7 - j62.4$

The positive- and negative- sequence voltages to neutral are

$$V_{na1} = \frac{1}{3}(V_{na} + \alpha V_{nb} + \alpha^2 V_{nc}) = -13.33 + j125.9$$

$|V_{na1}| = 126.60$ volts

$$V_{na2} = \frac{1}{3}(V_{na} + \alpha^2 V_{nb} + \alpha V_{nc}) = -13.33 - j1.11$$

$|V_{na2}| = 13.38$ volts

	Positive Sequence	Negative Sequence		Positive Sequence	Negative Sequence
s_1	0.10		x_{ab}	2.114	1.147
s_2		1.90	M	1.170	2.073
g_q	0.1779	0.1833	N	−0.186	0.0934
b_q	0.0401	0.786	$\lvert E_{na1} \rvert$	106.8	
g_{ab}	0.1805	0.186	$\lvert E_{na2} \rvert$		6.45
b_{ab}	0.0837	0.830	$3 \times T_{d1}$	35.75	
r_{ab}	4.560	0.257	$3 \times T_{d2}$		0.134

$$3 \times T_d = 35.62$$

Determination of Efficiency When Operating on Unbalanced 3-Phase Voltages

	Positive Sequence	Negative Sequence		Positive Sequence	Negative Sequence
s_1	0.05		b_{ab}	0.0542	0.8322
s_2		1.95	M	1.102	2.074
g_q	0.0931	0.1776	N	−0.0948	0.1012
b_q	0.0106	0.7886	$\lvert E_{na1} \rvert$	114.8	
g_{ab}	0.0957	0.1802	$\lvert E_{na2} \rvert$		6.44

$P_{p1} = 114.8^2 \times 0.0931 \times 0.95 - 48.7 = 1{,}116.9$ watts per phase

Windage and friction = 48.7 watts per phase

$P_{p2} = 6.44^2 \times 0.1776 \times (-0.95) = -7.0$ watts per phase
$P_{i1} = 114.8^2(1.102 \times 0.0957 - 0.0948 \times 0.0542) = 1{,}320.5$ watts per phase
$P_{i2} = 6.44^2(2.074 \times 0.1802 + 0.1012 \times 0.8322) = 18.99$ watts per phase
$P_p = 1{,}116.9 - 7.0 = 1{,}109.9$ watts per phase
$P_i = 1{,}320.5 + 18.99 = 1{,}339.5$ watts per phase
Efficiency = 1,109.9/1,339.5 = 82.8 per cent
Horsepower = 3 × 1,109.9 /746 = 4.46

or

$$T_1 = \frac{0.00587}{0.95}(114.8^2 \times 0.0931 \times 0.95 - 48.7) = 6.90 \text{ pound-feet per phase}$$

$$T_2 = \frac{0.00587}{-0.95}[6.44^2 \times 0.1776(-0.95)] = 0.04 \text{ pound-feet per phase}$$

$P_p = (6.90 - 0.04) \times 0.95 \times 1200 \times 2\pi \times 746/33{,}000 = 1{,}110$ watts per phase

The efficiency when operating with 220 volts balanced 3 phase at 5 per cent slip is 84.5 per cent.

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Tensor Algebra in Transformer Circuits

The application of tensor algebra to the solution of multiwinding transformer circuits is discussed in this paper for comparison with the methods more commonly used for the solution of such circuits. By means of the applications to transformer circuits given herein, elementary tensor methods are illustrated.

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THE object of this paper is to apply tensor algebra to the solution of the circuits of multiwinding transformers. As a practical means for becoming familiar with elementary tensor methods, such circuits offer variety, and are of interest on their own account. They are unique among static networks in that the ignoration of the magnetizing currents reduces by one the number of simultaneous equations involved per magnetic circuit; and this in turn leads to the concept of "leakage impedances," and to voltage differences rather than absolute voltages in the canonical equations.

It is assumed throughout this paper that the reader is familiar with the methods described in the first 2 articles of the serial "The Application of Tensors to the Analysis of Rotating Electrical Machinery" by Gabriel Kron in the *General Electric Review* (volume 38, 1935, April and May, pages 181–91 and 230–43).

In any network having n branches, there are n branch currents and n branch voltages, or a total of $2n$ variables. To determine all these, a total of $2n$ simultaneous equations would be required, although this does not preclude the possibility of determining certain of the variables from fewer than $2n$ equations. There are 3 common procedures for solving such networks. The steps in these methods will be stated so that a direct comparison can be made in order to show more clearly the position occupied by the tensor method. If it be supposed that the actual circuit contains m independent meshes or loops, the following methods are available:

Method *A*

1. Kirchhoff's current equations, $\Sigma i = 0$ at the junctions, yield $(n - m)$ independent relationships among the currents, and permit the elimination of any $(n - m)$ of them. In most cases, these equations are not actually written, but the identification is made on the connection diagram, which is exactly equivalent to writing the equations.

2. Kirchhoff's voltage equations, $\Sigma V = 0$ for each independent mesh, yield m equations. If these are written in terms of all the branch currents, then it is first necessary to eliminate $(n - m)$ of the currents by the use of the current equations just mentioned. These m equations then contain m unknowns (the mesh currents) and can be solved simultaneously.

3. The branch currents follow from the n equations relating the branch currents to the m currents solved for in paragraph 2, as identified on the connection diagram. Of these n equations $(n - m)$ are those given in paragraph 1.

4. The branch voltages are found from the n equations for the voltage drop of the branches.

Method *B*

This is a convenient variation of the previous method, in which

1. The n branch currents are expressed as a superposition of m loop currents.

2. Kirchhoff's m voltage equations are written in terms of the m loop currents and solved for these loop currents.

3. The n branch currents follow from the first statement.

4. The n branch voltages are given by the equations for the voltage drop of each branch.

Method *C*

This is a much older method, more familiar in dynamics, which is as follows:

1. The n equations of voltage drop for the branches are written in terms of the n branch voltages and the n branch currents. These are absolutely independent of interconnections, and will be called the *canonical equations* of the system.

2. The n branch currents are expressed in terms either of the m loop currents or of m noneliminated branch currents. These equations are called the *current constraints* of the system, and depend entirely on the connections.

3. Kirchhoff's equations for voltage are written for the m loops in terms of the n branch voltages e_k and the applied voltages. These are called the *voltage constraints* of the circuit, and depend entirely on the connections.

4. The n canonical equations of paragraph 1, the $(n - m)$ current constraints of paragraph 2, and the m voltage constraints of paragraph 3 provide the $2n$ simultaneous equations. Of course, the equations of constraint may be used to reduce the canonical equations to identically the same loop equations given in the previous methods, but this is not mandatory.

The advantage of method *A* or method *B* is that the minimum number of equations that must be solved simultaneously are written directly by inspection from the connection diagram, but these equations must be written anew for each change of connections. The advantage of method *C* is that it preserves a rigid distinction between the aspects of the network which are entirely independent of connections, and those which are entirely dependent on connections; however, more simultaneous equations must be handled. Thus, methods *A* and *B* are quicker, whereas method *C* gives a broader point of view.

The tensor method seeks to retain the advantages of each of the previous methods while eliminating

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their disadvantages, and in addition presents certain advantages unique to itself. Its procedure is as follows:

1. The same *canonical equations* as in method *C*, independent of the connections, are compressed into a single tensor equation:

$$e_j = Z_{jk} i^k \tag{1}$$

in which e_j and i^k are the branch voltage and current tensors, and Z_{jk} is the impedance tensor.

2. The *current constraints*, as identified on the connection diagram, are given by a *connection tensor* C_α^j. This tensor serves to express the "old" currents i^k (before interconnection) in terms of the "new" currents i^β (after interconnection) according to the tensor equation

$$i^k = C_\beta^k i^\beta \tag{2}$$

3. It is not necessary to identify separately the *voltage constraints* from the connection diagram, because, by virtue of the invariancy of the power input, the conjugate $(\bar{C}_\alpha^j)$ of the connection tensor gives the "new" voltages in terms of the "old"

$$e_\alpha = \bar{C}_\alpha^j e_j \tag{3}$$

(In all except the last example of this paper, the connection tensor does not contain imaginary quantities and, therefore, does not differ from its conjugate.)

4. By equations 2 and 3 equation 1 reduces to

$$e_\alpha = (\bar{C}_\alpha^j C_\beta^k Z_{jk}) i^\beta = Z_{\alpha\beta} i^\beta \tag{4}$$

which is identical with the Kirchhoff voltage equations of methods *A* and *B*. This is solved for the currents i^β by multiplying both sides of the equation by the inverse of $Z_{\alpha\beta}$. The individual branch currents follow from equation 2, and the branch voltages from equation 1.

Making a critical comparison between the tensor method and the ordinary methods, it is clear that it will be simpler in many cases to write equation 4 directly as Kirchhoff's voltage equations, by inspection of the connection diagram, rather than derive the connection tensor and use it to convert the "old" impedance into the "new" impedance tensor.

Nor does there appear to be any actual space saving in simple problems. Of course, a tensor equation like equation 1 is shorter than the set of n simultaneous equations which it supplants, but it must be remembered that a set of simultaneous equations also can be legitimately written and manipulated in the form

$$e_j = \sum_{k=1}^{n} Z_{jk} i_k \quad (j = 1, 2, \ldots . n)$$

and in any event, when the term Z_{jk} of equation 1 is written out in matrix form it takes up as much room as the set of simultaneous equations.

Basic Definitions

Consider 2 windings a and b of a multiwinding transformer, having turns N_a and N_b, respectively, and select the reference winding of the transformer of N_0 turns. Let

$$n_a = \frac{N_a}{N_0} = \frac{\text{turns on winding } a}{\text{turns on winding } 0}$$

$z_{aa} = R_{aa} + j\omega L_{aa}$ = self-impedance of winding a

$z_{ab} = z_{ba} = j\omega M_{ab}$ = mutual impedance between windings a and b

Then, ignoring the magnetizing current, there are

$$0 = n_a i^a + n_b i^b \tag{5}$$

$$e_a = z_{aa} i^a + z_{ab} i^b = (z_{aa} - \frac{n_a}{n_b} z_{ab}) i^a \tag{6}$$

$$e_b = z_{ba} i^a + z_{bb} i^b = (z_{ba} - \frac{n_a}{n_b} z_{bb}) i^a \tag{7}$$

Multiplying equation 6 by $(1/n_a)$ and equation 7 by $(1/n_b)$ and subtracting,

$$\frac{e_a/n_a - e_b/n_b}{n_a i^a} = \left(\frac{z_{aa}}{n_a^2} + \frac{z_{bb}}{n_b^2} - \frac{2z_{ab}}{n_a n_b}\right) = Z_{a-b} \tag{8}$$

= leakage impedance referred to reference winding 0

It is clear from the construction of equation 8 that the voltages and currents have been reduced to the reference winding, hence the right-hand side of equation 8 is the "leakage impedance of windings a and b referred to the reference winding."

The Canonical Equations

Consider a multiwinding transformer with $n + 1$ windings numbered from 0 to n inclusive, and let winding 0 be selected as the reference winding. Let indices j and k range from 1 to n inclusive. Then, ignoring the magnetizing ampere-turns,

$$0 = i^0 + n_k i^k \tag{9}$$

and hereby

$$e_j = z_{j0} i^0 + z_{jk} i^k = (z_{jk} - n_k z_{j0}) i^k \tag{10}$$

$$e_0 = z_{00} i^0 + z_{0k} i^k = (z_{0k} - n_k z_{00}) i^k \tag{11}$$

Subtracting n_j times equation 11 from equation 10 there is

$$\begin{aligned} e_j - n_j e_0 &= (z_{jk} - n_k z_{j0} - n_j z_{0k} + n_j n_k z_{00}) i^k \\ &= n_j n_k \left[\frac{1}{2} \left(z_{00} + \frac{z_{jj}}{n_j^2} - 2\frac{z_{j0}}{n_j} \right) + \right. \\ &\quad \frac{1}{2} \left(z_{00} + \frac{z_{kk}}{n_k^2} - 2\frac{z_{0k}}{n_k} \right) - \\ &\quad \left. \frac{1}{2} \left(\frac{z_{jj}}{n_j^2} + \frac{z_{kk}}{n_k^2} - 2\frac{z_{jk}}{n_j n^k} \right) \right] i^k \end{aligned} \tag{12}$$

Comparing the terms in round brackets of equation 12 with equation 8 it may be seen that they are the leakage impedances, and the complete expression in square brackets may be recognized as the familiar 3-winding impedances of multiwinding transformer theory. Then equation 12 may be rewritten as

$$e_j - n_j e_0 = Z_{jk} i^k \tag{13}$$

or expressed in terms of per unit quantities based on E and I,

$$\left(\frac{e_j}{E}\right) - n_j \left(\frac{e_0}{E}\right) = \left(\frac{I\, Z_{jk}}{E}\right) \left(\frac{i^k}{I}\right) \tag{14}$$

The strictly tensor method of arriving at equation 13 is longer and more awkward. Equation 9 would be written, in terms of indices α and β ranging from 0 to n inclusive, as

$$n_\alpha i^\alpha = 0 \tag{9a}$$

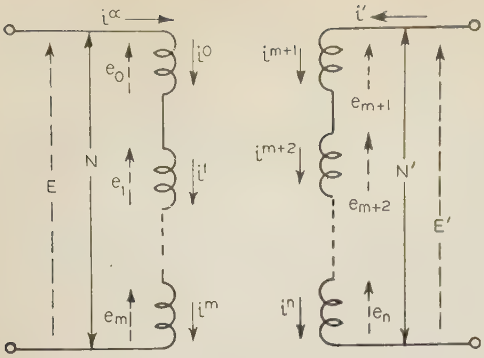

Fig. 1. Diagram of transformer with windings in series

and equations 10 and 11 as

$$e_\alpha = Z_{\alpha\beta} i^\beta \tag{10a}$$

in which

$$Z_{\alpha\beta} = \begin{array}{c|ccccc} \alpha \backslash \beta & 0 & 1 & 2 & \cdots & n \\ \hline 0 & z_{00} & z_{01} & z_{02} & \cdots & z_{0n} \\ 1 & z_{10} & z_{11} & z_{12} & \cdots & z_{01} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ n & z_{n0} & z_{n1} & z_{n2} & \cdots & z_{nn} \end{array} \tag{15}$$

The change of co-ordinates which eliminates the 0 axis is effected by the transformation

$$i^\alpha = C^\alpha_j i^j \tag{16}$$

where

$$C^\alpha_j = \begin{array}{c|ccccc} \alpha \backslash j & 1 & 2 & \cdots & n \\ \hline 0 & -n_1 & -n_2 & \cdots & -n_n \\ 1 & 1 & 0 & \cdots & 0 \\ 2 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ n & 0 & 0 & 0 & 1 \end{array} \tag{17}$$

It is not necessary to express C^α_j as a matrix. In fact, it is preferable to specify it as

$$C^\alpha_j = \begin{cases} -n_j & \text{if} \quad \alpha = 0 \\ \delta^\alpha_j & \text{if} \quad \alpha \neq 0 \end{cases} \tag{17a}$$

in which δ^α_j is the Kronecker delta (equal to zero if $\alpha \neq j$, and equal to unity if $\alpha = j$).

The "new" voltage tensor then is

$$C^\alpha_j e_\alpha = -n_j e_0 + \delta^\alpha_j e_\alpha = -n_j e_0 + e_j \tag{12a}$$

and the transformed impedance tensor becomes

$$\begin{aligned} Z_{jk} &= C^\alpha_j C^\beta_k Z_{\alpha\beta} \\ &= n_j n_k z_{00} - n_j \delta^\beta_k z_{0\beta} - \delta^\alpha_j n_k z_{\alpha 0} + \delta^\alpha_j \delta^\beta_k z_{\alpha\beta} \\ &= n_j n_k z_{00} - n_j z_{0k} - n_k z_{j0} + z_{jk} \end{aligned} \tag{12b}$$

and it may be seen that equations 12*a* and 12*b* agree with equation 12. The superiority of equation 17*a* over equation 17 is evident in arriving at equation 12*b*.

Windings in Series

Figure 1 shows a transformer having windings numbered from 0 to m, of total turns N, in series as a primary, carrying current i^α, and windings $1 + m$ to n of total turns N' in series as a secondary, carrying current i'. Let indices j and k range from 1 to m and indices r and s range from $1 + m$ to n. Then the balance of ampere turns requires that

$$i' = -\frac{N}{N'} i^\alpha = -n i^\alpha \tag{18}$$

The connection tensor which converts the current i^α into the currents i^j and i^r in the individual windings has the components

$$C^j_\alpha = g^j_\alpha \qquad C^r_\alpha = -n g^r_\alpha \tag{19}$$

where $g^j_\alpha = 1$ and $g^r_\alpha = 1$ within the respective ranges of j and r. The "new" reference voltage is

$$\begin{aligned} e_\alpha &= C^j_\alpha (e_j - n_j e_0) + C^r_\alpha (e_r - n_r e_0) \\ &= g^j_\alpha e_j - g^j_\alpha n_j e_0 - n\, g^r_\alpha e_r + n g^r_\alpha n_r e_0 \\ &= (E - e_0) - \left(\frac{N - N_0}{N_0}\right) e_0 - \left(\frac{N}{N'}\right) E' + \frac{N}{N'} N' e_0 \\ &= E - \frac{N}{N'} E' \end{aligned} \tag{20}$$

where E and E' are the total voltages across the primary and secondary, respectively. The "new" impedance then is

$$\begin{aligned} Z_{\alpha\beta} &= C^j_\alpha C^k_\beta Z_{jk} + C^j_\alpha C^s_\beta Z_{js} + C^r_\alpha C^k_\beta Z_{rk} + C^r_\alpha C^s_\beta Z_{rs} \\ &= g^j_\alpha g^k_\beta Z_{jk} - 2n\, g^j_\alpha g^s_\beta Z_{js} + n^2 g^r_\alpha g^s_\beta Z_{rs} \end{aligned} \tag{21}$$

For one winding as primary against 2 in series as secondary,

$$\begin{aligned} Z_{\alpha\beta} &= n^2 g^r_\alpha g^s_\beta Z_{rs} = \frac{N_0^2}{(N_1 + N_2)^2} [Z_{11} + 2Z_{12} + Z_{22}] \\ &= \frac{N_1}{N_1 + N_2} Z_{0-1} + \frac{N_2}{N_1 + N_2} Z_{0-2} - \frac{N_1 N_2}{(N_1 + N_2)^2} Z_{1-2} \end{aligned} \tag{22}$$

Windings in Parallel

Figure 2 shows 2 independent windings 0 and 1 and $n - 2$ windings paralleled with a common load impedance Z_{n+1}. Let indices j and k and indices α and β both range from 1 to n. Then the current i^{n+1} flowing into the load impedance Z_{n+1} is eliminated by the connection tensor whose components are

$$C^j_\alpha = \delta^j_\alpha \qquad C^{n+1}_\alpha = (\delta^1_\alpha - g^{n+1}_\alpha) \tag{23}$$

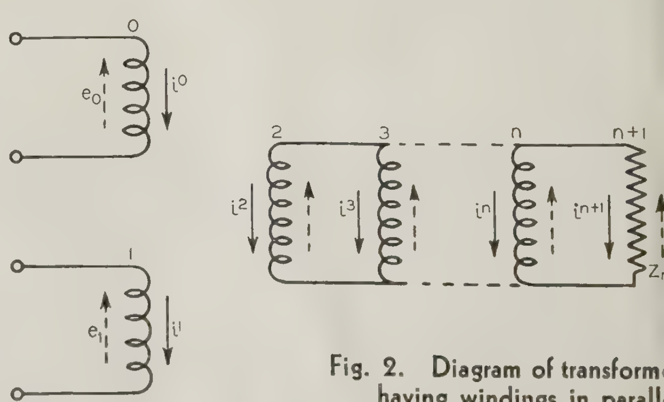

Fig. 2. Diagram of transformer having windings in parallel

The "new" voltages are ($e_2 = \ldots = e_n = e_{n+1}$)

$$e_\alpha = C_\alpha^j(e_j - n_j e_0) + C_\alpha^{n+1} e_{n+1} = \delta_\alpha^j(e_j - n_j e_0) + (\delta_\alpha^1 - g_\alpha^{n+1}) e_{n+1} \tag{24}$$

The "new" impedance tensor is

$$\begin{aligned} Z_{\alpha\beta} &= C_\alpha^j C_\beta^k Z_{jk} + C_\alpha^{n+1} C_\beta^{n+1} Z_{n+1} \\ &= \delta_\alpha^j \delta_\beta^k Z_{jk} + (\delta_\alpha^1 - g_\alpha^{n+1})(\delta_\beta^1 - g_\beta^{n+1}) Z_{n+1} \end{aligned} \tag{25}$$

or arranged as a matrix

$$Z_{\alpha\beta} = \begin{array}{c|cccc} \beta \backslash \alpha & 1 & 2 & \ldots & n \\ \hline 1 & Z_{11} & Z_{12} & \ldots & Z_{1n} \\ 2 & Z_{21} & (Z_{22} + Z_{n+1}) & \ldots & (Z_{2n} + Z_{n+1}) \\ . & \ldots & \ldots & \ldots & \ldots \\ n & Z_{n1} & (Z_{n2} + Z_{n+1}) & \ldots & (Z_{nn} + Z_{n+1}) \end{array} \tag{26}$$

The "new" equations are

$$e_\alpha = Z_{\alpha\beta} i^\beta \tag{27}$$

which can be solved for the currents i^α either by writing out the equations and solving by successive eliminations, or else by taking the inverse $Y^{\alpha\beta}$ of matrix 22 (which is exactly equivalent to solving the equations by determinants):

$$i^\alpha = Y^{\alpha\beta} e_\beta \tag{28}$$

For example, if there are 4 windings ($n = 3$) and the turns of the second and third are equal, $n_2 = n_3 = n$, the equations are, putting $Z_{n+1} = Z$,

$$\left.\begin{aligned} e_1 - n_1 e_0 &= Z_{11} i^1 + Z_{12} i^2 + Z_{13} i^3 \\ -n\ e_0 &= Z_{21} i^1 + (Z_{22} + Z) i^2 + (Z_{23} + Z) i^3 \\ -n\ e_0 &= Z_{31} i^1 + (Z_{32} + Z) i^2 + (Z_{33} + Z) i^3 \end{aligned}\right\} \tag{29}$$

Coupling Windings

A multiwinding transformer with a number of coupling windings is shown in figure 3. It is convenient to number the coupling windings $2k$ and $2k + 1$. The connection tensor which eliminates the currents in the coupling windings $2k + 1$ has the components $k = 1, 2, \ldots n$:

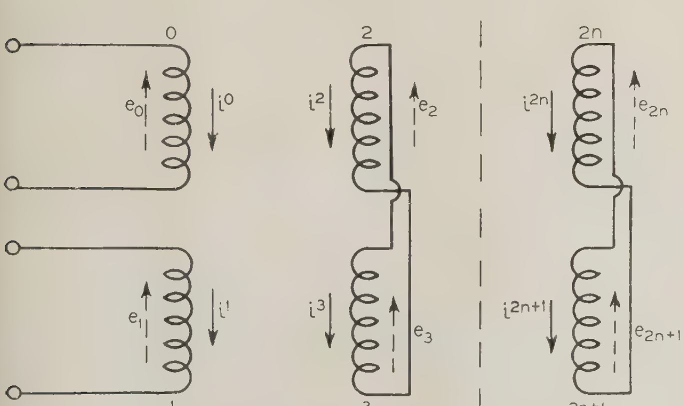

Fig. 3. Diagram of multiwinding transformer with coupling windings

$$C_\alpha^1 = \delta_\alpha^1 \qquad C_\alpha^{2k} = \delta_\alpha^{2k} \qquad C_\alpha^{2k+1} = -\delta_\alpha^{2k} \tag{30}$$

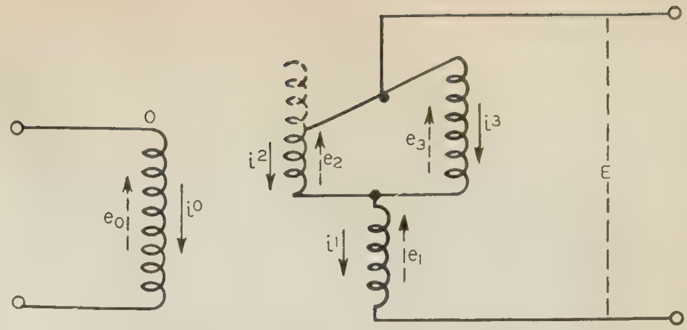

Fig. 4. Diagram of transformer for load ratio control

The "new" voltages are

$$\begin{aligned} e_\alpha &= C_\alpha^1(e_1 - n_1 e_0) + C_\alpha^{2k}(e_{2k} - n_2\, e_0) + C_\alpha^{2k+1}(e_{2k+1} - n_{2k+1} e_0) \\ &= \delta_\alpha^1(e_1 - n_1 e_0) + \delta_\alpha^{2k}(e_{2k} - n_{2k} e_0) - \delta_\alpha^{2k}(e_{2k+1} - n_{2k+1} e_0) \\ &= \delta_\alpha^1(e_1 - n_1 e_0) + \delta_\alpha^{2k}(n_{2k+1} - n_{2k}) e_0 \end{aligned} \tag{31}$$

This is a case in which the connection tensor is not in itself entirely sufficient to obtain the simplest form of the new voltages—it is necessary to observe from the connection diagram that $e_{2k+1} = e_k$. If the turns of the coupled windings are equal, as is usually the case, it is seen to simplify further. The "new" impedance tensor is

$$\begin{aligned} Z_{\alpha\beta} = {} & \delta_\alpha^1 \delta_\beta^1 Z_{11} + (\delta_\alpha^1 \delta_\beta^{2k} + \delta_\alpha^{2k} \delta_\beta^1)(Z_{1,2k} - Z_{1,2k+1}) \\ & + \delta_\alpha^{2j} \delta_\beta^{2k} (Z_{2j,2k} + Z_{2j+1,2k+1} - Z_{2j,2k+1} - Z_{2j+1,2k}) \end{aligned} \tag{32}$$

For example, a transformer with 2 pairs of coupling windings, and equal turns on each winding of a pair, would be represented by the equations

$$\left.\begin{aligned} e_1 - n_1 e_0 &= Z_{11} i^1 + (Z_{12} - Z_{13}) i^2 + (Z_{14} - Z_{15}) i^4 \\ 0 &= (Z_{12} - Z_{13}) i^1 + (Z_{22} + Z_{33} - 2Z_{23}) i^2 \\ &\quad + (Z_{24} + Z_{35} - Z_{25} - Z_{34}) i^4 \\ 0 &= (Z_{14} - Z_{15}) i^1 + (Z_{42} + Z_{53} - Z_{52} - Z_{43}) i^2 \\ &\quad + (Z_{44} + Z_{55} - 2Z_{45}) i^4 \end{aligned}\right\} \tag{33}$$

Load Ratio Control Circuit

The connection diagram for a typical load ratio control transformer with the connections in an off-ratio position is shown in figure 4. The connection tensor for eliminating the common current i' is

$$C_\alpha^j = \begin{array}{c|cc} j \backslash \alpha & 2 & 3 \\ \hline 1 & 1 & 1 \\ 2 & 1 & 0 \\ 3 & 0 & 1 \end{array}$$

The impedance matrix is

$$Z_{jk} = \begin{array}{c|ccc} j \backslash k & 1 & 2 & 3 \\ \hline 1 & Z_{11} & Z_{12} & Z_{13} \\ 2 & Z_{21} & Z_{22} & Z_{23} \\ 3 & Z_{31} & Z_{32} & Z_{33} \end{array}$$

The "new" impedance matrix then is

$$Z_{\alpha\beta} = C_\alpha^j C_\beta^k Z_{jk} = \begin{array}{c|cc} \alpha \backslash \beta & 2 & 3 \\ \hline 2 & Z_{11} + 2Z_{12} + Z_{22} & Z_{11} + Z_{12} + Z_{13} + Z_{23} \\ 3 & Z_{11} + Z_{12} + Z_{23} + Z_{13} & Z_{11} + 2Z_{13} + Z_{33} \end{array}$$

and the voltage matrix (putting $E = e_1 + e_2 = e_1 + e_3$) is

$$e_\alpha = C^j_\alpha(e_j - n_j e_0) = \begin{array}{cc} 2 & 3 \\ \hline \multicolumn{1}{|c|}{E - (n_2 + n_1)e_0} & \multicolumn{1}{c|}{E - (n_3 + n_1)e_0} \\ \hline \end{array} = \begin{array}{cc} 2 & 3 \\ \hline \multicolumn{1}{|c|}{E_2} & \multicolumn{1}{c|}{E_3} \\ \hline \end{array}$$

The admittance matrix found by taking the inverse of $Z_{\alpha\beta}$ is

$$y^{\alpha\beta} = \begin{array}{c|c|c|} \alpha \backslash \beta & 2 & 3 \\ \hline 2 & (Z_{11} + 2Z_{13} + Z_{33})/D & -(Z_{11} + Z_{12} + Z_{13} + Z_{23})/D \\ 3 & -(Z_{11} + Z_{12} + Z_{13} + Z_{23})/D & (Z_{11} + 2Z_{12} + Z_{22})/D \\ \hline \end{array}$$

where

$$D = (Z_{11} + 2Z_{12} + Z_{22})(Z_{11} + 2Z_{13} + Z_{33}) - (Z_{11} + Z_{12} + Z_{13} + Z_{23})^2$$

and finally, the currents are

$$i^\alpha = y^{\alpha\beta}e_\beta = \begin{array}{c|c|} \alpha & \\ \hline 2 & (Z_{11}+2Z_{13}+Z_{33})E_2/D-(Z_{11}+Z_{12}+Z_{13}+Z_{23})E_3/D \\ 3 & -(Z_{11}+Z_{12}+Z_{13}+Z_{23})E_2/D+(Z_{11}+2Z_{12}+Z_{22})E_3/D \\ \hline \end{array}$$

In a simple problem of this nature, the tensor or matrix method does not compare favorably with the ordinary method of solution. It is easier and shorter to identify the circuit constraints and insert them directly into the canonical equations.

Forked Autotransformer

Figure 5 shows the connection diagram for a forked autotransformer. The ampere-turn balance requires that

$$i_0 = -n_1 i^1 - n_2 i^2 = i^1 - i^2$$

Therefore

$$i^1 = \frac{1 - n_2}{1 + n_1} i^2 = \frac{-1 + n_2}{1 + n_1} i^3 = n i^3$$

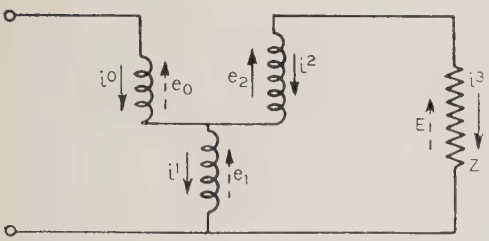

Fig. 5. Diagram of forked autotransformer

The connection matrix which eliminates the current in the common winding and the impedance matrix then are:

$$C^j_\alpha = \begin{array}{c|c|} j \backslash \alpha & 3 \\ \hline 1 & n \\ 2 & -1 \\ 3 & 1 \\ \hline \end{array} \qquad Z_{jk} = \begin{array}{c|c|c|c|} j \backslash k & 1 & 2 & 3 \\ \hline 1 & Z_{11} & Z_{12} & 0 \\ 2 & Z_{21} & Z_{22} & 0 \\ 3 & 0 & 0 & Z \\ \hline \end{array}$$

The "new" impedance matrix then is

$$Z_{\alpha\beta} = C^j_\alpha C^k_\beta Z_{jk} = (Z + Z_{22}) - 2nZ_{12} + n^2 Z_{11}$$

The voltage is

$$e_\alpha = C^j_\alpha(e_j - n_j e_0) = n(e_1 - n_1 e_0) - (e_2 - n_2 e_0) + e_3$$
$$= \left(\frac{n_1 + n_2}{n_1 + 1}\right) e_0 + \left(\frac{n_2 - 1}{n_1 + 1}\right) e_1 - e_2 + e_3$$
$$= \left(\frac{n_1 + n_2}{n_1 + 1}\right)(e_0 + e_1) = \left(\frac{n_1 + n_2}{n_1 + 1}\right) E$$

which is, of course, nothing but the line voltage referred to the load side.

Group of Transformers

The connection diagram for a 3-winding transformer, I, with regulating units consisting of an exciting transformer, II, and a series transformer, III, is shown in figure 6. The current constants are specified by the connection tensor

$$C^j_\alpha = \begin{array}{c|c|} j \backslash \alpha & 3 \\ \hline 1 & n_1' n_1'' \\ 2 & 1 \\ 1' & n_1'' \\ 1'' & 1 \\ 3 & 1 \\ \hline \end{array} \qquad Z_{jk} = \begin{array}{c|c|c|c|c|c|} j \backslash k & 1 & 2 & 1' & 1'' & 3 \\ \hline 1 & Z_{11} & Z_{12} & 0 & 0 & 0 \\ 2 & Z_{21} & Z_{22} & 0 & 0 & 0 \\ 1' & 0 & 0 & Z_{11}' & 0 & 0 \\ 1'' & 0 & 0 & 0 & Z_{11}'' & 0 \\ 3 & 0 & 0 & 0 & 0 & Z \\ \hline \end{array}$$

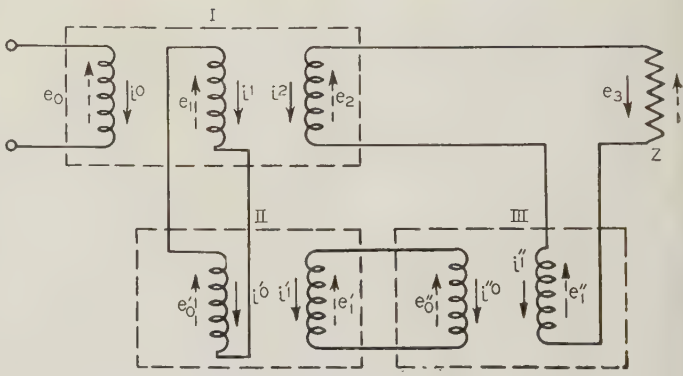

Fig. 6. Diagram of group of transformers

The over-all impedance then is

$$Z_{\alpha\beta} = C^j_\alpha C^k_\beta Z_{jk} = (n_1'n_1'')^2 Z_{11} + 2n_1'n_1''Z_{21} + Z_{22} + (n_1'')^2 Z_{11}' + Z_{11}'' + Z$$

and the "new" voltage is

$$e_\alpha = C^j_\alpha(e_j - n_j e_0)$$
$$= C^j_\alpha \begin{array}{ccccc} 1 & 2 & 1' & 1'' & 3 \\ \hline \multicolumn{1}{|c|}{e_1 - n_1 e_0} & \multicolumn{1}{c|}{e_2 - n_2 e_0} & \multicolumn{1}{c|}{e_1' - n_1' e_0'} & \multicolumn{1}{c|}{e_1'' - n_1'' e_0''} & \multicolumn{1}{c|}{-(e_2 + e_1'')} \\ \hline \end{array}$$
$$= -(n_1 n_1' n_1'' + n_2)e_1 + n_1'n_1''(e_1 - e_0') + n_1''(e_1' - e_0'')$$
$$= -(n_1 n_1' n_1'' + n_2)e_1$$

Wye-Delta-Zigzag Connection

Figure 7 gives the connection diagram for a star-delta-zigzag transformer with a load on the zigzag winding. There are 4 windings on each leg, but each "electrical" phase involves, through the zigzag, 2 "magnetic" phases (legs). This is typical of all phase-shifting connections, and requires some care in

setting up the transformation tensor. Let 1, a, and a^2 be the 3 roots of unity. Then

$$i^1 = -i''^2 = -a^2 i^2$$

or

$$i^2 = -a i^1 = a i^4$$

The transformation matrix and its conjugate for the "magnetic" phase, therefore, are

$$C^j_\alpha = \begin{array}{c|cc|} j \diagdown \alpha & 3 & 4 \\ \hline 1 & 0 & -1 \\ 2 & 0 & a \\ 3 & 1 & 0 \\ 4 & 0 & 1 \\ \hline \end{array} \qquad \bar{C}^j_\alpha = \begin{array}{c|cc|} j \diagdown \alpha & 3 & 4 \\ \hline 1 & 0 & -1 \\ 2 & 0 & a^2 \\ 3 & 1 & 0 \\ 4 & 0 & 1 \\ \hline \end{array}$$

If this be multiplied by the voltage vectors on the same magnetic phase there results (putting $e_4 = e_1 + e_2'' = e_1 - a^2 e_2$)

$$\bar{C}^j_\alpha = \begin{array}{|c|c|c|c|} \multicolumn{1}{c}{1} & \multicolumn{1}{c}{2} & \multicolumn{1}{c}{3} & \multicolumn{1}{c}{4} \\ \hline e_1 - n_1 e_0 & e_2 - n_2 e_0 & e_3 - n_3 e_0 & e_1 - a^2 e_2 \\ \hline \end{array}$$

$$= \begin{array}{|c|c|} \multicolumn{1}{c}{3} & \multicolumn{1}{c}{4} \\ \hline e_3 - n_3 e_0 & (n_1 - a^2 n_2) e_0 \\ \hline \end{array}$$

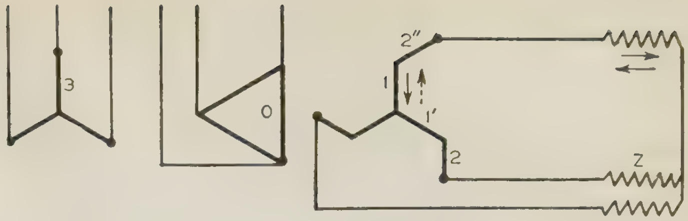

Fig. 7. Diagram of star-delta-zigzag transformer

The "old" impedance matrix is

$$Z_{jk} = \begin{array}{c|cccc|} j \diagdown k & 1 & 2 & 3 & 4 \\ \hline 1 & Z_{11} & Z_{12} & Z_{13} & 0 \\ 2 & Z_{21} & Z_{22} & Z_{23} & 0 \\ 3 & Z_{31} & Z_{32} & Z_{33} & 0 \\ 4 & 0 & 0 & 0 & Z \\ \hline \end{array}$$

and the "new" impedance matrix becomes

$$Z_{\alpha\beta} = C^j_\alpha \bar{C}^k_\beta Z_{jk} = \begin{array}{c|cc|} \alpha \diagdown \beta & 3 & 4 \\ \hline 3 & Z_{33} & -Z_{31} + a Z_{32} \\ 4 & -Z_{13} + a^2 Z_{23} & Z_{11} + Z_{12} + Z_{22} + Z \\ \hline \end{array}$$

"A Conference to Prevent War"

Conversations with foreign delegates to the Third World Power Conference held in Washington, D. C., during the early part of September, prompted Editor L. W. W. Morrow (A'13, F'25, director) of "Electrical World" to write the accompanying editorial, which is reprinted by permission from the September 26, 1936, issue of that publication.

"YOUR troubles over here are negligible things. How would you like to live in Europe these days? Every sane man there knows that the energies of each nation are devoted to war preparations, that war will come any day and that this war will destroy the world as we know it. Yet no one seems able to stop the torrent and you people seem blind to the fact that your country also will be brought into the world whirlpool sooner or later.'

"Such is the comment of the foreign delegates to the Third World Power Conference. This situation abroad is the background for their formal proposal that the United States call an international conference of leading business executives and industrialists to see if war cannot be prevented. Power, machinery and materials are necessary to war. Would it not be possible for business and industry to stop the deadly political tide that threatens to engulf the world in war? Such is the hope.

"For unanimous agreement exists that Fascist and Communist world leaders are determined to wage war. They have built up nationalistic pride and spirit to fever pitch. This can be sustained only so long. War is the inevitable consequence of this political leadership. All attempts to stop this war movement by diplomatic means have failed. Great Britain tried, but only burned her fingers. In desperation she too has now gone to work to prepare for war.

"We doubt that the United States would make any official diplomatic endeavor to avert this world war. The political answer given is: 'If Great Britain couldn't stop the trend how could we, and why should we put fingers into the fire. Also, the American public does not approve of our mixing into the foreign mess!' This is about the official reaction to the proposed conference as given in the statements of Secretary Hull.

"But why not by-pass officialdom? An international conference of business leaders needs no political support or surroundings. It is suggested that leading men could get together and state publicly that they will not support war. They will turn off the power. They will stop production of munitions. They will not pay the taxes to support war preparations. If this were done could the political leaders make war?

"Probably not. But the plan is idealistic and impracticable. *Business leaders of the key war countries are not free agents.* They speak and move under political orders. They are already enmeshed in a spider web. They have no power to act. We believe the proposed conference is the only alternative to a coming world war, and we regret that it cannot be held. It is too late. Now we too must get ready for war.

"And back of this coming war lies electric power. This fact is the major reason that each European country is putting on political pressure to hasten power developments. Political leaders realize that the use of an abundance of power multiplies man-power for war purposes.

"Foreign delegates paint a black but realistic picture of world affairs. They seek our help, but have little faith that we can do anything to avert a world-wide catastrophe. What a commentary on the evolution of society toward higher ideals and advanced social standards!"

Tensor Analysis of Multielectrode-Tube Circuits

The method of tensors, as applied to the analysis of electrical systems, has gained favor because of its versatility. The most common application of the method in electrical engineering is the analysis of the dynamics of rotating machinery, but a further application is presented in this paper. The attack includes flexible analyses of the characteristics of triodes, tetrodes, and pentodes, and their use as amplifiers, oscillators, and modulators.

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TENSOR analysis is considered the language of the theoretical physicist. It is advocated by him as the language in which all physical phenomena in nature should be expressed, since it unifies and simplifies their description and calculation. Experience has shown amply that tensor analysis is of even greater help to the engineer in the formulation, setting up, and solution of some of his problems than it is to the physicist, because of the greater need for unifying and simplifying concepts in his much wider variety of, and much more complex, problems.

Tensor analysis offers a new method of reasoning to analyze interrelated and to correlate divers physical phenomena; it offers new concepts to visualize the phenomena in any complex structure (rotating machines, multielectrode thermionic tube circuits, transmission lines, etc.) or combinations of structures, and to maintain this visualization without any effort throughout the analysis; it offers a shorthand representation of the set of equations describing the performance of interrelated systems that otherwise are difficult or almost impossible to analyze; and it offers a routine procedure to get the answer in the simplest form in terms of the design or test constants, provided the equations can be solved.

From the point of view of the engineer, whose main job is to analyze the performance of a wide variety of system arrangements (electrical, mechanical, thermal, etc., systems) the labor saving and thought saving qualifications of tensor analysis can be grouped under 2 main headings:

1. *It enables the engineer to use the same scalar equation developed for a simple system having one degree of freedom to describe the behavior of the most complex system with any number of degrees of freedom, by replacing each scalar by an appropriate polyadic (this is the "first generalization theorem").*

The formulation of the complex problem and the whole physical and mathematical analysis is carried through in terms of these symbols (polyadics) only, analogously to a simple system with one variable. Each symbol corresponds to a definite physical characteristic of the system and is not an arbitrary collection of a set of numbers. Once the symbolic equation is established, the solution becomes a routine manipulation of a set of numbers according to certain rules, without any additional thought on the problem.

2. *The most important contribution of tensor analysis is that it enables the engineer to use the same symbolic equation, with the aid of a group of "transformation tensors" C_α^m, for an infinite variety of problems in which the physical laws are fundamentally the same, only the system arrangement, or the point of view, is different.*

For instance, the same symbolic equation can be used for the analysis of all rotating electrical machinery, but with the usual methods of attack the equations for each new system must be established from the beginning.

It is this last property to which tensor analysis owes its usefulness in the analysis of the problems of both pure and applied physics. The method of tensors supplies a multitude of concepts and routine procedures for the simultaneous analysis of many physical systems, by which:

1. Those characteristics in which the systems are identical to each other are separated and given separate symbols called "geometric objects."
2. Those characteristics in which the systems differ from each other also are separated and given separate symbols called "transformation tensors" C_α^m (or "connection tensors").

By this separation all nonessential features of the problem are removed from the essential ones and because of their absence the physical analysis, the setting up, and the solution of the equations are performed in an organized and systematic manner in terms of symbols, each symbol representing a great many scalars.

In this paper it is shown that the scalar equations of a single nonlinear element, say a crystal detector in series with a single impedance, also are valid for an n electrode thermionic tube, or tubes, connected to any complicated network and acting as an amplifier or oscillator, modulator or detector, provided

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each scalar expression in the equations is replaced by an appropriate polyadic. Once the equations are set up in terms of polyadics, the substitution of the test constants into these polyadic equations becomes a routine manipulation that can be handled by anyone familiar with the multiplication of polyadics, but otherwise ignorant of tube circuit analysis. The final answer containing the test constants follows automatically. It checks the results of other conventional types of analysis that require continuous attention to the physics of the problem, and which become extremely complex and almost unmanageable after the first few steps.

The foundations for the understanding of this paper are given in the first 2 parts of a series of articles[1A] in which all the necessary definitions are stated and tensor bibliography is given.

Summary of Results

The results of this paper can be summarized in 2 sets of equations, one set containing scalars, the other polyadics of various ranks. Let the first derivative at the operating point of the static characteristic curve of a crystal detector be Y and one half of its second derivative be M. The change of current Δi due to a small change of voltage Δe across the crystal is

$$\Delta i = Y\Delta e + M(\Delta e)^2 \tag{1}$$

If an impedance Z is connected in series with the detector and a set of small voltage changes $\Delta e_{(\alpha)}$ of different frequencies is impressed across both of them, the fundamental frequency current changes $\Delta i^{(\epsilon)}$ and the product frequency current changes $\Delta i^{(\alpha)(\pm\beta)}$ are

$$\Delta i^{(\epsilon)} + \Delta i^{(\alpha)(\pm\beta)} = Y^{(\epsilon)(\sigma)}\Delta e_{(\sigma)} + M^{(\alpha)(\pm\beta)(\gamma)(\delta)}\Delta e_{(\gamma)}\Delta e_{(\delta)} \tag{2}$$

where $Y^{(\epsilon)(\sigma)}$ and $M^{(\alpha)(\pm\beta)(\gamma)(\delta)}$ are found from the known quantities Z, Y, and M by the formulas

$$Y^{(\epsilon)(\sigma)} = (Y^{-1} + Z)_{(\epsilon)(\sigma)}{}^{-1} \tag{3}$$

$$M^{(\alpha)(\pm\beta)(\gamma)(\delta)} = M(Y^{-1})^3 Y^{(\alpha)(\pm\beta)} Y^{(\gamma)} Y^{(\delta)} \tag{4}$$

where the components of $Y^{(\delta)}$ are calculated for the various impressed frequencies and the components of $Y^{(\alpha)(\pm\beta)}$ for the various product frequencies.

Now let the first derivatives at the operating points of all the static characteristic curves of an n electrode tube (or tubes) be represented in the form of a square by a polyadic $\boldsymbol{Y}$ of rank 2 and half of their second derivatives in the form of a cube by a polyadic $\boldsymbol{M}$ of rank 3. If all of the current changes are represented in the form of a vector as $\Delta\boldsymbol{i}$ and all applied voltage changes across the tube as $\Delta\boldsymbol{e}$, the current changes are found by the formula

$$\Delta\boldsymbol{i} = \boldsymbol{Y}\cdot\Delta\boldsymbol{e} + \Delta\boldsymbol{e}\cdot\boldsymbol{M}\cdot\Delta\boldsymbol{e} \tag{5}$$

If an outside network, represented by a polyadic $\boldsymbol{Z}$, of rank 2, is connected to the tube (or tubes) and several sets of small voltage changes of different frequencies $\Delta\boldsymbol{e}_{(\alpha)}$ are applied anywhere in the outside network, then the fundamental and product frequency current changes in the various circuits are

$$\Delta\boldsymbol{i}^{(\epsilon)} + \Delta\boldsymbol{i}^{(\alpha)(\pm\beta)} = \boldsymbol{Y}^{(\epsilon)(\sigma)}\cdot\Delta\boldsymbol{e}_{(\sigma)} + \Delta\boldsymbol{e}_{(\gamma)}\cdot\boldsymbol{M}^{(\alpha)(\pm\beta)(\gamma)(\delta)}\cdot\Delta\boldsymbol{e}_{(\delta)} \tag{6}$$

where $\boldsymbol{Y}^{(\epsilon)(\sigma)}$ and $\boldsymbol{M}^{(\alpha)(\pm\beta)(\gamma)(\delta)}$ are found from the known polyadics $\boldsymbol{Z}$, $\boldsymbol{Y}$, and $\boldsymbol{M}$ by the formulas

$$\boldsymbol{Y}^{(\epsilon)(\sigma)} = [\boldsymbol{C}_t\cdot(\boldsymbol{Y}^{-1} + \boldsymbol{Z})\cdot\boldsymbol{C}]_{(\epsilon)(\sigma)}{}^{-1} \tag{7}$$

$$\boldsymbol{M}^{(\alpha)(\pm\beta)(\gamma)(\delta)} = \boldsymbol{Y}_t{}^{(\gamma)}\cdot\boldsymbol{C}_t\cdot\boldsymbol{Y}_t{}^{-1}\cdot(\boldsymbol{Y}^{(\alpha)(\pm\beta)}\cdot\boldsymbol{C}\cdot\boldsymbol{Y}^{-1}\cdot\boldsymbol{M})\cdot\boldsymbol{Y}^{-1}\cdot\boldsymbol{C}\cdot\boldsymbol{Y}^{(\delta)} \tag{8}$$

where $\boldsymbol{C}$ is the transformation tensor representing the manner of interconnection of the multielectrode tube (or tubes) with the outside network.

With the aid of these equations, the otherwise extremely complicated physical analysis becomes a routine substitution of polyadics into the formulas.

The first part of the paper gives the calculation of the fundamental-frequency currents, the third part that of the product-frequency currents. The second part is a mathematical digression developing the polyadic form of Taylor's series and its inverse for complex variables.

It should be most emphatically understood that the foregoing equations are not merely indications of the form of the solutions in a customary symbolic manner. They are the final solutions; that is, they give the values of the magnitude and frequency of the current changes in the circuits due to small voltage variations of various frequencies applied anywhere in the outside network, when the constants of the tube and the network are known at the operating point. The remaining work of substituting the test constants into the equations, which may be short or long, depending on the complexity of the circuit, can be done by any computer trained in the elementary procedure of multiplying a set of numbers together in a certain way.

That there are no new results in this paper is emphasized; however, a new method of reasoning is introduced for organizing sets of linear and quadratic equations, and equations of higher degree, that become algebraically unmanageable for a complex system. All of the limitations for the validity of the final equations apply equally, whether the results are found by means of the customary slow scalar methods or with the aid of the powerful tensor method. The final results with both methods are identical, as an example at the end of the paper shows.

It is also emphasized that this paper does not advocate the utilization of Taylor's series expansion for tube circuit analysis. A part of this paper gives a labor saving procedure, to be applied only when the series expansion method is used.

The Method of Attack

To avoid any misunderstanding, it is proper to explain that the expressions for the various polyadics, such as Z_{mn} and M^{mnk} under no circumstances should be interpreted as representing the *general* term of a polyadic, that is, one component of n^2 or n^3 components. Each expression Z_{mn} or M^{mnk} is a "geometric object," a mathematical entity having properties entirely different from those of its components. In the expression Z_{mn}, the single symbol

1A. For all numbered references see list at end of paper.

Z, the carrier letter, represents all of the n^2 components, and the variable indices m and n are auxiliary symbols that show, among other things (using the "index" notation and the terminology of Shouten and Struik[2]):

1. The rank of Z.
2. The particular reference system in which the components of Z are to be measured.
3. The formula of transformation of Z, showing how to find in a routine manner the components of Z in any other reference system.
4. The rule of operation by which the components of Z are combined with the components of other geometric objects.

In order to emphasize that Z_{mn} stands for n^2 components, and not for one component, the so-called "direct" notation also is used throughout, representing each geometric object by a single symbol without any indices, as $\mathbf{Z} = Z_{mn}$. Contrary to the assumptions of vector analysis or matrix algebra, however, a *permanent* formula of transformation is associated with every symbol used, thereby making the equations valid for an infinite number of analogous systems instead of for only one particular system. Both direct and index notations will be used parallel.

Since the illustrations used in this paper are thermionic tube circuits, the group of transformation tensors C_α^m is of simple form, having only constant components. As a result, the transformation formulas of all geometric objects also have simple forms; however, this treatment is intended to serve only as a foundation for the analysis of similar *nonlinear* problems of more complex systems, such as rotating machines or other dynamic systems, in which the group of transformation tensors C_α^m assumes more complex forms. All equations and methods of reasoning are independent of thermionic tube circuits; they serve only as illustrations of the concepts and reasonings developed.

This property of the equations of tensor analysis, namely, their validity for a host of analogous physical phenomena makes the method of tensors a most useful analytical tool in the hands of the engineer dealing with a large variety of analogous physical problems.

The underlying principle of the method of attack of this paper is to allow the addition of extra complications needed in the future, so that the analysis given in the paper need not be repeated, but could be enlarged in the same manner as additional floors are added to a building already finished. The reasonings and results definitely should not be interpreted in the language of matrix algebra or vector analysis, which would make them incapable of future extensions and development.

This expansive quality of the formulas of tensor analysis with the increasing complexity of the physical problem fits naturally the tendency of engineering problems to increase in physical complexity and in degrees of freedom. The increase of the number of mechanical calculating devices, such as network analyzers and differential analyzers, makes practicable the solution of engineering problems with many variables; hence, the setting up of such equations becomes increasingly important in engineering practice.

In special problems various labor saving devices can be used, and should be introduced from the theory of determinants, the theory of matrices, etc. These special cases are not included in this discussion.

Considering the method of attack of this paper from another point of view, it deals with a typical engineering set-up, in which 2 physically and analytically different types of structures are interconnected. In one type of structure (the network), the impedances are known; in the other type (the tubes), the admittances are known. The first structure is linear, the other nonlinear. The method of tensor analysis enables the engineer to divide the whole set-up into its several organic parts (in the present case into networks and tubes), to analyze each part separately (since each structure itself is built up from several more fundamental elements) as if the other parts were absent, and then to combine them into one unit.

While the present example is a particularly simple case of 2 dissimilar stationary electromagnetic structures, identical fundamental reasoning applies to the analysis of more complex structures, in which mechanical, electromagnetic, acoustic, optical, and other structures are combined in any manner into one set-up. Since theoretical physicists do not analyze in general such *combined* dissimilar structures, only engineers do, this particular unifying feature of the method of tensors should appeal more to the engineer than to the physicist.

Summarizing, the central objective of this paper (and of the other papers on the application of tensor analysis) is to present a method of reasoning applicable to as large a variety of practical engineering problems as possible and employing as few concepts and symbols as possible. Every effort has been made at the same time to introduce only such concepts and symbols that are parts of the mathematical structure known as the "absolute differential calculus." In other words, multielectrode tubes, rotating electrical machines, and so on are intended to serve only as *illustrations* for the method of attack. Similarly the method of tensor analysis is used only as a *vehicle* for the presentation of the method of reasoning to be employed in formulating and analyzing interrelated physical phenomena. The primary interest of these papers is engineering and not mathematics; their primary objective is the presentation of a new method of reasoning and not of a new type of mathematics.

The central feature of the method of attack is the process of *transformation of the reference frame;* its backbone is the group of *transformation tensors* C_α^m and its goal is the setting up of *invariant equations* valid for the routine analysis of an infinite type of analogous engineering structures.

I—Multielectrode Tubes as Amplifiers and Oscillators

N-Electrode Tubes

As an introduction to the study of the behavior of thermionic tubes as influenced by the curvature of their voltage-current characteristics, the effect of

curvature will be neglected. The equations for a pentode will be set up as representing a general n electrode vacuum tube.

In a pentode there are 3 grids and one plate. From an analytical point of view there is no difference between the actions of the grids and the action of the plate. To each of them a unit vector is assigned, say ***a, b, c,*** and ***p***. The voltage across ***a*** represents the difference of potential between grid ***a*** and the filament, and the current through ***a*** represents the current between grid ***a*** and the filament; hence, an n electrode tube can be represented as an n–1 winding transformer with n–1 pairs of terminals, as shown in figure 1. However, one of each pair of terminals are connected together, the connection representing the filament.

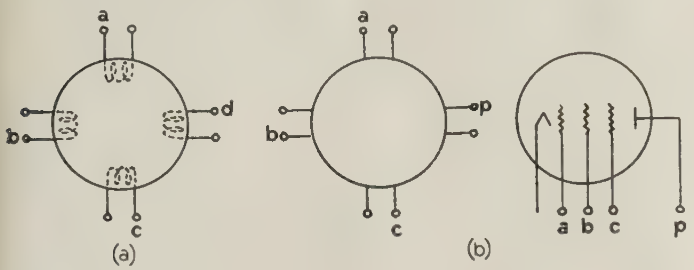

Fig. 1. Diagram showing voltages applied to a 4 winding transformer, and to an analytically equivalent pentode

(*a*)—Transformer with 4 windings (*b*)—Pentode

It should be noted that the filament heating current does not appear in the diagram or the equations, for its value is not pertinent to the analysis to follow.

Where various constant potentials are applied to each pair of terminals, that is, between the terminals, ***a, b, c, p,*** and the filament, a constant current flows in each of the terminals. As the applied d-c voltage at any one of the terminals varies, the currents in the other terminals also vary, but no linear relation exists between the voltage and the currents. For any given filament heating current and any given $n-1$ constant potentials, the variation of the currents with the variation of the nth terminal voltage will be assumed to be known, that is, the n curves $i = f(e)$.

Equations of the Tube

On a pentode let 4 d-c terminal voltages be applied:

$$e_m = \begin{array}{|c|c|c|c|} \multicolumn{1}{c}{a} & \multicolumn{1}{c}{b} & \multicolumn{1}{c}{c} & \multicolumn{1}{c}{p} \\ \hline e_a & e_b & e_c & e_p \\ \hline \end{array} \qquad (9)$$

The 4 d-c currents are

$$i^m = \begin{array}{|c|c|c|c|} \multicolumn{1}{c}{a} & \multicolumn{1}{c}{b} & \multicolumn{1}{c}{c} & \multicolumn{1}{c}{p} \\ \hline i^a & i^b & i^c & i^p \\ \hline \end{array} \qquad (10)$$

Assume now that one of the terminal voltages, say e_a varies by a small amount from e_a to $e_a + \Delta e_a$. Then all 4 currents also will vary. One of the currents, say i^a changes from i^a to $i^a + \Delta i^a$. If the curvature of the curve $i^a = f(e_a)$ in the neighborhood of the given d-c values of e_a and i^a is neglected, then Δi^a is found from the given value of Δe_a by the formula

$$\Delta i^a = \frac{\partial i^a}{\partial e_a} \Delta e_a \qquad (11)$$

$\partial i^a / \partial e_a$ represents the slope of the curve $i^a = f(e_a)$ at the given d-c values of i^a and e_a. This slope also is assumed to be known (figure 2).

Similarly, with the variation of e_a, the other currents also vary by an amount depending on Δe_a and the slope of the particular $i = f(e_a)$ curve, that is,

$$\begin{aligned} \Delta i^b &= \frac{\partial i^b}{\partial e_a} \Delta e_a \\ \Delta i^c &= \frac{\partial i^c}{\partial e_a} \Delta e_a \\ \Delta i^p &= \frac{\partial i^p}{\partial e_a} \Delta e_a \end{aligned} \qquad (12)$$

If all 4 voltages vary, the change in i^a is

$$\Delta i^a = \frac{\partial i^a}{\partial e_a} \Delta e_a + \frac{\partial i^a}{\partial e_b} \Delta e_b + \frac{\partial i^a}{\partial e_c} \Delta e_c + \frac{\partial i^a}{\partial e_p} \Delta e_p \qquad (13)$$

because each small Δe produces its own Δi irrespective of the presence of other small voltage changes.

Similar equations apply for other current changes, so that the following 4 linear equations can be set up, representing the change in the currents due to changes in the voltages

$$\begin{aligned} \Delta i^a &= \frac{\partial i^a}{\partial e_a} \Delta e_a + \frac{\partial i^a}{\partial e_b} \Delta e_b + \frac{\partial i^a}{\partial e_c} \Delta e_c + \frac{\partial i^a}{\partial e_p} \Delta e_p \\ \Delta i^b &= \frac{\partial i^b}{\partial e_a} \Delta e_a + \frac{\partial i^b}{\partial e_b} \Delta e_b + \frac{\partial i^b}{\partial e_c} \Delta e_c + \frac{\partial i^b}{\partial e_p} \Delta e_p \\ \Delta i^c &= \frac{\partial i^c}{\partial e_a} \Delta e_a + \frac{\partial i^c}{\partial e_b} \Delta e_b + \frac{\partial i^c}{\partial e_c} \Delta e_c + \frac{\partial i^c}{\partial e_p} \Delta e_p \\ \Delta i^p &= \frac{\partial i^p}{\partial e_a} \Delta e_a + \frac{\partial i^p}{\partial e_b} \Delta e_b + \frac{\partial i^p}{\partial e_c} \Delta e_c + \frac{\partial i^p}{\partial e_p} \Delta e_p \end{aligned} \qquad (14)$$

Building New Polyadics by Differentiation

So far only 2 polyadics i^m and e_m have been introduced. New polyadics can be constructed from known polyadics in many different ways. In this paper all new polyadics will be built from the vector i^m by successive differentiation with respect to the vector e_m. In general it can be stated that:

1. A polyadic is differentiated with respect to a vector by differentiating each component of the polyadic with respect to each component of the vector. For instance, if the polyadic is a dyadic having n^2 components, each of its components is differentiated n times, if the vector has n components, giving a new polyadic with n^3 components.

2. Whenever a polyadic of rank n is differentiated with respect to a vector, the rank of the resultant polyadic is $n + 1$.

3. If the vector is a covariant vector, that is, having a lower index, the resultant polyadic has one additional contravariant (upper) index and *vice versa*.

For instance

$$\frac{\partial Y^{mn}}{\partial e_k} = M^{mnk} \qquad (15)$$

(That is, let a vector e_m be differentiated with respect to i^n. If m, n, and k represent indices of the old coordinate axes and α, β, and γ of the new axes, then

$$\frac{\partial e_m}{\partial i^n} = \frac{\partial e_m}{\partial i^\alpha}\frac{\partial i^\alpha}{\partial i^n} \qquad (16)$$

However, as demonstrated in a previous paper,[1Ca]

$$\frac{\partial i^\alpha}{\partial i^n} = C_n^\alpha \qquad (17)$$

therefore

$$\frac{\partial e_m}{\partial i^n} = \frac{\partial e_m}{\partial i^\alpha} C_n^\alpha$$

If $\partial e_m/\partial i^\alpha$ is replaced by a symbol as $A_{m\alpha}$ the second index α must be a lower index to balance correctly the upper index of C_n^α; in other words, any index α that is transformed by C_n^α to n must be a lower index. Then

$$\frac{\partial e_m}{\partial i^n} = \frac{\partial e_m}{\partial i^\alpha} C_n^\alpha = A_{m\alpha}C_n^\alpha = A_{mn}$$

or

$$\frac{\partial e_m}{\partial i^n} = A_{mn} \qquad (18)$$

Hence, the new polyadic formed from e_m by differentiation with respect to i^n has $1 + 1 = 2$ indices, the second extra index being a covariant (lower) index n. Its other index m is the same as that of the original vector.)

If a tensor is differentiated with respect to a vector, in general the resultant polyadic is not a

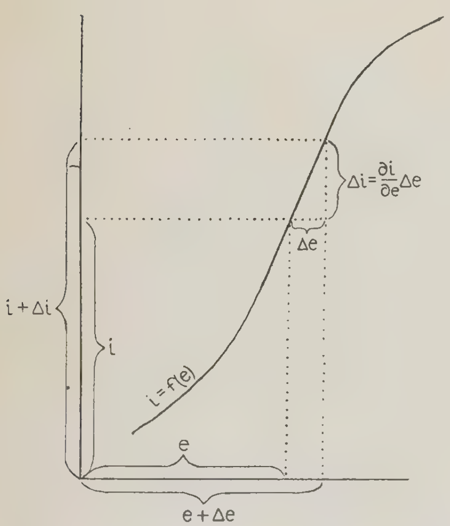

Fig. 2. Static characteristic curve

tensor. The difference between hypercomplex numbers, polyadics, geometric objects, and tensors should be noted.[1Da]

(For instance, let a tensor e_m be differentiated. That e_m is a tensor is expressed by writing its formula of transformation, and all manipulations are done on that formula.

Let

$$e_m = e_\alpha C_m^\alpha \qquad (19)$$

Differentiating both sides of the equation with respect to i^n

$$\frac{\partial e_m}{\partial i^n} = \frac{\partial e_\alpha}{\partial i^n} C_m^\alpha + e_\alpha \frac{\partial C_m^\alpha}{\partial i^n}$$

for a product of polyadics is differentiated by differentiating each polyadic separately

$$\frac{\partial e_m}{\partial i^n} = \frac{\partial e_\alpha}{\partial i^\beta}\frac{\partial i^\beta}{\partial i^n} C_m^\alpha + e_\alpha \frac{\partial C_m^\alpha}{\partial i^n}$$

By equation 17

$$\frac{\partial e_m}{\partial i^n} = \frac{\partial e_\alpha}{\partial i^\beta} C_n^\beta C_m^\alpha + e_\alpha \frac{\partial C_m^\alpha}{\partial i^n}$$

and by equation 18

$$Z_{mn} = Z_{\alpha\beta}C_m^\alpha C_n^\beta + e_\alpha \frac{\partial C_m^\alpha}{\partial i^n} \qquad (20)$$

The transformation formula of $\partial e_m/\partial i^n = Z_{mn}$ is not that of a tensor of rank 2, due to the presence of the last term.)

Where the transformation is linear, that is, where the transformation tensor has only constant components, the derivatives of all tensors are tensors, because all additional expressions in their transformation formulas contain $\partial C_m^\alpha/\partial i^n$ which are all zero.

In this presentation all transformation tensors have only constant components, hence all polyadics are transformed as tensors.

The differential dA of a tensor A similarly is not a tensor, except for a linear transformation. For instance, in the general case[1E]

$$\Delta i^m = C_\alpha^m \Delta i^\alpha + \frac{\partial C_\alpha^m}{\partial i^\beta} i^\alpha \Delta i^\beta \qquad (21)$$

In the present analysis the last term drops out for $\partial C_\alpha^m/\partial i^\beta = 0$, showing that Δi^α is transformed here as a contravariant tensor of rank one.

The Admittance Matrix

The transformations to be considered have only constant components; therefore, in the 4 linear equations 14 the various Δi can be considered as the components of a contravariant vector $\Delta \boldsymbol{i} = \Delta i^m$, that is,

$$\Delta i^m = \begin{array}{c|c|c|c} a & b & c & p \\ \hline \Delta i^a & \Delta i^b & \Delta i^c & \Delta i^p \end{array} \qquad (22)$$

The various Δe_m can be considered as the components of a covariant vector $\Delta \boldsymbol{e} = \Delta e_m$, that is

$$\Delta e_m = \begin{array}{c|c|c|c} a & b & c & p \\ \hline \Delta e_a & \Delta e_b & \Delta e_c & \Delta e_p \end{array} \qquad (23)$$

The coefficients of Δe_m form the components of a twice contravariant dyadic $\boldsymbol{Y} = Y^{mn}$, called the

"admittance matrix," since they are the derivatives of the vector i^m with respect to the vector e_n

$$
Y^{mn} = \begin{array}{c|cccc} m \backslash n & a & b & c & p \\ \hline a & \frac{\partial i^a}{\partial e_a} & \frac{\partial i^a}{\partial e_b} & \frac{\partial i^a}{\partial e_c} & \frac{\partial i^a}{\partial e_p} \\ b & \frac{\partial i^b}{\partial e_a} & \frac{\partial i^b}{\partial e_b} & \frac{\partial i^b}{\partial e_c} & \frac{\partial i^b}{\partial e_p} \\ c & \frac{\partial i^c}{\partial e_a} & \frac{\partial i^c}{\partial e_b} & \frac{\partial i^c}{\partial e_c} & \frac{\partial i^c}{\partial e_p} \\ p & \frac{\partial i^p}{\partial e_a} & \frac{\partial i^p}{\partial e_b} & \frac{\partial i^p}{\partial e_c} & \frac{\partial i^p}{\partial e_p} \end{array} \tag{24}
$$

so that equations 14 can be written in index notation as

$$
\boxed{\Delta i^m = Y^{mn} \Delta e_n} \tag{25}
$$

The dyadic Y^{mn} can also be written as

$$
\boxed{Y^{mn} = \frac{\partial i^m}{\partial e_n}} \tag{26}
$$

so that equation 25 can be written

$$
\boxed{\Delta i^m = \frac{\partial i^m}{\partial e_n} \Delta e_n} \tag{27}
$$

This polyadic equation, representing the n linear equations 14 with n variables, is similar to equation 11 representing one equation with one variable.

Each term in the admittance matrix represents the slope of an $i = f(e)$ curve at the operating point, and each term is a real, and not a complex, number. These constants are called self and mutual conductances (reflex and transconductances, respectively) of the various circuits and are denoted by G^{mn}. The admittance matrix 24 is written in terms of the conductances

$$
Y^{mn} = \begin{array}{c|cccc} m \backslash n & a & b & c & p \\ \hline a & G^{aa} & G^{ab} & G^{ac} & G^{ap} \\ b & G^{ba} & G^{bb} & G^{bc} & G^{bp} \\ c & G^{ca} & G^{cb} & G^{cc} & G^{cp} \\ p & G^{pa} & G^{pb} & G^{pc} & G^{pp} \end{array} \tag{28}
$$

Many components may be zero, of course.

It is customary to define the inverse of a self conductance as a resistance; that is, $G^{mm} = 1/r_{mm}$ and the ratio of a mutual conductance to a self conductance is defined as the amplification factor.

$$
\mu_a{}^b = \frac{G^{ab}}{G^{aa}} = \frac{\dfrac{\partial i^a}{\partial e_b}}{\dfrac{\partial i^a}{\partial e_a}} = -\frac{de_a}{de_b} \tag{29}
$$

and

$$
\mu_b{}^a = \frac{G^{ba}}{G^{bb}} = -\frac{de_b}{de_a}
$$

so that

$$
\boxed{G^{ba} = \frac{\mu_b{}^a}{r_{bb}}} \tag{30}
$$

Hence, in terms of the amplification factors and resistances, the admittance matrix of a pentode is

$$
Y^{mn} = \begin{array}{c|cccc} m \backslash n & a & b & c & p \\ \hline a & \frac{1}{r_{aa}} & \frac{\mu_a{}^b}{r_{aa}} & \frac{\mu_a{}^c}{r_{aa}} & \frac{\mu_a{}^p}{r_{aa}} \\ b & \frac{\mu_b{}^a}{r_{bb}} & \frac{1}{r_{bb}} & \frac{\mu_b{}^c}{r_{bb}} & \frac{\mu_b{}^p}{r_{bb}} \\ c & \frac{\mu_c{}^a}{r_{cc}} & \frac{\mu_c{}^b}{r_{cc}} & \frac{1}{r_{cc}} & \frac{\mu_c{}^p}{r_{cc}} \\ p & \frac{\mu_p{}^a}{r_{pp}} & \frac{\mu_p{}^b}{r_{pp}} & \frac{\mu_p{}^c}{r_{pp}} & \frac{1}{r_{pp}} \end{array} \tag{31}
$$

The admittance matrix of a tetrode is found by omitting the row and column of c. If the customary notation is used,

$$
Y^{mn} = \begin{array}{c|ccc} m \backslash n & a & b & p \\ \hline a & \frac{1}{r_a} & \frac{\eta_a}{r_a} & \frac{\nu_a}{r_a} \\ b & \frac{\eta_b}{r_b} & \frac{1}{r_b} & \frac{\nu_b}{r_b} \\ p & \frac{\mu_a}{r_p} & \frac{\mu_b}{r_p} & \frac{1}{r_p} \end{array} = \begin{array}{c|ccc} m \backslash n & a & b & p \\ \hline a & G^{aa} & G^{ab} & G^{ap} \\ b & G^{ba} & G^{bb} & G^{bp} \\ p & G^{pa} & G^{pb} & G^{pp} \end{array} \tag{32}
$$

where

μ is the plate amplification factor.
ν is the grid amplification factor.
η is the cross amplification factor.

The admittance matrix of a triode is found by omitting the rows and columns of b and c.

$$
Y^{mn} = \begin{array}{c|cc} m \backslash n & g & p \\ \hline g & \frac{1}{r_g} & \frac{\mu_g}{r_g} \\ p & \frac{\mu_p}{r_p} & \frac{1}{r_p} \end{array} = \begin{array}{c|cc} m \backslash n & g & p \\ \hline g & G^{gg} & G^{gp} \\ p & G^{pg} & G^{pp} \end{array} \tag{33}
$$

Impedance Matrix of Nonlinear Systems

One important factor should be emphasized: In most network analysis usually the impedances of the circuit elements are known, but in vacuum tubes the admittances of the tube elements are given instead of impedances. The difficulty in analyzing tube circuits is that in one part of the system the impedances are known, but in another part only the admittances are known. The difficulty in the analysis disappears if the concept of the impedance matrix of a tube is introduced.

The impedance matrix of a tube is the inverse of its admittance matrix, that is

$$
Z = Y^{-1} \tag{34}
$$

which, of course, follows from the formula

$$
Y = Z^{-1} \tag{35}
$$

hence the impedance matrix of a tube is found simply by calculating the inverse of the admittance matrix given in equations 31 to 33, according to the rules given elsewhere.[1B]

The impedance matrix of the tetrode is

$$Z_{mn} = \begin{array}{c|ccc} m \diagdown n & a & b & p \\ \hline a & \dfrac{1 - \mu_b\nu_b}{r_b r_p D} & \dfrac{\eta_b\nu_a - \eta_a}{r_a r_p D} & \dfrac{\eta_a\nu_b - \nu_a}{r_a r_b D} \\ b & \dfrac{\mu_a\nu_b - \eta_b}{r_b r_p D} & \dfrac{1 - \mu_a\nu_a}{r_a r_p D} & \dfrac{\eta_b\nu_a - \nu_b}{r_a r_p D} \\ p & \dfrac{\eta_b\mu_b - \mu_a}{r_b r_p D} & \dfrac{\mu_a\eta_a - \mu_b}{r_a r_p D} & \dfrac{1 - \eta_a\eta_b}{r_a r_p D} \end{array} \tag{36}$$

where

$$D = \frac{1 + \mu_a(\eta_a\nu_b - \nu_a) + \mu_b(\eta_b\nu_a - \nu_b) - \eta_a\eta_b}{r_a r_a r_p}$$

The impedance matrix of a triode is

$$Z_{mn} = \begin{array}{c|cc} m \diagdown n & g & p \\ \hline g & \dfrac{r_g}{1 - \mu_g\mu_p} & \dfrac{-\mu_g r_p}{1 - \mu_g\mu_p} \\ p & \dfrac{-\mu_p r_g}{1 - \mu_g\mu_p} & \dfrac{r_p}{1 - \mu_g\mu_p} \end{array} \tag{37}$$

It should be noted that the definition of the admittance matrix as given in equation 26; that is, $Y^{mn} = \partial i^m/\partial e_n$ and the analogous definition of the impedance matrix

$$Z_{mn} = \frac{\partial e_m}{\partial i^n} \tag{39}$$

are valid not only for thermionic tubes, but also for any nonlinear system having several degrees of freedom.

In certain cases Y^{mn} is given; in others Z_{mn} is given. In saturated rotating machines it is easier to determine the saturation curves $e = f(i)$ or $\psi = f(i)$ hence it is easier to find the impedances from the definition

$$\Delta e_m = \frac{\delta e_m}{\delta i^n}\Delta i^n \tag{40}$$

by finding the change in the resultant fluxes due to the change in one of the currents.

The Equations of Transformation

With every geometric object 2 equations are permanently associated in tensor analysis. One is their "equation of definition" shown above. The other is their "equation of transformation," giving their components in any reference frame if they are known in one reference frame.

If a new reference frame α is introduced by a transformation tensor $\boldsymbol{C} = C_\alpha^m$ with constant components, the equations of transformation of the polyadics hitherto introduced[1Cb] are

$$\begin{array}{ll|ll} \boldsymbol{i} & = \boldsymbol{C}\cdot\boldsymbol{i}' & i^m & = i^\alpha C_\alpha{}^m \\ \boldsymbol{e} & = \boldsymbol{C}_t{}^{-1}\cdot\Delta\boldsymbol{e}' & e_m & = e_\alpha C_m{}^\alpha \\ \Delta\boldsymbol{i} & = \boldsymbol{C}\cdot\Delta\boldsymbol{i}' & \Delta i^m & = \Delta i^\alpha C_\alpha{}^m \\ \Delta\boldsymbol{e} & = \boldsymbol{C}_t{}^{-1}\cdot\Delta\boldsymbol{e}' & \Delta e_m & = \Delta e_\alpha C_m{}^\alpha \\ \boldsymbol{Z}' & = \boldsymbol{C}_t\cdot\boldsymbol{Z}\cdot\boldsymbol{C} & Z_{\alpha\beta} & = Z_{mn}C_\alpha{}^m C_\beta{}^n \\ \boldsymbol{Y}' & = \boldsymbol{C}^{-1}\cdot\boldsymbol{Y}\cdot\boldsymbol{C}_t{}^{-1} & Y^{\alpha\beta} & = Y^{mn}C_m{}^\alpha C_n{}^\beta \end{array} \tag{38}$$

where $C_m{}^\alpha$ is the inverse of $C_\alpha{}^m$. If the componen of $C_m{}^\alpha$ are functions of the variables, the case nc considered here, the equations of transformation ai more complicated.

Comparisons With N-Winding Transformers

It is interesting to compare the impedance an admittance matrices of a multielectrode thermion tube with those of a multiwinding transformer, with those of any passive network.

1. In a thermionic tube the admittances are known and the ir pedances must be calculated; in a network the impedances of tl elements are known, and their admittances must be calculated.

2. In a multiwinding transformer, or in any passive network, bo the impedance and admittance matrices are symmetrical; that i the rows and columns can be interchanged. In a thermionic tube tl mutual conductances are different in the 2 directions; in oth words, G^{ab} is different from G^{ba}, and the effect of a change in tl plate voltage upon a grid current is different from the same chan in the grid voltage upon the plate current. The theorem of rec procity consequently does not apply to vacuum tubes.

3. In a transformer or network each component in the matrix is real or a complex number during steady state conditions. In a tul each component is a real number representing conductances. may be mentioned that in a high frequency tube where the time flight of the electrons between electrodes must be considered, the re numbers in the admittance matrix are replaced by complex nun bers.

4. In networks the matrices are equally valid for any applie terminal voltage or for a change in the applied voltage, but in a tul or in other nonlinear systems the matrices are valid for small chang in the voltages or currents.

Equations of the Interconnected System

With the concept of the impedance matrix of a n electrode tube once established, the solution of th interconnected system consisting of a tube and a ne work closely follows the solution of any other inte connected system.[1Cc]

If the impedance matrix of the tube is $\boldsymbol{Z}_1$ the matri of the outside circuit $\boldsymbol{Z}_2$, and the transformatio tensor of the system is $\boldsymbol{C}$, representing the manne of interconnection of the tube and the networl then the resultant impedance matrix of the inte connected system is

$$\boxed{\boldsymbol{Z} = \boldsymbol{C}_t\cdot(\boldsymbol{Z}_1 + \boldsymbol{Z}_2)\cdot\boldsymbol{C}} \quad (4$$

From which the admittance matrix of the system

$$\boxed{\boldsymbol{Y} = \boldsymbol{Z}^{-1}} \quad (4$$

$\boldsymbol{Y}$ can be written

$$\boldsymbol{Y} = [\boldsymbol{C}_t\cdot(\boldsymbol{Z}_1 + \boldsymbol{Z}_2)\cdot\boldsymbol{C}]^{-1} \quad (4$$

and the change of currents due to voltage chang applied anywhere in the outside network is

$$\boxed{\Delta\boldsymbol{i} = \boldsymbol{Y}\cdot\Delta\boldsymbol{e}} \quad (4$$

$$\boxed{\Delta i^m = Y^{mn}\Delta e_n} \quad (4$$

When in an axis a set of sinusoidal voltages different frequencies is applied, for each differe

frequency of voltage the expression $p = d/dt$ is replaced by the term $j\omega$ and there must be as many admittance matrices (replacing p by $j\omega_1, j\omega_2, j\omega_3 \ldots$ in succession) as there are different frequencies applied. If all applied voltages have the same frequency, only one steady-state admittance matrix is set up. The transient form of the resultant impedance or admittance matrices is ideally suited for the investigation of the criteria for oscillation, maximum amplification, and so on.

Provided special assumptions are made about the tube performance, some of the conductances in the tube admittance matrix $\boldsymbol{Y}'$ may be equated to zero.

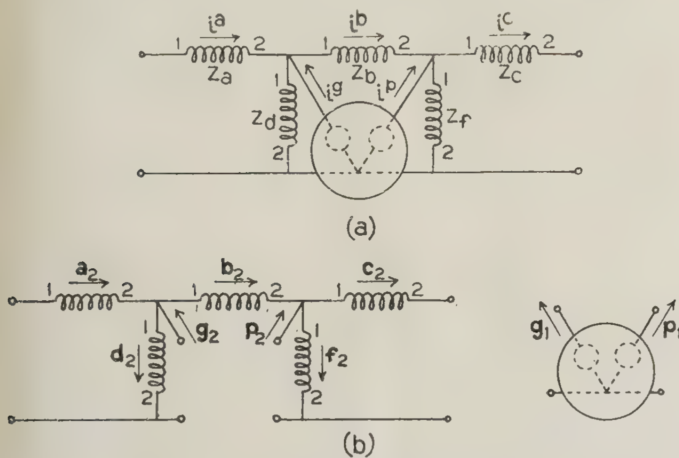

Fig. 3. Triode component systems and their resultant system for determining the impedance matrix

(a)—Resultant system (b)—Component systems

In general, various labor saving devices can be introduced from the theory of matrices.[1F]

Equation 45, representing the relation between Δi^m and the voltages applied in the outside circuit, looks the same as equations 25 representing the relation between Δi^m and the voltages applied at the terminals of the tube without the presence of the outside network.

It should be expressly noted that impedance matrices Z_{mn}, belonging to different circuits, can be added; on the contrary admittance matrices Y^{mn} cannot be added. This fundamental difference in the manipulation (and also in the physical and geometrical representation) of covariant and contravariant geometrical objects is intimately connected with the theory of *subspaces* to be treated in detail in another publication.[1] For the present it should be noted that the polyadics $\boldsymbol{Z}_1$, $\boldsymbol{Z}_2$, and $\boldsymbol{Z}$ have less rows and columns that the polyadic $\boldsymbol{Z}_1 + \boldsymbol{Z}_2$, showing that the spaces in which $\boldsymbol{Z}_1$, $\boldsymbol{Z}_2$, and $\boldsymbol{Z}$ represent some geometric object have less number of dimensions than the space represented by $\boldsymbol{Z}_1 + \boldsymbol{Z}_2$. Hence the 3 spaces are different types of subspaces of the enveloping space of $\boldsymbol{Z}_1 + \boldsymbol{Z}_2$.

In passing from one subspace into another, or into the enveloping space and back, covariant indices behave in a different manner than contravariant indices. From purely electrical analogies it also is obvious that only geometric objects with all covariant indices, such as Z_{mn} or Q_{mnk}, can be added, since they represent quantities measured with all terminals open-circuited. On the contrary, contravariant quantities, such as Y^{mn} or M^{mnk}, cannot be added since they represent quantities measured with all terminals short-circuited.

Example of Triode Circuit Analysis

Consider the triode circuit of figure 3. To find its impedance matrix the system is divided into 2 components as shown in figure 3*b*, one comprising the tube, the other the outside network.

First, the impedance matrix of the outside network, shown again in figure 4*b*, must be set up by following the steps given elsewhere.[1Cd]

The impedance matrix of the generalized network (figure 4*a*) is

$$\boldsymbol{Z} = \begin{array}{c|ccccccc|} & a & b & c & g & p & d & f \\ \hline a & Z_a & 0 & 0 & 0 & 0 & 0 & 0 \\ b & 0 & Z_b & 0 & 0 & 0 & 0 & 0 \\ c & 0 & 0 & Z_c & 0 & 0 & 0 & 0 \\ g & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ p & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ d & 0 & 0 & 0 & 0 & 0 & Z_d & X \\ f & 0 & 0 & 0 & 0 & 0 & X & Z_f \\ \hline \end{array} \qquad (46)$$

where each diagonal component may be of the form $R + Lp + 1/Cp$. In a more general circuit several mutual inductances may exist between the various circuits by replacing some of the zero components in the foregoing matrix.

Although Z_g and Z_p are zero, the corresponding rows and columns are reserved for them, since axes $\boldsymbol{g}$ and $\boldsymbol{p}$ will occur in the final matrix.

There are 5 closed circuits; hence, 5 new currents will be used. If i^d and i^f are eliminated, the relations between the old and the new currents are

$$\begin{aligned} i^a &= i^{a2} &&= i^{a2} \\ i^b &= i^{b2} &&= \quad i^{b2} \\ i^c &= i^{c2} &&= \qquad i^{c2} \\ i^g &= i^{g2} &&= \qquad\quad i^{g2} \\ i^p &= i^{p2} &&= \qquad\qquad i^{p2} \\ i^d &= i^{a2} - i^{b2} + i^{g2} &&= i^{a2} - i^{b2} + i^{g2} \\ i^f &= i^{b2} - i^{c2} + i^{p2} &&= i^{b2} - i^{c2} + i^{p2} \end{aligned} \qquad (47)$$

The transformation tensor consists of the coefficients of the new currents, that is, $\boldsymbol{C} = \partial i^{old}/\partial i^{new} =$

$$\boldsymbol{C} = \begin{array}{c|ccccc|} & a_2 & b_2 & c_2 & g_2 & p_2 \\ \hline a & 1 & 0 & 0 & 0 & 0 \\ b & 0 & 1 & 0 & 0 & 0 \\ c & 0 & 0 & 1 & 0 & 0 \\ g & 0 & 0 & 0 & 1 & 0 \\ p & 0 & 0 & 0 & 0 & 1 \\ d & 1 & -1 & 0 & 1 & 0 \\ f & 0 & 1 & -1 & 0 & 1 \\ \hline \end{array} \qquad (48)$$

Now using the equation of transformation of $\boldsymbol{Z}$,

$$\boxed{\boldsymbol{Z}_2 = \boldsymbol{C}_t \cdot \boldsymbol{Z} \cdot \boldsymbol{C}} \qquad (49)$$

the impedance matrix is found again in 2 steps. The first step is

		a_2	b_2	c_2	g_2	p_2
	a	Z_a	0	0	0	0
	b	0	Z_b	0	0	0
	c	0	0	Z_c	0	0
$Z \cdot C =$	g	0	0	0	0	0
	p	0	0	0	0	0
	d	Z_d	$X - Z_d$	$-X$	Z_d	X
	f	X	$Z_f - X$	$-Z_f$	X	Z_f

The second step $C_t \cdot (Z \cdot C)$ gives the final Z_2 of the network as

		a_2	b_2	c_2	g_2	p_2
	a_2	$Z_a + Z_d$	$X - Z_d$	$-X$	Z_d	X
	b_2	$X - Z_d$	$Z_b + Z_d + Z_f - 2X$	$X - Z_f$	$X - Z_d$	$Z_f - X$
$Z_2 =$	c_2	$-X$	$X - Z_f$	$Z_c + Z_f$	$-X$	$-Z_f$
	g_2	Z_d	$X - Z_d$	$-X$	Z_d	X
	p_2	X	$Z_f - X$	$-Z_f$	X	Z_f

(50)

The impressed voltage vector is by $e_2 = C_t \cdot e$

	a_2	b_2	c_2	g_2	p_2	
$e_2 =$	$e_a + e_d$	$e_b - e_d + e_f$	$e_c - e_f$	$e_g + e_d$	$e_p + e_f$	(51)

showing the 5 closed circuits of which the voltage equations are given by $e_2 = Z_2 \cdot i_2$; namely, circuits *a–d*, *b–f–d*, *c–f*, *g–d*, and *p–f*.

The impedance matrix of a triode is given in equation 37 as

		g_1	p_1
$Z_1 =$	g_1	$\frac{r_g}{D}$	$\frac{-\mu_g r_p}{D}$
	p_1	$\frac{-\mu_p r_g}{D}$	$\frac{r_p}{D}$

(52)

where $D = 1 - \mu_g \mu_p$

Figure 3*a* shows that the 2 g and the 2 p axes are connected in series. The effect of the transformation tensor would be to change g_1 and g_2 to g and p_1 and p_2 to p. It is not necessary in this simple proces to progress through the steps of setting up relation between the currents to find a transformation tensor All of these steps can be replaced simply by desig nating both axes in equations 50 and 52 as g or p instead of g_1 and g_2 or p_1 and p_2. The resultan impedance matrix is then the sum of the 2 matrice 50 and 52. In the new matrix the dyad gg wil contain $Z_d + r_g/D$. Hence, the resultant impedanc matrix of the combined system of figure 3 is

		a	b	c	g	p
	a	$Z_a + Z_d$	$X - Z_d$	$-X$	Z_d	X
	b	$X - Z_d$	$Z_b + Z_d + Z_f - 2X$	$X - Z_f$	$X - Z_d$	$Z_f - X$
$Z =$	c	$-X$	$X - Z_f$	$Z_c + Z_f$	$-X$	$-Z_f$
	g	Z_d	$X - Z_d$	$-X$	$Z_d + r_g/D$	$X - \mu_g r_p/D$
	p	X	$Z_f - X$	$-Z_f$	$X - \mu_p r_g/D$	$Z_f + r_p/D$

(53

The currents are found by the formula

$$\Delta i = Z^{-1} \cdot \Delta e$$

In calculating the currents caused by applie voltages by the formula $i = Y \cdot e$ or $\Delta i = Y \cdot \Delta e$ usually not every terminal has a voltage impresse upon it and all of the currents are not needed. In most cases only one voltage is impressed and only one current is to be calculated. In such cases it i not necessary to calculate the whole admittanc matrix by finding the cofactors of all the n^2 com ponents. The calculation of the cofactor of only one component is sufficient. That component lies in th row of the impressed voltage and in the column o the desired current. If, for instance, a voltage i impressed in terminal b and the grid current i desired, the cofactor of the component in the secon row and the fourth column is calculated. With 2 impressed voltages for each current, 2 cofactors nee to be calculated.

If, instead of simply adding the matrices 50 an 52 to find the matrix 53, the 2 networks are adde according to equation 41, that is, according to

$$Z = C_t \cdot (Z_1 + Z_2) \cdot C \tag{54}$$

then the sum of the 2 matrices is $Z_1 + Z_2 =$

	a_2	b_2	c_2	g_2	p_2	g_1	p_1
a_2	$Z_a + Z_d$	$X - Z_d$	$-X$	Z_d	X	0	0
b_2	$X - Z_d$	$Z_b + Z_d + Z_f - 2X$	$X - Z_f$	$X - Z_d$	$Z_f - X$	0	0
c_2	$-X$	$X - Z_f$	$Z_c + Z_f$	$-X$	$-Z_f$	0	0
g_2	Z_d	$X - Z_d$	$-X$	Z_d	X	0	0
p_2	X	$Z_f - X$	$-Z_f$	X	Z_f	0	0
g_1	0	0	0	0	0	$\frac{r_g}{D}$	$\frac{-\mu_g r_p}{D}$
p_1	0	0	0	0	0	$\frac{-\mu_p r_g}{D}$	$\frac{r_p}{D}$

(55)

The transformation tensor can be established by a simple inspection of figure 3*a*, in which it is shown that i^{g_1} and i^{g_2} are both replaced by i^g; also i^{p_1} and i^{p_2} are replaced by i^p; all other currents remaining unchanged. Hence, writing the 7 old axes in a verti-

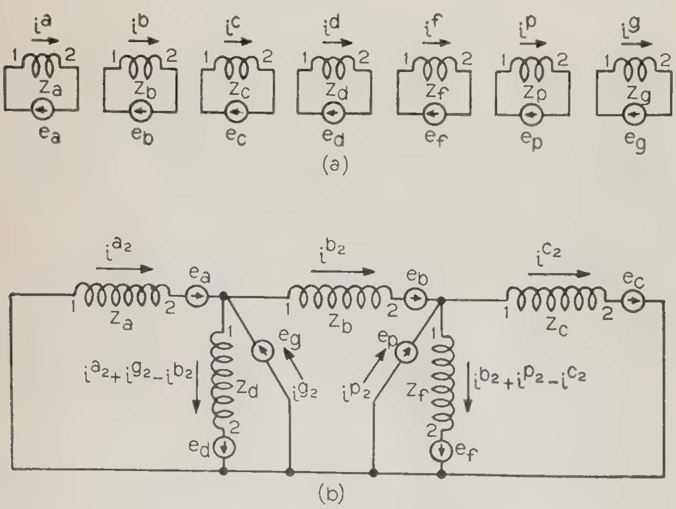

Fig. 4. External network of figure 3

(a)—Generalized network (b)—Component systems

cal column and the 5 new axes in a horizontal row, the transformation tensor of the system is

$$
\boldsymbol{C} = \begin{array}{c|ccccc|}
 & a & b & c & g & p \\
\hline
a_2 & 1 & 0 & 0 & 0 & 0 \\
b_2 & 0 & 1 & 0 & 0 & 0 \\
c_2 & 0 & 0 & 1 & 0 & 0 \\
g_2 & 0 & 0 & 0 & 1 & 0 \\
p_2 & 0 & 0 & 0 & 0 & 1 \\
g_1 & 0 & 0 & 0 & 1 & 0 \\
p_1 & 0 & 0 & 0 & 0 & 1 \\
\hline
\end{array} \qquad (56)
$$

If matrix 55 is multiplied by matrix 56, and by its transpose, the resultant is matrix 53.

Example of Screen-Grid Tube Circuits

Consider the tetrode circuit of figure 5*a*. It is divided into 2 components, one containing the tube

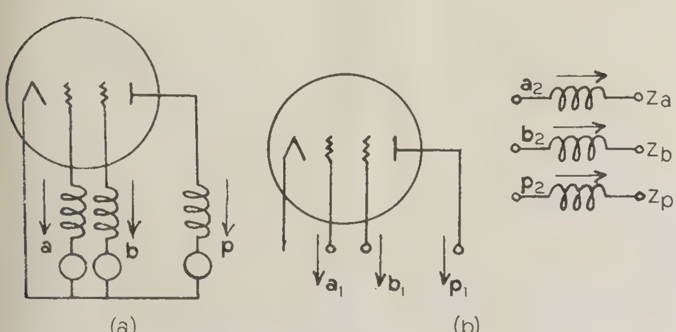

Fig. 5. Tetrode component systems and their resultant system for determining the impedance matrix

(*a*)—Resultant system (*b*)—Component systems

itself, the other containing all the outside impedances, as shown in figure 5*b*. The impedance matrix of the outside circuit is

$$
\boldsymbol{Z}_2 = \begin{array}{c|ccc|}
 & a_2 & b_2 & p_2 \\
\hline
a_2 & Z_a & 0 & 0 \\
b_2 & 0 & Z_b & 0 \\
p_2 & 0 & 0 & Z_b \\
\hline
\end{array} \qquad (57)
$$

where

$Z = R + Lp + 1/Cp.$

The impedance matrix of the tetrode has been given in equation 36.

The transformation tensor of the system is

$$
\boldsymbol{C} = \begin{array}{c|ccc|}
 & a & b & p \\
\hline
a_1 & 1 & 0 & 0 \\
b_1 & 0 & 1 & 0 \\
p_1 & 0 & 0 & 1 \\
a_2 & 1 & 0 & 0 \\
b_2 & 0 & 1 & 0 \\
p_2 & 0 & 0 & 1 \\
\hline
\end{array} \qquad (58)
$$

The resultant impedance matrix is by $\boldsymbol{C}_t \cdot \boldsymbol{Z} \cdot \boldsymbol{C}$ is equal to

$$
\boldsymbol{Z}' = \begin{array}{c|ccc|}
 & a & b & p \\
\hline
a & \dfrac{1 - \mu_b \nu_b}{r_b r_p D} + Z_a & \dfrac{\eta_b \nu_a - \eta_a}{r_a r_p D} & \dfrac{\eta_a \nu_b - \nu_a}{r_a r_b D} \\
b & \dfrac{\mu_a \nu_b - \eta_b}{r_b r_p D} & \dfrac{1 - \mu_a \nu_a}{r_a r_p D} + Z_b & \dfrac{\eta_b \nu_a - \nu_b}{r_a r_p D} \\
p & \dfrac{\eta_b \mu_b - \mu_a}{r_b r_p D} & \dfrac{\mu_a \eta_a - \mu_b}{r_a r_p D} & \dfrac{1 - \eta_a \eta_b}{r_a r_p D} + Z_p \\
\hline
\end{array} \qquad (59)
$$

$$
\boldsymbol{Z}' = \begin{array}{c|ccc|}
 & a & b & p \\
\hline
a & A & B & C \\
b & D & E & F \\
p & G & H & K \\
\hline
\end{array}
$$

and the admittance matrix of the whole system is

$$
\boldsymbol{Y}' = \begin{array}{c|ccc|}
 & a & b & p \\
\hline
a & \dfrac{EK - HF}{Det} & \dfrac{HC - BK}{Det} & \dfrac{BF - EC}{Det} \\
b & \dfrac{GF - DK}{Det} & \dfrac{AK - GC}{Det} & \dfrac{DC - AF}{Det} \\
p & \dfrac{DH - GE}{Det} & \dfrac{GB - AH}{Det} & \dfrac{AE - BD}{Det} \\
\hline
\end{array} \qquad (60)
$$

From the admittance matrix the current changes in the 2 grid or plate circuits due to voltage changes in any one of the circuits can be read immediately.

II—Taylor's Series and Its Inverse in Polyadic Form

The Index Apparatus of Tensor Analysis

Where the curvature of the current-voltage characteristics of tubes also is taken into consideration, then the equation of currents is not of such a simple form as $\Delta \boldsymbol{i} = \boldsymbol{Y} \cdot \Delta \boldsymbol{e}$, for additional terms also occur. Although the foregoing treatment of tubes is a special case of the following, all the results (impedance matrices, transformation tensors, etc.) are utilized in the extension. In the usual methods of attack the whole analysis would have to be started all over again.

The systematic treatment of multielectrode tube circuits offers an easily visualized example, where the index apparatus of modern tensor analysis and spinor analysis finds an extensive application. The simplicity of the transformation tensors $\boldsymbol{C}$ in stationary networks shifts the emphasis from certain concepts, important say in rotating machinery, to other types of concepts that play a secondary part in rotating machine studies. In the following treatment careful distinction must be made between the following types of indices:[1Db]

1. Fixed and variable.
2. Free and dummy.
3. Upper and lower.
4. Open and closed.
5. Tensor and spin.
6. Dotted and undotted.

Each of these indices denotes a different type of transformation in tensor analysis.

In this part the expansion of functions $y = f(x)$ in Taylor's series is developed and the following cases are considered: (1) functions of real variables in one variable and in several variables and (2)

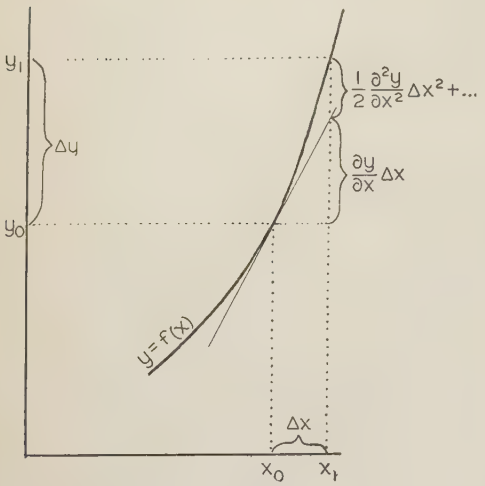

Fig. 6. Graph of any equation $y = f(x)$

functions of sets of complex variables in one set of variables and in several sets of variables.

Afterward the inverse relation $x = f^{-1}(y)$ are developed for the same cases.

Throughout the development, emphasis will be laid upon the important fact that as the complexity of the physical system increases the equations and reasonings accompanying them do not change, only the number of indices associated with each symbol increases. The increase in the complexity of the physical set-up consists of the following steps:

1. One circuit, having one current.
2. One circuit, having several frequencies of currents.
3. Several circuits, each having several frequencies of currents.

The step from 1 to 2 introduces a set of closed indices for each symbol and the step from 2 to 3 adds an additional set of open indices.

Taylor's Series

Let any equation $y = f(x)$ be given. Its graph is shown in figure 6. Suppose that for a given value of x, say x_0, the value of y is known to be y_0, but it is not known for any other value of x. Suppose also that for the given value of x not only y but also all of its derivatives are known.

The problem is whether it is possible to find the value of y at another point x_1 in the neighborhood of x_0, even though the value of y is known only at point x_0.

Taylor's series gives the value of y at point x_1 in terms of the value of y at x_0, of all its derivatives at x_0, and of the distance $x_1 - x_0 = \Delta x$, all of which are known, provided certain conditions are satisfied. The value of y at x_1 is

$$y_0 + \Delta y = y_0 + \frac{\partial y}{\partial x}\Delta x + \frac{1}{2}\frac{\partial^2 y}{\partial x^2}(\Delta x)^2 + \frac{1}{6}\frac{\partial^3 y}{\partial x^3}(\Delta x)^3 + \ldots \quad (61)$$

where all derivatives are taken at point y_0. Onl
Δy, the change in y is needed and equation 61 b
comes

$$\boxed{\Delta y = \frac{\partial y}{\partial x}\Delta x + \frac{1}{2!}\frac{\partial^2 y}{\partial x^2}(\Delta x)^2 + \frac{1}{3!}\frac{\partial^3 y}{\partial x^3}(\Delta x)^3 + \ldots} \quad (6$$

If the slope of the curve in the neighborhood
y_0 is constant, the curvature and $\partial^2 y/\partial x^2$ are zer
leaving

$$\boxed{\Delta y = \frac{\partial y}{\partial x}\Delta x} \quad (6$$

which was assumed in equation 11.

Taylor's Series in Polyadic Form

Another example will be shown of the theorem th
the polyadic equation for several degrees of freedo
usually has the same form as the scalar equation ha

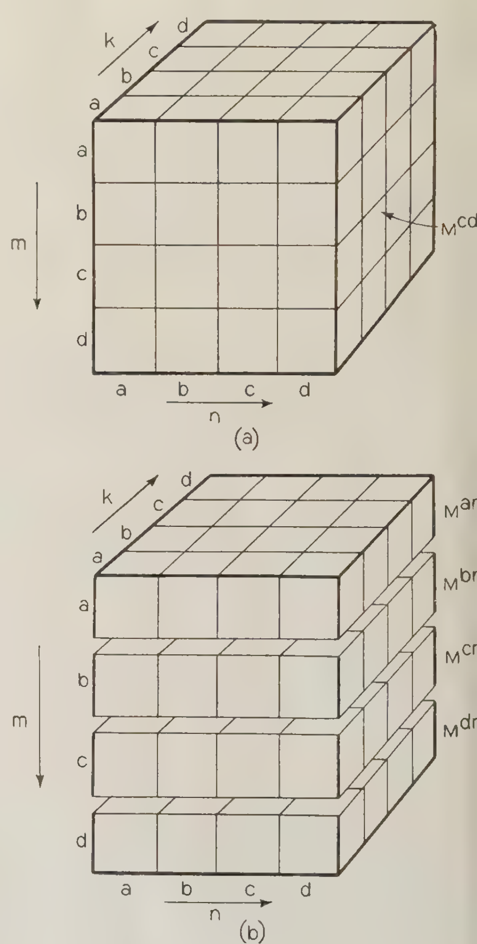

Fig. 7. Graphical representation of a triadic

(a)—The triadic M^{mnk} represented as n^3 numbers arranged in a cube

(b)—Component matrices of the triadic

for one degree of freedom, provided each scalar
replaced by a polyadic.[1G] Let each of 2 variables b
a function of 2 other independent variables; that i
let

$$y^a = f^a(x_a, x_b)$$
$$y^b = f^b(x_a, x_b) \quad (6$$

The position of the indices varies in various problem
depending upon what the variables y and x represen

If each of the independent variables changes by an amount Δx_a and Δx_b, the change in the dependent variables is

$$
\begin{aligned}
\Delta y^a = {} & \left[\frac{\partial y^a}{\partial x_a}\Delta x_a + \frac{\partial y^a}{\partial x_b}\Delta x_b\right] + \\
& \frac{1}{2}\left[\frac{\partial^2 y^a}{\partial x_a \partial x_a}\Delta x_a \Delta x_a + \frac{\partial^2 y^a}{\Delta x_a \Delta x_b}\Delta x_a \Delta x_b + \frac{\partial^2 y^a}{\partial x_b \partial x_a}\Delta x_b \Delta x_a + \frac{\partial^2 y^a}{\partial x_b \partial x_b}\Delta x_b \Delta x_b\right] + \\
& \frac{1}{6}\left[\frac{\partial^3 y^a}{\partial x_a \partial x_a \partial x_a}\Delta x_a \Delta x_a \Delta x_a + \ldots\right] + \ldots . \\
\Delta y^b = {} & \left[\frac{\partial y^b}{\partial x_a}\Delta x_a + \frac{\partial y^b}{\partial x_b}\Delta x^b\right] + \\
& \frac{1}{2}\left[\frac{\partial^2 y^b}{\partial x_a \partial x_a}\Delta x_a \Delta x_a + \frac{\partial^2 y^b}{\partial x_a \partial x_b}\Delta x_a \Delta x_b + \frac{\partial^2 y^b}{\partial x_b \partial x_a}\Delta x_b \Delta x_a + \frac{\partial^2 y^b}{\partial x_b \partial x_b}\Delta x_b \Delta x_b\right] + \\
& \frac{1}{6}\left[\frac{\partial^3 y^b}{\partial x_a \partial x_a \partial x_a}\Delta x_a \Delta x_a \Delta x_a + \ldots .\right] + \ldots
\end{aligned}
\tag{65}
$$

If, instead of 2 functions of 2 variables, there are n functions of n variables,

$$
\begin{aligned}
y^a &= f^a\,(x_a, x_b \ldots . x_n) \\
y^b &= f^b\,(x_a, x_b \ldots . x_n) \\
& \cdots\cdots\cdots\cdots \\
y^n &= f^n\,(x_a, x_b \ldots . x_n)
\end{aligned}
\tag{66}
$$

then there are n such equations as the foregoing, each parenthesis containing n, n^2, or n^3 terms instead of 2, 2^2, or 2^3. In index notation the n equations are written as one polyadic equation

$$
\boxed{\Delta y^m = \frac{\partial y^m}{\partial x_n}\Delta x_n + \frac{1}{2!}\frac{\partial^2 y^m}{\partial x_n \partial x_k}\Delta x_n \Delta x_k + \frac{1}{3!}\frac{\partial^3 y^m}{\partial x_n \partial x_k \partial x_h}\Delta x_n \Delta y_k \Delta x_h + \ldots .}
\tag{67}
$$

where the indices $m, n, k, \ldots$ may assume any one of the values $a, b, c^3, \ldots$ (Equation 67 would be a tensor equation for any group of transformations, if $\partial y^m/\partial x^n$ were an absolute derivative.[4])

Triadics and Polyadics of Higher Rank

In the polyadic equation 67 Δy^m is a contravariant vector and Δx_m is a covariant vector: $\partial y^m/\partial x_n$ is a doubly contravariant dyadic, as has been shown in equation 26, and is represented, say, by Y^{mn}.

The expression

$$M^{mnk} = \frac{1}{2}\frac{\partial^2 y^m}{\partial x_n \partial x_k}$$

is a triadic where each term is a partial derivative of a component in Y^{mn}. It is represented by n^3 numbers arranged in a cube as shown in figure 7a. On paper for say $n = 4$ it can be represented by 4 matrices, each containing 4^2 components by assuming that in M^{mnk} the variable index m assumes the fixed indices a, b, c, and d in succession. One of the 4 matrices is (when m assumes the value a)

$$
2M^{ank} = \begin{array}{c|cccc|}
n \diagdown k & a & b & c & d \\
\hline
a & \dfrac{\partial^2 y^a}{\partial x_a \partial x_a} & \dfrac{\partial^2 y^a}{\partial x_a \partial x_b} & \dfrac{\partial^2 y^a}{\partial x_a \partial x_c} & \dfrac{\partial^2 y^a}{\partial x_a \partial x_d} \\
b & \dfrac{\partial^2 y^a}{\partial x_b \partial x_a} & \dfrac{\partial^2 y^a}{\partial x_b \partial x_b} & \dfrac{\partial^2 y^a}{\partial x_b \partial x_c} & \dfrac{\partial^2 y^a}{\partial x_b \partial x_d} \\
c & \dfrac{\partial^2 y^a}{\partial x_c \partial x_a} & \dfrac{\partial^2 y^a}{\partial x_c \partial x_b} & \dfrac{\partial^2 y^a}{\partial x_c \partial x_c} & \dfrac{\partial^2 y^a}{\partial x_c \partial x_d} \\
d & \dfrac{\partial^2 y^a}{\partial x_d \partial x_a} & \dfrac{\partial^2 y^a}{\partial x_d \partial x_b} & \dfrac{\partial^2 y^a}{\partial x_d \partial x_c} & \dfrac{\partial^2 y^a}{\partial x_d \partial x_d} \\
\hline
\end{array}
\tag{68}
$$

In matrix $2M^{bnk}$ in each numerator y^b occurs instead of y^a; similarly, in the others, y^c and y^d occur respectively.

Instead of m of course n or k might have assumed the fixed indices a, b, c, d, resulting in matrices that represent different cuts of the original cube, or the same 4^3 components might have been arranged in 2 other ways.

The expression

$$D^{mnkh} = \frac{1}{6}\frac{\partial^3 y^m}{\partial x_n \partial x_k \partial x_h}$$

is a polyadic of rank 4. It has n^4 components, each a partial derivative of a component in M^{mnk} with respect to x_h. They may be arranged in n cubes, each with n^3 components, or, as a figure of speech, in a 4 dimensional cube.

On paper a polyadic of rank 4 may be represented by n^2 matrices, each with n^2 components, by replacing 2 of its variable indices by their fixed values in succession. For $n = 4$ one of the 16 matrices is (when $m = b$ and $n = c$)

$$
6D^{bckh} = \begin{array}{c|cccc|}
k \diagdown h & a & b & c & d \\
\hline
a & \dfrac{\partial^3 y^b}{\partial x_c \partial x_a \partial x_a} & \dfrac{\partial^3 y^b}{\partial x_c \partial x_a \partial x_b} & \dfrac{\partial^3 y^b}{\partial x_c \partial x_a \partial x_c} & \dfrac{\partial^3 y^b}{\partial x_c \partial x_a \partial x_d} \\
b & \dfrac{\partial^3 y^b}{\partial x_c \partial x_b \partial x_a} & \dfrac{\partial^3 y^b}{\partial x_c \partial x_b \partial x_b} & \dfrac{\partial^3 y^b}{\partial x_c \partial x_b \partial x_c} & \dfrac{\partial^3 y^b}{\partial x_c \partial x_b \partial x_d} \\
c & \dfrac{\partial^3 y^b}{\partial x_c \partial x_c \partial x_a} & \dfrac{\partial^3 y^b}{\partial x_c \partial x_c \partial x_b} & \dfrac{\partial^3 y^b}{\partial x_c \partial x_c \partial x_c} & \dfrac{\partial^3 y^b}{\partial x_c \partial x_c \partial x_d} \\
d & \dfrac{\partial^3 y^b}{\partial x_c \partial x_d \partial x_a} & \dfrac{\partial^3 y^b}{\partial x_c \partial x_d \partial x_b} & \dfrac{\partial^3 y^b}{\partial x_c \partial x_d \partial x_c} & \dfrac{\partial^3 y^b}{\partial x_c \partial x_d \partial x_d} \\
\hline
\end{array}
\tag{69}
$$

The polyadic form of Taylor's series, equation 67, is similar to its scalar form in equation 62, except the nth power of Δx, that is, $(\Delta x)^n$ is replaced by $\Delta x_a \Delta x_b \ldots \Delta x_n$.

In terms of these polyadics Taylor's series can be written

$$
\boxed{\Delta y^m = Y^{mn}\Delta x_n + M^{mnk}\Delta x_n \Delta x_k + D^{mnkh}\Delta x_n \Delta x_k \Delta x_h + \ldots}
\tag{70}
$$

It should be noted that the free index in each term is m. The indices for Δy and Δx may be upper or lower indices, depending on the problem. Accordingly, the position of the indices of the other polyadics also varies in the various problems, as is shown later. For instance, when the series represents a transformation of the variables (when Δy^m represents the variables in the old reference system and Δx^m in the new reference system, a type of transformation that occurs frequently in tensor analysis) both y and x have upper indices and equation 70 becomes

$$\Delta x^m = Y^{m}_{\cdot\,\alpha}\Delta x^\alpha + M^{m}_{\cdot\,\alpha\beta}\Delta x^\alpha \Delta x^\beta + D^{m}_{\cdot\,\alpha\beta\gamma}\Delta x^\alpha \Delta x^\beta \Delta x^\gamma + \ldots$$

If small voltages are applied at the various terminals of a multielectrode tube in addition to the constant battery voltages, the small current changes are given by equation 70, where y is replaced by i and x by e; that is,

$$\boxed{\Delta i^m = Y^{mn}\Delta e_n + M^{mnk}\Delta e_n \Delta e_k + D^{mnkh}\Delta e_n \Delta e_k \Delta e_h + \ldots} \qquad (71)$$

The polyadic D^{mnkh} of rank 4 introduces currents that cause distortion; hence it may be called the distortion polyadic.

Usually it is sufficient to consider the first curvature of the i-e curve; hence, the last term is neglected, leaving

$$\boxed{\Delta i^m = Y^{mn}\Delta e_n + M^{mnk}\Delta e_n \Delta e_k} \qquad (72)$$

Since the triadic M^{mnk} determines the modulation characteristics of tubes it may be called the modulation triadic. If the curvature is neglected, equation 72 simplifies to

$$\boxed{\Delta i^m = Y^{mn}\Delta e_n} \qquad (73)$$

which has been already given in equation 25.

The admittance matrix of thermionic tubes and other nonlinear systems differs from the admittance matrix of linear networks in the important respect that it represents the admittance of a system to a small change of voltage only, and not to any applied voltage. Because of this difference the admittance matrix of nonlinear networks may be called the amplification matrix.

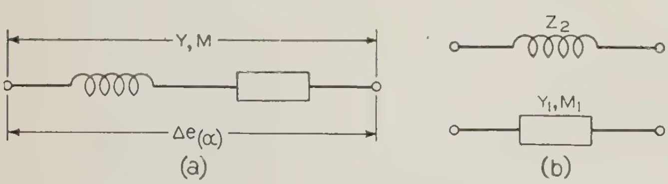

Fig. 8. A crystal detector connected in series with an impedance

(a)—Resultant system (b)—Component systems

For any tube

$$Y^{mn} = \frac{\partial i^m}{\partial e_n}$$

$$M^{mnk} = \frac{1}{2!}\frac{\partial Y^{mn}}{\partial e_k} = \frac{1}{2!}\frac{\partial^2 i^m}{\partial e_n \partial e_k} \qquad (74)$$

$$D^{mnkh} = \frac{1}{3!}\frac{\partial M^{mnk}}{\partial e_h} = \frac{1}{3!}\frac{\partial^2 Y^{mn}}{\partial e_k \partial e_h} = \frac{1}{3!}\frac{\partial^3 i^m}{\partial e_n \partial e_k \partial e_h}$$

Where a new reference frame α is introduced by a transformation tensor $\boldsymbol{C} = C_\alpha^{\ m}$ with constant components, the equations of transformation are

$$Y^{\alpha\beta} = Y^{mn}C_m^{\ \alpha}C_n^{\ \beta}$$

$$M^{\alpha\beta\gamma} = M^{mnk}C_m^{\ \alpha}C_n^{\ \beta}C_k^{\ \gamma} \qquad (75)$$

$$D^{\alpha\beta\gamma\delta} = D^{mnkh}C_m^{\ \alpha}C_n^{\ \beta}C_k^{\ \gamma}C_h^{\ \delta}$$

For a pentode the 4 matrices of the modulation triadic are given in equation 68 by replacing y by i and x by e. For a screen-grid tube the 3 matrices of the modulation triadic are

$$2M^{ank} = \begin{array}{c|ccc} n \backslash k & a & b & p \\ \hline a & -\frac{1}{r_a^2}\frac{\partial r_a}{\partial e_a} & -\frac{1}{r_a^2}\frac{\partial r_a}{\partial e_b} & -\frac{1}{r_a^2}\frac{\partial r_a}{\partial e_p} \\ b & \frac{\partial G^{ab}}{\partial e_a} & \frac{\partial G^{ab}}{\partial e_b} & \frac{\partial G^{ab}}{\partial e_p} \\ p & \frac{\partial G^{ap}}{\partial e_a} & \frac{\partial G^{ap}}{\partial e_b} & \frac{\partial G^{ap}}{\partial e_p} \end{array}$$

$$2M^{bnk} = \begin{array}{c|ccc} n \backslash k & a & b & p \\ \hline a & \frac{\partial G^{ba}}{\partial e_a} & \frac{\partial G^{ba}}{\partial e_b} & \frac{\partial G^{ba}}{\partial e_p} \\ b & -\frac{1}{r_b^2}\frac{\partial r_b}{\partial e_a} & -\frac{1}{r_b^2}\frac{\partial r_b}{\partial e_b} & -\frac{1}{r_b^2}\frac{\partial r_b}{\partial e_p} \\ p & \frac{\partial G^{bp}}{\partial e_a} & \frac{\partial G^{bp}}{\partial e_b} & \frac{\partial G^{bp}}{\partial e_p} \end{array} \qquad (7$$

$$2M^{pnk} = \begin{array}{c|ccc} n \backslash k & a & b & p \\ \hline a & \frac{\partial G^{pa}}{\partial e_a} & \frac{\partial G^{pa}}{\partial e_b} & \frac{\partial G^{pa}}{\partial e_p} \\ b & \frac{\partial G^{pb}}{e_a} & \frac{\partial G^{pb}}{e_b} & \frac{\partial G^{pb}}{e_p} \\ p & -\frac{1}{r_p^2}\frac{\partial r_p}{\partial e_a} & -\frac{1}{r_p^2}\frac{\partial r_p}{\partial e_b} & -\frac{1}{r_p^2}\frac{\partial r_p}{\partial e_p} \end{array}$$

Since $M^{mnk} = \partial Y^{mn}/\partial e_k$ the components o Y^{mn} are differentiated with respect to the thir index, whatever that happens to be. The thir index k is given above the columns; hence, eac component is differentiated with respect to th index above its column. The first 2 indices i M^{mnk} give the components of Y^{mn}. For instanc M^{cpa} must be equal to half of the partial derivativ of Y^{cp} with respect to e_a.

For a triode, the 2 matrices of the modulatio triadic are

$$M^{gnk} = \frac{1}{2} \times \begin{array}{c|cc} n \backslash k & g & p \\ \hline g & -\frac{1}{r_g^2}\frac{\partial r_g}{\partial e_g} & -\frac{1}{r_g^2}\frac{\partial r_g}{\partial e_p} \\ p & \frac{\partial G^{gp}}{\partial e_g} & \frac{\partial G^{gp}}{\partial e_p} \end{array}$$

$$M^{pnk} = \frac{1}{2} \times \begin{array}{c|cc} n \backslash k & g & p \\ \hline g & \frac{\partial G^{pg}}{\partial e_g} & \frac{\partial G^{pg}}{\partial e_p} \\ p & -\frac{1}{r_p^2}\frac{\partial r_p}{\partial e_g} & -\frac{1}{r_p^2}\frac{\partial r_p}{\partial e_p} \end{array} \qquad (7$$

In direct notation equation 72 can be writte

$$\boxed{\Delta \boldsymbol{i} = \boldsymbol{Y}\cdot\Delta \boldsymbol{e} + \Delta \boldsymbol{e}\cdot\boldsymbol{M}\cdot\Delta \boldsymbol{e}} \qquad (7$$

if the middle unit vector of the triadic $\boldsymbol{M}$ is assume to correspond to the free index m in M^{mnk}, represen ing the circuit of the current $\Delta \boldsymbol{i}$, and the other unit vectors are dot-multiplied by $\Delta \boldsymbol{e}$.

If y is replaced by e and x by i in Taylor's serie for small current changes the voltage chang

appearing across the terminals are, by interchanging upper and lower indices in Taylor's series:

$$\left.\begin{array}{l} \boxed{\Delta e_m = Z_{mn}\,\Delta i^n + Q_{mnk}\,\Delta i^n\,\Delta i^k} \\ \boxed{\Delta \boldsymbol{e} = \boldsymbol{Z}\cdot\Delta \boldsymbol{i} + \Delta \boldsymbol{i}\cdot \boldsymbol{Q}\cdot \Delta \boldsymbol{i}} \end{array}\right\} \quad (79)$$

where

$$Z_{mn} = \frac{\partial e_m}{\partial i^n}$$

$$Q_{mnk} = \frac{1}{2}\,\frac{\partial Z_{mn}}{\partial i^k} = \frac{1}{2}\,\frac{\partial^2 e_m}{\partial i^n \partial i^k}$$

The transformation formula of Q_{mnk} for a linear group of transformations is

$$Q_{\alpha\beta\gamma} = Q_{mnk} C_\alpha{}^m C_\beta{}^n C_\gamma{}^k \quad (80)$$

It should be remembered that these equations are valid only if some of the voltage changes are applied right at the terminals of the tube while the other terminals are short-circuited. If impedances exist between the tube and the applied voltages, or if the terminals are short-circuited through impedances, the equations still are valid, provided all the Δe represents the difference of potential across all the tube terminals. However, in most cases tube terminal voltages are not known, but the voltages applied in the outside network are known. It is shown later that in such cases an equation of the same form as equation 72 or 79 applies, but the new amplification and modulation polyadics are some functions of the foregoing polyadics and contain complex numbers instead of real numbers.

Complex Taylor's Series

In this section Taylor's series will be investigated where the variables and the coefficients are complex numbers instead of real numbers.

Let a set of n sinusoidal voltages be impressed on a nonlinear circuit, say a crystal detector in series with an impedance, each voltage being of different frequency; that is, let

$$\begin{array}{c|cccc} (\alpha) & (1) & (2) & & (n) \\ \hline \Delta e_{(\alpha)} = & \boxed{\Delta\hat{e}_{(\omega_1)}} & \boxed{\Delta\hat{e}_{(\omega_2)}} & \cdots & \boxed{\Delta\hat{e}_{(\omega_n)}} \end{array}$$

each $\Delta\hat{e}$ being a complex number

$$\Delta\hat{e}_1 = \Delta e_1 + j\Delta e_1' = \sqrt{2}\,(E_1 \cos \omega_1 t - E_1' \sin \omega_1 t)$$

The n voltage components may be arranged in a row to form a vector. Since transformation of the voltage components to other voltages is not intended in this paper, the closed index α in $e_{(\alpha)}$ shows that the components of $e_{(\alpha)}$ are arranged in a row, but no formula of transformation is associated with the index.[1Dc] Hence, $\Delta e_{(\alpha)}$ is a set of n scalars arranged in a row, a hypercomplex number of rank 1, and not a polyadic.

Due to the application of the n voltages with n different frequencies, the following sets of currents flow in the nonlinear circuit:

1. A set of n currents $\Delta i^{(\alpha)}$, each current having the frequency of the corresponding voltage

$$\begin{array}{c|cccc} (\alpha) & (1) & (2) & & (n) \\ \hline \Delta i^{(\alpha)} = & \boxed{\Delta\hat{i}^{(\omega_1)}} & \boxed{\Delta\hat{i}^{(\omega_2)}} & \cdots & \boxed{\Delta\hat{i}^{(\omega_n)}} \end{array}$$

2. A set of n^2 currents $\Delta i^{(\alpha)(\beta)}$, each having the frequency of the sum of 2 impressed voltage frequencies, including the case $\alpha = \beta$. These currents can be arranged in a square. They are caused by the curvature of the e-i curve.

3. A set of n^2 currents $\Delta i^{(\gamma)(\delta)}$, each having the frequency of the difference of 2 impressed voltage frequencies. The 2 preceding matrices are denoted as $\Delta i^{(\alpha)(\pm\beta)}$ so that

$$\Delta i^{(\alpha)(+\beta)} = \begin{array}{c|cccc} (\alpha)\backslash(+\beta) & (1) & (2) & & (n) \\ \hline (1) & \boxed{\Delta\hat{i}^{(\omega_1+\omega_1)}} & \boxed{\Delta\hat{i}^{(\omega_1+\omega_2)}} & \vdots & \boxed{\Delta\hat{i}^{(\omega_1+\omega_n)}} \\ (2) & \boxed{\Delta\hat{i}^{(\omega_2+\omega_1)}} & \boxed{\Delta\hat{i}^{(\omega_2+\omega_2)}} & \vdots & \boxed{\Delta\hat{i}^{(\omega_2+\omega_n)}} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ (n) & \boxed{\Delta\hat{i}^{(\omega_n+\omega_1)}} & \boxed{\Delta\hat{i}^{(\omega_n+\omega_2)}} & \vdots & \boxed{\Delta\hat{i}^{(\omega_n+\omega_n)}} \end{array}$$

A similar matrix exists for $\Delta i^{(\alpha)(-\beta)}$. The $2n^2$ currents are said to be product frequency currents, and $\Delta i^{(\alpha)(+\beta)}$ is a set of n^2 scalars arranged in a square, instead of a polyadic of rank 2.

4. A set of $4n^3$ currents $\Delta i^{(\alpha)(\pm\beta)(\pm\gamma)}$ arranged in 4 cubes, each having the frequency of the sums or differences of 3 of the impressed frequencies.

5. A set of $8n^4$ currents $\Delta i^{(\alpha)(\pm\beta)(\pm\gamma)(\pm\delta)}$ and so on.

Only the sets $\Delta i^{(\alpha)}$ and $\Delta i^{(\alpha)(\pm\beta)}$ are calculated here, giving $n + 2n^2$ components of current.

The first set of n currents $\Delta i^{(\alpha)}$ is calculated by the formula

$$\Delta i^{(\alpha)} = Y^{(\alpha)(\beta)}\,\Delta e_{(\beta)} \quad (81)$$

where $Y^{(\alpha)(\beta)}$ usually is a matrix having only diagonal components, each component being calculated for the frequency of the applied voltage; hence, in equation 81, $\alpha = \beta$. In the most general case, such an equality is not necessarily true, however.

The second set of $2n^2$ currents is calculated by the formula

$$\Delta i^{(\alpha)(\pm\beta)} = M^{(\alpha)(\pm\beta)(\gamma)(\delta)}\,\Delta e_{(\gamma)}\,\Delta e_{(\delta)} \quad (82)$$

where usually $\gamma = \alpha$, and $\delta = \beta$ and M are 2 hypercomplex numbers of rank 4. Their calculation is shown later.

The hypernumber $\Delta e_{(\gamma)}\,\Delta e_{(\delta)}$ represents $2n^2$ complex numbers (2 matrices) formed by all possible products of the components of $\Delta e_{(\gamma)}$, forming sum and difference frequency quantities.

In multiplying 2 complex numbers $\Delta\hat{e}_1 = A + jB$ and $\Delta\hat{e}_2 = C + jD$, each representing a sinusoidal function of different frequencies ω_1 and ω_2, the following must be noted:[1H]

1. The product with $\omega_1 + \omega_2$ frequencies is found by $(A + jB) \times (C + jD)$.

2. The product with $\omega_1 - \omega_2$ frequencies is found by $(A + jB) \times (C - jD)$.

3. The component complex numbers represent square-root-of-mean-square values, and the products represent peak values.

It should be noted that $M^{(\alpha)(\beta)(\gamma)(\delta)}$ with closed indices is a hypernumber of rank 4, but with open indices it is of rank 3. The additional rank is due to the additional rank of the currents it produces.

Equations 81 and 82 (a vector and a matrix equation) may be combined as

$$\boxed{\Delta i^{(\epsilon)} + \Delta i^{(\alpha)(\pm\beta)} = Y^{(\epsilon)(\sigma)}\,\Delta e_{(\sigma)} + M^{(\alpha)(\pm\beta)(\gamma)(\delta)}\,\Delta e_{(\gamma)}\,\Delta e_{(\delta)}} \quad (83)$$

representing $n + 2n^2$ equations. Since the components of a vector $\Delta i^{(\epsilon)}$ and a matrix $\Delta i^{(\alpha)(\beta)}$ cannot be added, the various sets of currents are kept separate.

Compound Series

Assume now that additional sets of voltages with $(\alpha \pm \beta)$, $(\alpha \pm \beta \pm \gamma)$ ... frequencies also are impressed on the system. The set of $2n^2$ voltages with product frequencies then produces a set of $2n^2$ currents of the same frequency

$$\Delta i^{(\alpha)(\pm\beta)} = Y^{(\alpha)(\pm\beta)(\gamma)(\pm\delta)} \Delta e_{(\gamma)(\pm\delta)} \tag{84}$$

where usually $\gamma = \alpha$, $\delta = \beta$, and the components of $Y^{(\alpha)(\pm\beta)(\gamma)(\pm\delta)}$ are calculated at the various product frequencies. The additional currents due to the curvature of the e-i curves will not be calculated here.

The hypernumber $\Delta e_{(\gamma)(\pm\delta)}$ contains $2n^2$ complex numbers representing product frequency impressed voltages, but they are not formed by products of fundamental frequency impressed voltages (as are the components of $\Delta e_{(\gamma)} \Delta e_{(\delta)}$), being independent of them.

Hence, if both fundamental frequency and product frequency voltages are impressed, the resultant fundamental and product frequency currents are

$$\begin{aligned}\Delta i^{(\epsilon)} + \Delta i^{(\alpha)(\pm\beta)} + \ldots = {} & Y^{(\epsilon)(\sigma)} \Delta e_{(\sigma)} + \\ & M^{(\alpha)(+\beta)(\gamma)(\delta)} \Delta e_{(\gamma)} \Delta e_{(\delta)} + \ldots + \\ & Y^{(\alpha)(\pm\beta)(\gamma)(\pm\delta)} \Delta e_{(\gamma)(\pm\delta)} + \ldots + \ldots\ldots\ldots\ldots\end{aligned} \tag{85}$$

There is a hypernumber form of Taylor's series for each set of impressed voltages, that is, one for $\Delta e_{(\alpha)}$, and another for $\Delta e_{(\alpha)(\pm\beta)}$, each set of voltage producing an infinite set of currents. Such a series is called a compound series. If only fundamental frequency voltages are impressed, the last term of equation 85 drops out.

Conversely, if both fundamental frequency and product frequency currents flow, the resultant fundamental frequency and product frequency terminal voltages are

$$\begin{aligned}\Delta e_{(\epsilon)} + \Delta e_{(\alpha)(\pm\beta)} + \ldots = {} & Z_{(\epsilon)(\sigma)} \Delta i^{(\sigma)} + \\ & Q_{(\alpha)(\pm\beta)(\gamma)(\delta)} \Delta i^{(\gamma)} \Delta i^{(\delta)} + \ldots + \\ & Z_{(\alpha)(\pm\beta)(\gamma)(\pm\delta)} \Delta i^{(\gamma)(\pm\delta)} + \ldots + \ldots\ldots\ldots\ldots\end{aligned} \tag{86}$$

where usually $\gamma = \alpha$, $\delta = \beta$, and $\epsilon = \sigma$.

The important fact should be noted that the variables Δi are not necessarily always arranged in a row to form a vector $\Delta i^{(\alpha)}$. Here some of the variables are arranged in squares, $\Delta i^{(\alpha)(\pm\beta)}$, and some in cubes. In modern matrix mechanics the variables are arranged in a square forming a matrix.

It is emphasized that all symbols occurring in this section, such as $Z_{(\epsilon)(\sigma)}$, are not "polyadics" (as defined in reference 1*Da*) but "hypercomplex numbers" or "hypernumbers," that is set of numbers arranged in a row, or a square, or a cube, etc., having, however, no equation of transformation associated with them, only rules of manipulation. The reason is that the various frequencies represented by the indices are not going to be transformed into another set of frequencies in the present problem.

Complex Series in Polyadic Form

Instead of a crystal detector let a multielectro
tube be connected to a network containing impe
ances. If along several co-ordinate axes (circui
a set of fundamental voltages $\Delta e_{(\alpha)}$ is impresse
then in each circuit several sets of currents of vario
frequencies flow.

Here 2 sets of indices must be introduced: (
open indices m, n, k, representing the various c
cuits of the system a, b, c, g, p and (2) closed indic
(α), (β), (γ), representing the various impressed fr
quencies ω_1, ω_2 ... ω_n.

In interconnecting networks by a transformati
tensor C_p^m only the open indices are transforme
For this reason, the closed indices are also call
dead indices. That is, the various circuits (repr
sented by the open indices) are going to be interco
nected by a transformation tensor C_α^m, but the va
ous frequencies (represented by the closed indice
will not be transformed, but will remain unchang
throughout the whole analysis.

In direct notation the open indices are not show
With both fundamental and product frequen
voltages applied along the various circuits, t
currents are (see equations 78 and 85)

$$\begin{aligned}\Delta \boldsymbol{i}^{(\epsilon)} + \Delta \boldsymbol{i}^{(\alpha)(\pm\beta)} + \ldots = {} & \boldsymbol{Y}^{(\epsilon)(\sigma)} \cdot \Delta \boldsymbol{e}_{(\sigma)} + \\ & \Delta \boldsymbol{e}_{(\gamma)} \cdot \boldsymbol{M}^{(\alpha)(\pm\beta)(\gamma)(\delta)} \cdot \Delta \boldsymbol{e}_{(\delta)} + \ldots + \\ & \boldsymbol{Y}^{(\alpha)(\pm\beta)(\gamma)(\pm\delta)} \cdot \Delta \boldsymbol{e}_{(\gamma)(\pm\delta)} + \ldots + \ldots\ldots\ldots \qquad (8\end{aligned}$$

The order of the polyadics is the same as in equatio
78 and the order of the closed indices is the same as
equation 85.

In index notation

$$\begin{aligned}\Delta i^{m(\epsilon)} + \Delta i^{m(\alpha)(\pm\beta)} + \ldots = {} & Y^{mn(\epsilon)(\sigma)} \Delta e_{n(\sigma)} + \\ & M^{mnk(\alpha)(\pm\beta)(\gamma)(\delta)} \Delta e_{n(\gamma)} \Delta e_{k(\delta)} + \ldots + \\ & Y^{mn(\alpha)(\pm\beta)(\gamma)(\pm\delta)} \Delta e_{n(\gamma)(\pm\delta)} + \ldots \qquad (8 \\ & + \ldots\ldots\ldots\end{aligned}$$

The order of the "open" indices is the same as
equation 72 and the order of the "closed" indices as
equation 85.

The open indices have covariant and contravaria
meaning, but the closed indices have no such si
nificance; hence, for a more compact notatio
the position of the closed indices may be chang
and the previous equation may be written

$$\boxed{\begin{aligned}\Delta i^{\cdot m}_{(\epsilon)} + \Delta i^{\cdot m}_{(\alpha)(\pm\beta)} + \ldots = {} & Y^{m\ n}_{(\epsilon)(\sigma)} \Delta e_n^{(\sigma)} + \\ & M^{mnk}_{(\alpha)(\pm\beta)(\gamma)(\delta)} \Delta e_n^{(\gamma)} \Delta e_k^{(\delta)} + \ldots + \\ & Y^{mn}_{(\alpha)(\pm\beta)(\gamma)(\pm\delta)} \Delta e_n^{(\gamma)(\pm\delta)} + \ldots + \ldots\ldots\ldots\end{aligned}} \qquad (8$$

The polyadics containing both open and clos
indices are compound polyadics.[1Dc] For instance:

1. $\Delta \boldsymbol{i}^{(\epsilon)}$ is a set of n vectors arranged in a column.
2. $\Delta \boldsymbol{e}_{(\alpha)(\beta)}$ is a set of n^2 vectors arranged in a square (figure 11
3. $\boldsymbol{Y}^{(\alpha)(\beta)}$ is a set of n^2 dyadics arranged in a square.
4. $\boldsymbol{M}^{(\alpha)(\beta)(\gamma)(\delta)}$ is a set of n^4 triadics arranged in n cubes, or one 4 dimensional cube.

Instead of writing n^2 different dyadics to repr
sent $\boldsymbol{Y}^{(\alpha)(\beta)}$ the procedure in calculation is
write first one dyadic $\boldsymbol{Y}$, with each component st
containing the closed indices, as $A^{(\alpha)(\beta)}$, thereby, t
operations represented by the open indices a

disposed of, leaving the variable closed indices. The closed indices thereafter are assumed to vary through their own range, each variable closed index assuming the range of fixed indices in succession.

Spinor Notation

If the components of the polyadics contain complex numbers $a + jb$, it is advantageous to use spin indices instead of tensor indices, that is, indices with dots over some of them.[1Ce,1Db] In calculating power, or in introducing new reference axes, spinor notation plays an important part. The dots over some of the indices also help to keep the correct order in multiplying several polyadics of various ranks. Spinor notation is valid, however, only if all complex numbers represent quantities of the same frequency.

Assuming the open indices as fixed indices and the closed ones as variable indices (if in one particular circuit all possible currents are considered), each complex number represents a quantity of different frequency. Hence, the closed indices cannot be spin indices in the present analysis.

Assuming the closed indices as fixed indices and the open ones as variable indices (that is, if the currents of one particular frequency in all circuits are considered), each complex number in a polyadic represents a quantity of the same frequency. Therefore, the open indices may be considered as spin indices.

In terms of spin indices the polyadics hitherto introduced are

$$e_{\dot{m}},\ i^m,\ \Delta e_{\dot{m}},\ \Delta i^m,\ Y^{m\dot{n}},\ Z_{\dot{m}n},\ M^{m\dot{n}\dot{k}},\ Q_{\dot{m}nk},\ D^{m\dot{n}\dot{k}\dot{h}}$$

If a voltage of the same frequency is applied in all the circuits, equation 72 can be written as 2 equations

$$\begin{aligned}\Delta i^m &= Y^{m\dot{n}}\,\Delta e_n\\ \Delta i^m &= M^{m\dot{n}\dot{k}}\,\Delta e_{\dot{n}}\,\Delta e_{\dot{k}}\end{aligned}$$

Once the position of the dots in e_m and i^m is determined by the invariance of $e_m i^m$, their position in the other polyadics automatically follows in the equations by the rule that the 2 dummy indices must be either both dotted or both undotted indices.

Where closed indices also exist, the foregoing 2 equations may be combined into one, since the closed indices separate the fundamental frequency currents from the product frequency currents and equation 89 may be written

$$\boxed{\begin{aligned}&\Delta \dot{i}^{m}_{(\epsilon)} + \Delta \dot{i}^{m}_{(\alpha)(\pm\beta)} + \ldots\\ &= Y^{m\ \dot{n}}_{(\epsilon)(\sigma)}\,\Delta e_{\dot{n}}^{(\sigma)} + M^{m\ \dot{n}\dot{k}}_{(\alpha)(\pm\beta)(\gamma)(\pm\delta)}\,\Delta e_{\dot{n}}^{(\gamma)}\,\Delta e_{\dot{k}}^{(\pm\delta)} + \ldots\\ &+ Y^{m\dot{n}}_{(\alpha)(\pm\beta)(\gamma)(\pm\delta)}\,\Delta e_{\dot{n}}^{(\gamma)(\pm\delta)} + \ldots\\ &+ \ldots\ldots\ldots\end{aligned}} \tag{90}$$

The Inverse of Taylor's Series

In the foregoing discussion, the different forms of Taylor's series were established. In the remaining treatment, the method of finding Δi as a function of Δe, if Δe is a known function of Δi, is investigated. If the Taylor's series

$$\Delta e = Z\Delta i + Q(\Delta i)^2 \tag{91}$$

is given, the analysis includes the method of finding the coefficients Y and M of the inverse Taylor's series

$$\Delta i = Y\Delta e + M(\Delta e)^2 \tag{92}$$

in terms of the known coefficients Z and Q.[5]

To find Y and M, substitute the second equation into the first. Neglecting terms of order higher than the second,

$$\begin{aligned}\Delta e &= Z(Y\Delta e + M\Delta e^2) + Q(Y\Delta e)^2\\ &= ZY\Delta e + (ZM + QY^2)\Delta e^2\end{aligned}$$

Equating corresponding coefficients of Δe and Δe^2 on both sides of the equation

$$\begin{aligned}1 &= ZY\\ 0 &= ZM + QY^2\end{aligned}$$

Solving for the Y and M

$$\boxed{Y = Z^{-1}} \tag{93}$$

$$\boxed{M = -QY^3} \tag{94}$$

To save space, M will be called the inverse of Q and Y the inverse of Z. Similarly, the current equation 92 will be called the inverse of the voltage equation 91. It should be noted that in finding the inverse of Q by Equation 94, the inverse of Z also must be known.

Inverse of the Polyadic Series

The same reasoning as in the previous analysis will be followed, except that each scalar will be replaced by a polyadic.

Where a triadic $\boldsymbol{M}$ and a dyadic $\boldsymbol{Z}$ are enclosed in parentheses as $(\boldsymbol{Z}\cdot\boldsymbol{M})$ it will be understood that the middle (free) unit vector of $\boldsymbol{M}$ is to be multiplied by the second unit vector of the dyadic $\boldsymbol{Z}$. The vectors or dyadics outside the parenthesis are to be multiplied by the first and third unit vectors of $\boldsymbol{M}$ as usual. In index notation the expression $\boldsymbol{e}' = \boldsymbol{e}\cdot(\boldsymbol{Z}\cdot\boldsymbol{M})\cdot\boldsymbol{e}$ would read as $e_\epsilon = M^{\alpha\beta\gamma}e_\beta e_\gamma Z_{\epsilon\alpha}$.

Hence, let the inverse of

$$\left.\begin{aligned}\Delta e &= \boldsymbol{Z}\cdot\Delta \boldsymbol{i} + \Delta \boldsymbol{i}\cdot\boldsymbol{Q}\cdot\Delta \boldsymbol{i}\\ \Delta e_{\dot{m}} &= Z_{\dot{m}n}\Delta i^n + Q_{\dot{m}nk}\Delta i^n\Delta i^k\end{aligned}\right\} \tag{95}$$

be defined as

$$\left.\begin{aligned}\Delta i &= \boldsymbol{Y}\cdot\Delta \boldsymbol{e} + \Delta \boldsymbol{e}\cdot\boldsymbol{M}\cdot\Delta \boldsymbol{e}\\ \Delta i^m &= Y^{m\dot{n}}\Delta e_{\dot{n}} + M^{m\dot{n}\dot{k}}\Delta e_{\dot{n}}\Delta e_{\dot{k}}\end{aligned}\right\} \tag{96}$$

where $\boldsymbol{Y}$ and $\boldsymbol{M}$ are unknown functions of $\boldsymbol{Z}$ and $\boldsymbol{Q}$.

Substituting the second equation into the first, since $\boldsymbol{Y}\cdot\Delta\boldsymbol{e} = \Delta\boldsymbol{e}\cdot\boldsymbol{Y}_t$,

$$\Delta e = \boldsymbol{Z}\cdot\boldsymbol{Y}\cdot\Delta\boldsymbol{e} + \Delta\boldsymbol{e}\cdot\boldsymbol{Y}_t\cdot\boldsymbol{Q}\cdot\boldsymbol{Y}\cdot\Delta\boldsymbol{e} + \Delta\boldsymbol{e}\cdot(\boldsymbol{Z}\cdot\boldsymbol{M})\cdot\Delta\boldsymbol{e}$$

Equating corresponding coefficients of $\Delta\boldsymbol{e}$

$$\begin{aligned}\boldsymbol{I} &= \boldsymbol{Z}\cdot\boldsymbol{Y}\\ 0 &= \boldsymbol{Y}_t\cdot\boldsymbol{Q}\cdot\boldsymbol{Y} + (\boldsymbol{Z}\cdot\boldsymbol{M})\end{aligned}$$

Solving for $\boldsymbol{Y}$ and $\boldsymbol{M}$

$$\boxed{\boldsymbol{Y} = \boldsymbol{Z}^{-1}} \tag{97}$$

$$\boxed{\boldsymbol{M} = -\ \boldsymbol{Y}_t\cdot(\boldsymbol{Y}\cdot\boldsymbol{Q})\cdot\boldsymbol{Y}} \tag{98}$$

In spinor notation

$$\boxed{M^{m\dot{n}k} = -\ Q_{\dot{h}fg}\, Y^{m\dot{h}}\, Y^{f\dot{n}}\, Y^{gk}}$$

It should be noted that the free index $\dot{h}$ of $Q_{\dot{h}fg}$ is multiplied by the second index of $Y^{m\dot{h}}$, but the other indices of $Q_{\dot{h}fg}$ are multiplied by the first indices of $Y^{f\dot{n}}$. This is important, for $Y^{m\dot{h}}$ is not a symmetrical matrix, although $Y^{(\alpha)(\beta)}$ is a symmetrical matrix, and any change in the correct order would produce an incorrect result. Spinor notation automatically accounts for this order in the multiplication.

Inverse of the Complex Series

Let the voltage equation

$$\Delta e_{(\epsilon)} + \Delta e_{(\alpha)(\pm\beta)} = Z_{(\epsilon)(\sigma)}\,\Delta i^{(\sigma)} + Q_{(\alpha)(\pm\beta)(\gamma)(\delta)}\,\Delta i^{(\gamma)}\,\Delta i^{(\delta)} + Z_{(\alpha)(\pm\beta)(\gamma)(\pm\delta)}\,\Delta i^{(\gamma)(\pm\delta)} \tag{99}$$

be given and let its inverse be calculated

$$\Delta i^{(\epsilon)} + \Delta i^{(\alpha)(\pm\beta)} = Y^{(\epsilon)(\sigma)}\,\Delta e_{(\sigma)} + M^{(\alpha)(\pm\beta)(\gamma)(\delta)}\,\Delta e_{(\gamma)}\,\Delta e_{(\delta)} + Y^{(\alpha)(\pm\beta)(\gamma)(\pm\delta)}\,\Delta e_{(\gamma)(\pm\delta)} \tag{100}$$

where Y and M are unknown functions of Z and Q.

Following the steps of the previous sections, let the second equation be substituted into the first. Neglecting higher than second order terms

$$\begin{aligned}\Delta e_{(\epsilon)} + \Delta e_{(\alpha)(\pm\beta)} &= Z_{(\epsilon)(\sigma)}\,Y^{(\sigma)(\nu)}\,\Delta e_{(\nu)} + \\ &\quad Q_{(\alpha)(\pm\beta)(\gamma)(\delta)}\,Y^{(\gamma)(\sigma)}\,Y^{(\delta)(\nu)}\,\Delta e_{(\sigma)}\,\Delta e_{(\nu)} + \\ &\quad Z_{(\alpha)(\pm\beta)(\gamma)(\pm\delta)}\,M^{(\gamma)(\pm\delta)(\sigma)(\nu)}\,\Delta e_{(\sigma)}\,\Delta e_{(\nu)} + \\ &\quad Z_{(\alpha)(\pm\beta)(\gamma)(\pm\delta)}\,Y^{(\gamma)(\pm\delta)(\sigma)(\pm\nu)}\,\Delta e_{(\sigma)(\pm\nu)}\end{aligned}$$

Equating corresponding coefficients of Δe on both sides of the equation

$$I_{(\epsilon)}^{(\nu)} = Z_{(\epsilon)(\sigma)}\,Y^{(\sigma)(\nu)}$$

$$I_{(\alpha)(\pm\beta)}^{(\sigma)(\pm\nu)} = Z_{(\alpha)(\pm\beta)(\gamma)(\pm\delta)}\,Y^{(\gamma)(\pm\delta)(\sigma)(\pm\nu)}$$

$$0 = Q_{(\alpha)(\pm\beta)(\gamma)(\delta)}\,Y^{(\gamma)(\sigma)}\,Y^{(\delta)(\nu)} + Z_{(\alpha)(\pm\beta)(\gamma)(\pm\delta)}\,M^{(\gamma)(\pm\delta)(\sigma)(\nu)}$$

Solving for Y and M

$$\boxed{Y^{(\alpha)(\beta)} = \text{inverse of } Z_{(\beta)(\alpha)}} \tag{101}$$

$$\boxed{Y^{(\alpha)(\pm\beta)(\gamma)(\pm\delta)} = \text{inverse of } Z_{(\gamma)(\pm\delta)(\alpha)(\pm\beta)}} \tag{102}$$

$$\boxed{M^{(\alpha)(\pm\beta)(\gamma)(\delta)} = -\ Q_{(\sigma)(\pm\nu)(\epsilon)(\omega)}\ Y^{(\alpha)(\pm\beta)(\sigma)(\pm\nu)}\,Y^{(\epsilon)(\gamma)}\,Y^{(\omega)(\delta)}} \tag{103}$$

where $\gamma = \alpha$ and $\delta = \beta$.

Since in $\Delta e_{(\epsilon)} = Z_{(\epsilon)(\sigma)}\,\Delta i^{(\sigma)}$ the matrix $Z_{(\epsilon)(\sigma)}$ usually contains only diagonal components (one current producing only one voltage of its own frequency) its inverse $Y^{(\epsilon)(\sigma)}$ is calculated in such cases by taking the inverse of each component separately.

Similarly, since in $\Delta e_{(\alpha)(\pm\beta)} = Z_{(\alpha)(\pm\beta)(\gamma)(\pm\delta)}\,\Delta i^{(\gamma)(\pm\delta)}$ each current usually produces only one voltage of its own frequency (since $\gamma = \alpha$ and $\delta = \beta$) the inverse of $Z_{(\alpha)(\pm\beta)(\gamma)(\pm\delta)}$ is calculated by taking the inverse of each of its components.

Inverse of the Complex Polyadic Series

Let the equation

$$\Delta \boldsymbol{e}_{(\epsilon)} + \Delta \boldsymbol{e}_{(\alpha)(\pm\beta)} = \boldsymbol{Z}_{(\epsilon)(\sigma)}\cdot\Delta\boldsymbol{i}^{(\sigma)} + \Delta\boldsymbol{i}^{(\gamma)}\cdot\boldsymbol{Q}_{(\alpha)(\pm\beta)(\gamma)(\delta)}\cdot\Delta\boldsymbol{i}^{(\delta)} + \boldsymbol{Z}_{(\alpha)(\pm\beta)(\gamma)(\pm\delta)}\cdot\Delta\boldsymbol{i}_{(\gamma)(\pm\delta)} \quad (1$$

be given and let its inverse be calculated

$$\Delta\boldsymbol{i}^{(\epsilon)} + \Delta\boldsymbol{i}^{(\alpha)(\pm\beta)} = \boldsymbol{Y}^{(\epsilon)(\sigma)}\cdot\Delta\boldsymbol{e}_{(\sigma)} + \Delta\boldsymbol{e}_{(\gamma)}\cdot\boldsymbol{M}^{(\alpha)(\pm\beta)(\gamma)(\delta)}\cdot\Delta\boldsymbol{e}_{(\delta)} + \boldsymbol{Y}^{(\alpha)(\pm\beta)(\gamma)(\pm\delta)}\cdot\Delta\boldsymbol{e}_{(\gamma)(\pm\delta)} \quad (1$$

in which $\boldsymbol{Y}$ and $\boldsymbol{M}$ are unknown functions of $\boldsymbol{Z}$ a $\boldsymbol{Q}$.

Substituting the second equation into the fir

$$\begin{aligned}\Delta\boldsymbol{e}_{(\epsilon)} + \Delta\boldsymbol{e}_{(\alpha)(\pm\beta)} &= \boldsymbol{Z}_{(\epsilon)(\sigma)}\cdot\boldsymbol{Y}^{(\sigma)(\nu)}\cdot\Delta\boldsymbol{e}_{(\nu)} + \\ &\quad \Delta\boldsymbol{e}_{(\sigma)}\cdot\boldsymbol{Y}_t^{(\gamma)(\sigma)}\cdot\boldsymbol{Q}_{(\alpha)(\pm\beta)(\gamma)(\delta)}\cdot\boldsymbol{Y}^{(\delta)(\nu)}\cdot\Delta\boldsymbol{e}_{(\nu)} + \\ &\quad \Delta\boldsymbol{e}_{(\sigma)}\cdot(\boldsymbol{Z}_{(\alpha)(\pm\beta)(\gamma)(\pm\delta)}\cdot\boldsymbol{M}^{(\gamma)(\pm\delta)(\sigma)(\nu)})\cdot\Delta\boldsymbol{e}_{(\nu)} + \\ &\quad \boldsymbol{Z}_{(\alpha)(\pm\beta)(\gamma)(\pm\delta)}\cdot\boldsymbol{Y}^{(\gamma)(\pm\delta)(\sigma)(\pm\nu)}\cdot\Delta\boldsymbol{e}_{(\sigma)(\pm\nu)}\end{aligned}$$

Equating corresponding coefficients of $\Delta\boldsymbol{e}$

$$\boldsymbol{I}_{(\epsilon)}^{(\nu)} = \boldsymbol{Z}_{(\epsilon)(\sigma)}\cdot\boldsymbol{Y}^{(\sigma)(\nu)}$$

$$\boldsymbol{I}_{(\alpha)(\pm\beta)}^{(\sigma)(\pm\nu)} = \boldsymbol{Z}_{(\alpha)(\pm\beta)(\gamma)(\pm\delta)}\cdot\boldsymbol{Y}^{(\gamma)(\pm\delta)(\sigma)(\pm\nu)}$$

$$0 = \boldsymbol{Y}_t^{(\gamma)(\sigma)}\cdot\boldsymbol{Q}_{(\alpha)(\pm\beta)(\gamma)(\delta)}\cdot\boldsymbol{Y}^{(\delta)(\nu)} + (\boldsymbol{Z}_{(\alpha)(\pm\beta)(\gamma)(\pm\delta)}\cdot\boldsymbol{M}^{(\gamma)(\pm\delta)(\sigma)}$$

Solving for the unknown $\boldsymbol{Y}$ and $\boldsymbol{M}$

$$\boxed{\boldsymbol{Y}^{(\alpha)(\beta)} = \boldsymbol{Z}^{-1}_{(\alpha)(\beta)}} \quad (1$$

$$\boxed{\boldsymbol{Y}^{(\alpha)(\pm\beta)(\gamma)(\pm\delta)} = \boldsymbol{Z}^{-1}_{(\alpha)(\pm\beta)(\gamma)(\pm\delta)}}$$

$$\boldsymbol{M}^{(\alpha)(\pm\beta)(\gamma)(\delta)} = -\ \boldsymbol{Y}_t^{(\epsilon)(\gamma)}\cdot(\boldsymbol{Y}^{(\alpha)(\pm\beta)(\sigma)(\pm\nu)}\cdot\boldsymbol{Q}_{(\sigma)(\pm\nu)(\epsilon)(\omega)})\cdot\boldsymbol{Y}^{(\omega)} \quad (10$$

In spinor notation the last equation, by changi the position of the closed indices and using sp indices, is

$$\boxed{M^{m\dot{n}k}_{(\alpha)(\pm\beta)(\gamma)(\delta)} = -\ Q^{(\sigma)(\pm\nu)(\epsilon)(\omega)}_{\dot{h}fg}\ Y^{m\dot{h}}_{(\alpha)(\pm\beta)(\sigma)(\pm\nu)}\ Y^{f\dot{n}}_{(\epsilon)(\gamma)}\ Y^{g\dot{k}}_{(\omega)(\delta)}}$$

It should be noted that on both sides of the equ tion the dotted and undotted, open and closed, upp and lower indices are balanced.

It should also be noted that in the last equati representing the most complete polyadic form, t same number of symbols (basic letters) occur as equation 94 representing the simplest possible sca form (Y^3 actually stands for YYY).

III—Multielectrode Tubes as Modulators and Detectors

Interconnection of Linear Systems

How to add a linear network to a linear tube c cuit has been shown already. Since the design co stants of the tube are admittances $\boldsymbol{Y}_1$, while those the network are impedances $\boldsymbol{Z}_2$, their interconnecti consists of the following steps:

1. Given $\boldsymbol{Y}_1$ of the tube and $\boldsymbol{Z}_2$ of the network
2. Find the inverse of $\boldsymbol{Y}_1$, giving $\boldsymbol{Z}_1$
3. Add to it $\boldsymbol{Z}_2$ of the network as $\boldsymbol{Z}_1 + \boldsymbol{Z}_2$
4. Transform their sum by the transformati tensor $\boldsymbol{C}$ representing their manner of interco nection

$$\boldsymbol{Z} = \boldsymbol{C}_t\cdot(\boldsymbol{Z}_1 + \boldsymbol{Z}_2)\cdot\boldsymbol{C}$$

5. Find its inverse $\boldsymbol{Y} = \boldsymbol{Z}^{-1}$

These steps in terms of polyadics are equivalent to setting up the following series of equations:

1. Given the current equation of the tube

$\Delta \boldsymbol{i}_1 = \boldsymbol{Y}_1 \cdot \Delta \boldsymbol{i}_1$

and the voltage equation of the network

$\Delta \boldsymbol{e}_2 = \boldsymbol{Z}_2 \cdot \Delta \boldsymbol{i}_2$

2. Find the inverse equation of the tube

$\Delta \boldsymbol{e}_1 = \boldsymbol{Z}_1 \cdot \Delta \boldsymbol{i}_1$

3. Add the 2 voltage equations together

$\Delta \boldsymbol{e}_1 + \Delta \boldsymbol{e}_2 = (\boldsymbol{Z}_1 + \boldsymbol{Z}_2) \cdot (\Delta \boldsymbol{i}_1 + \Delta \boldsymbol{i}_2)$

4. Transform them by $\boldsymbol{C}$ to

$\Delta \boldsymbol{e} = \boldsymbol{Z} \cdot \Delta \boldsymbol{i}$

5. Find its inverse equation

$\Delta \boldsymbol{i} = \boldsymbol{Y} \cdot \Delta \boldsymbol{e}$

It should be noted that only voltage equations can be added, since all their known geometric objects, namely, Z_{mn} and Q_{mnk}, have only covariant indices.

Interconnection of Nonlinear Systems

Similar steps are followed when nonlinear networks, or a nonlinear network with a linear network, is interconnected. That is, the following 5 sets of equations will be established:

1. Given the current equation of the tube

$\Delta \boldsymbol{i}_1 = \boldsymbol{Y} \cdot \Delta \boldsymbol{e}_1 + \Delta \boldsymbol{e}_1 \cdot \boldsymbol{M}_1 \cdot \Delta \boldsymbol{e}_1$

and the voltage equation of a nonlinear (or linear) network

$\Delta \boldsymbol{e}_2 = \boldsymbol{Z}_2 \cdot \Delta \boldsymbol{i}_2 + \Delta \boldsymbol{i}_2 \cdot \boldsymbol{Q}_2 \cdot \Delta \boldsymbol{i}_2$

The last term is zero in a linear network, and the last equation may be a current equation instead of a voltage equation.

2. Find the inverse equation of the tube

$\Delta \boldsymbol{e}_1 = \boldsymbol{Z}_1 \cdot \Delta \boldsymbol{i}_1 + \Delta \boldsymbol{i}_1 \cdot \boldsymbol{Q}_1 \cdot \Delta \boldsymbol{i}_1$

3. Add the 2 voltage equations

$$\Delta \boldsymbol{e}_1 + \Delta \boldsymbol{e}_2 = (\boldsymbol{Z}_1 + \boldsymbol{Z}_2) \cdot (\Delta \boldsymbol{i}_1 + \Delta \boldsymbol{i}_2) + (\Delta \boldsymbol{i}_1 + \Delta \boldsymbol{i}_2) \cdot (\boldsymbol{Q}_1 + \boldsymbol{Q}_2) \cdot (\Delta \boldsymbol{i}_1 + \Delta \boldsymbol{i}_2)$$

4. Transform them by $\boldsymbol{C}$ representing the manner of interconnection of the 2 nonlinear systems, giving

$\Delta \boldsymbol{e} = \boldsymbol{Z} \cdot \Delta \boldsymbol{i} + \Delta \boldsymbol{i} \cdot \boldsymbol{Q} \cdot \Delta \boldsymbol{i}$

5. Find the inverse equation

$\Delta \boldsymbol{i} = \boldsymbol{Y} \cdot \Delta \boldsymbol{e} + \Delta \boldsymbol{e} \cdot \boldsymbol{M} \cdot \Delta \boldsymbol{e}$

For rapid calculation the following steps will be used, employing only polyadics, instead of equations:

1. Given $\boldsymbol{Y}_1$ and $\boldsymbol{M}_1$ of the tube and $\boldsymbol{Z}_2$ (and $\boldsymbol{Q}_2$ if there is any) of the network
2. Find the inverse of $\boldsymbol{Y}_1$ and $\boldsymbol{M}_1$ giving $\boldsymbol{Z}_1$ and $\boldsymbol{Q}_1$
3. Find $\boldsymbol{Z}_1 + \boldsymbol{Z}_2$ and $\boldsymbol{Q}_1 + \boldsymbol{Q}_2$ (if any $\boldsymbol{Q}_2$)
4. Transform them by the transformation tensor $\boldsymbol{C}$ as

$$\boldsymbol{Z} = \boldsymbol{C}_t \cdot (\boldsymbol{Z}_1 + \boldsymbol{Z}_2) \cdot \boldsymbol{C} \tag{108}$$

$$\boldsymbol{Q} = \boldsymbol{C}_t \cdot [\boldsymbol{C} \cdot (\boldsymbol{Q}_1 + \boldsymbol{Q}_2)] \cdot \boldsymbol{C} \tag{109}$$

5. Find the inverse of $\boldsymbol{Z}$ and $\boldsymbol{Q}$, giving $\boldsymbol{Y}$ and $\boldsymbol{M}$

This reasoning is not limited to the case considered. Similar steps are followed in the analysis:

1. If more than 2 terms of Taylor's series are used.
2. If more than 2 nonlinear or linear systems are interconnected.
3. If either the voltage or the current equations of the systems to be interconnected are given.
4. If the transformation tensor $\boldsymbol{C}$ is a function of the variables, instead of being constant. In that case the transformation formulas of $\boldsymbol{Z}$ and $\boldsymbol{Q}$ as given in equations 108 and 109 are more complicated.

The steps followed in part I are special cases of the steps followed here. In this part the equations of a nonlinear tube connected to a linear network are calculated in detail, but the reasoning is quite general. It is valid for the interconnection of any number and type of nonlinear and linear systems, not necessarily tubes or electric circuits.

When several tubes are interconnected through a network, then the set-up is divided into 2 groups. The first group comprises all the individual tubes without any interconnection between them, the second group comprises the linear network.

Simplifications in Tube Circuits

In thermionic tube circuits the polyadics of the previous part assume special forms, and the following simplifications may be introduced:

1. For a multielectrode tube $\boldsymbol{Y}_1^{(\alpha)(\beta)}$, $\boldsymbol{M}_1^{(\alpha)(\pm\beta)(\gamma)(\delta)}$ and their inverse contain only real numbers; hence, the closed indices may be removed from them, for their components are independent of the frequency of the impressed voltages.

2. In the impedance and admittance dyadics of the outside network all components are zero unless 2 of the closed indices are identical. For instance, the compound dyadic $\boldsymbol{Z}_{(\alpha)(\beta)}$ is

$$\boldsymbol{Z}_{(\alpha)(\beta)} = \begin{array}{c|cccc} {}_{(\alpha)}\diagdown^{(\beta)} & (1) & (2) & & (n) \\ \hline (1) & \boxed{\boldsymbol{Z}_{\omega 1}} & \boxed{0} & \vdots & \boxed{0} \\ (2) & \boxed{0} & \boxed{\boldsymbol{Z}_{\omega 2}} & \vdots & \boxed{0} \\ & \cdots & \cdots & \cdots & \cdots \\ (n) & \boxed{0} & \boxed{0} & \vdots & \boxed{\boldsymbol{Z}_{\omega n}} \end{array} \tag{110}$$

Hence $\boldsymbol{Z}_{(\alpha)(\beta)}$ may also be considered as a set of n dyadics arranged in a row $\boldsymbol{Z}_{(\alpha)}$

$$\boldsymbol{Z}_{(\alpha)} = \begin{array}{cccc} (1) & (2) & & (n) \\ \boxed{\boldsymbol{Z}_{\omega 1}} & \boxed{\boldsymbol{Z}_{\omega 2}} & \ldots & \boxed{\boldsymbol{Z}_{\omega n}} \end{array} \tag{111}$$

Similarly $\boldsymbol{Y}^{(\beta)(\alpha)}$ the inverse of $\boldsymbol{Z}_{(\alpha)(\beta)}$, may be considered as a set of n dyadics arranged in a row

$$\boldsymbol{Y}^{(\alpha)} = \begin{array}{cccc} (1) & (2) & & (n) \\ \boxed{\boldsymbol{Z}_{\omega 1}^{-1}} & \boxed{\boldsymbol{Z}_{\omega 2}^{-1}} & \ldots & \boxed{\boldsymbol{Z}_{\omega n}^{-1}} \end{array} \tag{112}$$

It should be noted that the inverse of $\boldsymbol{Z}_{(\alpha)(\beta)}$ is calculated by finding the inverse of each of its component dyadics. They, in turn, all have identical forms, except for frequencies; also $Y^{(\alpha)(\pm\beta)(\gamma)(\pm\delta)}$ where $\gamma = \alpha$ and $\delta = \beta$, may be replaced by $Y^{(\alpha)(\pm\beta)}$, each component of the 2 matrices containing $\boldsymbol{Z}_{\omega 1 + \omega 2}^{-1}$, all having identical form, unless calculated for different product frequencies.

3. If power is not calculated, the spin indices may be replaced by tensor indices (the dots over the indices may be omitted).

Let a crystal detector be connected in series with an impedance Z_2 (figure 8) and let a voltage vector $\Delta e_{(\alpha)}$ be impressed across the whole system. The 5 steps in its analysis are:

1. The impedance of the outside impedance is $Z_{2(\alpha)}$, forming a set of n impedances, say

$$Z_{2(\alpha)} = \begin{array}{cccc} (1) & (2) & & (n) \\ \boxed{R + j\omega_1 L} & \boxed{R + j\omega_2 L} & \cdots & \boxed{R + j\omega_n L} \end{array}$$

The constants of the crystal are $Y_1 = \partial i/\partial e$ and $M_1 = (^1/_2)\partial^2 i/\partial e^2$, both real numbers.

2. The inverse of Y_1 and M_1 is, by equations 93 and 94,

$$Z_1 = Y_1^{-1}$$

$$Q_1 = -M_1 Z_1^3$$

both being real numbers. Z_1 may also be written as $Z_{1(\alpha)}$

$$Z_{1(\alpha)} = \begin{array}{cccc} (1) & (2) & & (n) \\ \boxed{r} & \boxed{r} & \cdots & \boxed{r} \end{array}$$

3. The sum of the 2 impedances is $Z_{1(\alpha)} + Z_{2(\alpha)}$.

4. The transformation tensor is superfluous in this simple case. The corresponding components of $Z_{1(\alpha)}$ and $Z_{2(\alpha)}$ are simply added, as

$$Z_{(\alpha)} = \begin{array}{ccc} (1) & & (n) \\ \boxed{r + R + j\omega_1 L} & \cdots & \boxed{r + R + j\omega_n L} \end{array}$$

5. The inverses of $Z_{(\alpha)}$ and Q are, by equations 101 and 103,

$$\boxed{Y^{(\alpha)} = Z^{-1}_{(\alpha)}} \quad \text{or} \quad \boxed{Y^{(\alpha)(\beta)} = Z^{-1}_{(\beta)(\alpha)}}$$

$$\boxed{M^{(\alpha)(\beta)(\gamma)(\pm\delta)} = M_1 Z_1^3 Y^{(\alpha)} Y^{(\beta)} Y^{(\gamma)(\pm\delta)}}$$

where $\gamma = \alpha$ and $\delta = \beta$. In a still simpler notation

$$M^{(\alpha)(\beta)(\alpha\pm\beta)} = M_1 Z_1^3 Y^{(\alpha)} Y^{(\beta)} Y^{(\alpha\pm\beta)}$$

where α and β vary through the range of the impressed frequencies $1 \ldots n$.

For instance, if only the plate circuit of a triode, containing an outside impedance Z_p, is considered,

$$Z_1 = r_p;\ Z_{2(\alpha)} = Z_{p(\alpha)}$$

$$Y^{(\alpha)} = 1/(r_p + Z_{p(\alpha)})$$

$$M_1 = \frac{1}{2}\frac{\partial Y_1}{\partial e_p} = \frac{1}{2}\frac{\partial(1/r_p)}{\partial e_p} = -\frac{1}{2r_p^2}\frac{\partial r_p}{\partial e_p}$$

and the equations of the plate currents are:

1. For fundamental frequency currents

$$\Delta i^{p(\alpha)} = \frac{1}{r_p + Z_{p(\alpha)}}\Delta e_{p(\alpha)}$$

2. For product frequency currents

$$\Delta i^{p(\alpha)(\pm\beta)} = -\frac{r_p}{2[r_p + Z_{p(\alpha)}][r_p + Z_{p(\beta)}][r_p + Z_{p(\alpha)(\pm\beta)}]} \times \frac{\partial r_p}{\partial e_p}\Delta e_{p(\alpha)}\Delta e_{p(\beta)}$$

If the set of impressed voltages is

$$\Delta e_{(\alpha)} = \begin{array}{ccc} (1) & (2) & (3) \\ \boxed{\Delta \hat{e}_{\omega_1}} & \boxed{\Delta \hat{e}_{\omega_2}} & \boxed{\Delta \hat{e}_{\omega_3}} \end{array}$$

then, for instance when $\alpha = \omega_1$ and $\beta = \omega_3$ one of t
2 product frequency currents is

$$\Delta \hat{i}^{p(\omega_1+\omega_3)} = -\frac{r_p}{2(r_p + Z_{p\omega_1})(r_p + Z_{p\omega_3})(r_p + Z_{p(\omega_1+\omega_3)})} \times \frac{\partial r_p}{\partial e_p}\Delta \hat{e}_{\omega_1}\Delta \hat{e}$$

where $Z_{p\omega_1} = R_p + j\omega_1 L_p$. The other product fr
quency current is $\Delta \hat{i}^{p(\omega_1-\omega_3)}$.

Altogether there are $2n^2 = 18$ such $\Delta \hat{i}^p$ term
assuming for α and β the various frequencies
ω_2, and ω_3. These equations agree with equatio
98b and 100 of a book of McIlwain and Brainer

INTERCONNECTION OF MULTIELECTRODE TUBES

Let a multielectrode tube (or several tubes)
connected to an outside network $\mathbf{Z}_2$ and let a set
n voltages $\Delta \mathbf{e}_{(\alpha)}$ be impressed at several points k
the outside network. The 5 steps in the analysis ar

1. The constants of the tube (or tubes) are $\mathbf{Y}_1$ and $\mathbf{M}_1$, all their co
ponents containing only real numbers. The constants of the n
work are $\mathbf{Z}_{2(\alpha)}$.

2. The inverse of $\mathbf{Y}_1$ and $\mathbf{M}_1$ are, from equations 97 and 98,

$$\mathbf{Z}_1 = \mathbf{Y}_1^{-1}$$

$$\mathbf{Q}_1 = -\mathbf{Z}_{1t}\cdot(\mathbf{Z}_1\cdot\mathbf{M}_1)\cdot\mathbf{Z}_1$$

3. The sum of the 2 $\mathbf{Z}$ matrices is $\mathbf{Z}_{1(\alpha)} + \mathbf{Z}_{2(\alpha)}$.

4. Transforming by $\mathbf{C}$

$$\mathbf{Z}_{(\alpha)} = \mathbf{C}_t\cdot[\mathbf{Z}_{1(\alpha)} + \mathbf{Z}_{2(\alpha)}]\cdot\mathbf{C} \qquad (11$$

$$\mathbf{Q} = -\mathbf{C}_t\cdot\mathbf{Z}_{1t}\cdot(\mathbf{C}\cdot\mathbf{Z}_1\cdot\mathbf{M}_1)\cdot\mathbf{Z}_1\cdot\mathbf{C} \qquad (11$$

5. The inverse of $\mathbf{Z}_{(\alpha)}$ is, by equation 106,

$$\boxed{\mathbf{Y}^{(\alpha)} = \mathbf{Z}^{-1}_{(\alpha)}}$$

or

$$\boxed{\mathbf{Y}^{(\alpha)(\beta)} = \mathbf{Z}^{-1}_{(\beta)(\alpha)}}$$

Since by equation 107 the inverse of $\mathbf{Q}$ is

$$\mathbf{M}^{(\alpha)(\pm\beta)(\gamma)(\delta)} = -\mathbf{Y}_t^{(\gamma)}\cdot(\mathbf{Y}^{(\alpha)(\pm\beta)}\cdot\mathbf{Q})\cdot\mathbf{Y}^{(\delta)}$$

substituting $\mathbf{Q}$ from equation 114

$$\boxed{\mathbf{M}^{(\alpha)(\pm\beta)(\gamma)(\delta)} = \mathbf{Y}_t^{(\gamma)}\cdot\mathbf{C}_t\cdot\mathbf{Z}_{1t}\cdot(\mathbf{Y}^{(\alpha)(\pm\beta)}\cdot\mathbf{C}\cdot\mathbf{Z}_1\cdot\mathbf{M})\cdot\mathbf{Z}_1\cdot\mathbf{C}\cdot\mathbf{Y}^{(\delta)}} \qquad (11$$

where $\gamma = \alpha$ and $\delta = \beta$. Also $\mathbf{Y}^{(\gamma)}$ and $\mathbf{Y}^{(\delta)}$ are calculated
fundamental frequencies, and $\mathbf{Y}^{(\alpha)(\pm\beta)}$ at product frequencies.

It has been shown in part I that the effect of
is to change the unit vectors $\mathbf{a}_1, \mathbf{b}_1 \ldots \mathbf{p}_1$ of the tu
and $\mathbf{a}_2, \mathbf{b}_2 \ldots \mathbf{p}_2$ of the network to $\mathbf{a}, \mathbf{b} \ldots \mathbf{p}$. Th
transformation may be done on the original polyadi
without the aid of $\mathbf{C}$. With this understanding,
may be omitted from equation 115 in some case
simplifying it to

$$\mathbf{M}^{(\alpha)(\pm\beta)(\gamma)(\delta)} = \mathbf{Y}_t^{(\gamma)}\cdot\mathbf{Z}_{1t}\cdot(\mathbf{Y}^{(\alpha)(\pm\beta)}\cdot\mathbf{Z}_1\cdot\mathbf{M}_1)\cdot\mathbf{Z}_1\cdot\mathbf{Y}^{(\delta)} \qquad (11$$

The final currents consist of: (1) the fundament
frequency currents

$$\boxed{\Delta \mathbf{i}^{(\alpha)} = \mathbf{Y}^{(\alpha)(\beta)}\cdot\Delta \mathbf{e}_{(\beta)}}$$

which are needed in amplification and oscillation studies; and (2) the product frequency currents

$$\boxed{\Delta i^{(\alpha)(\pm\beta)} = \Delta e_{(\gamma)} \cdot M^{(\alpha)(\pm\beta)(\gamma)(\delta)} \cdot \Delta e_{(\delta)}}$$

which are needed in modulation and detection studies.

The compound polyadic $M^{(\alpha)(\pm\beta)(\gamma)(\delta)}$ (or rather $M^{(\alpha)(\pm\beta)(\gamma)(\pm\delta)}$) contains $4n^4$ triadics M. Since only those triadics in which $\alpha = \gamma$ and $\beta = \delta$ are not zero, there are $2n^2$ triadics M not equal to zero. They can be arranged in 2 squares each containing n^2 triadics; one giving $\alpha + \beta$, and the other $\alpha - \beta$

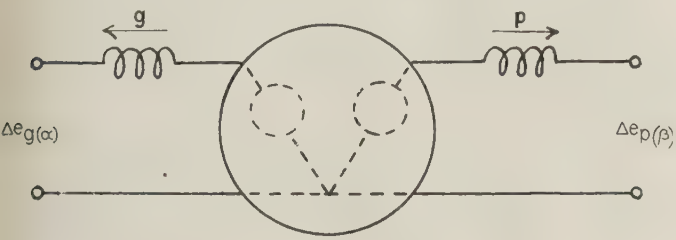

Fig. 9. Diagram showing voltages applied to a triode connected to 2 external impedances

frequency currents (figure 12). Each triadic contains currents of the same frequency flowing in the various circuits.

If k of the circuits has an impressed voltage vector $\Delta e_{(\alpha)}$, each circuit containing n voltages of different frequencies, each of the possible k^2 products produces $2n^2$ product frequency currents in each circuit; that is, in each circuit flow $2n^2k^2$ product frequency currents and nk fundamental frequency currents.

In spinor notation equation 116 is

$$M^{\dot{m}\dot{n}k}_{(\alpha)(\pm\beta)(\gamma)(\delta)} = M^{h\dot{f}\dot{g}} Z_{\dot{d}h} Z_{fc} Z_{\dot{g}b} Y^{\dot{m}d}_{(\alpha)(\pm\beta)} Y^{\dot{c}n}_{(\gamma)} Y^{bk}_{(\delta)}$$

Putting equation 115 in spinor notation, if b, c, d ... represent the old reference axes before interconnection and k, m, n, p ... the new reference axes after interconnection,

$$\boxed{M^{\dot{m}\dot{n}k}_{(\alpha)(\pm\beta)(\gamma)(\delta)} = M^{h\dot{f}\dot{g}} Z_{\dot{d}h} Z_{fc} Z_{\dot{g}b} C^{\dot{d}}_{p} C^{c}_{q} C^{b}_{r} Y^{\dot{m}p}_{(\alpha)(\pm\beta)} Y^{\dot{q}n}_{(\gamma)} Y^{rk}_{(\delta)}}$$

Example of a Triode Circuit

Consider a triode with 2 outside impedances (figure 9), and assume sets of voltages of various frequencies impressed on both grid and plate circuits.

Amplification. The impedance matrix of the tube is given in equation 37 as

$$Z_1 = \begin{array}{c|cc} & g_1 & p_1 \\ \hline g_1 & \dfrac{r_g}{1-\mu_p\mu_g} & \dfrac{-\mu_g r_p}{1-\mu_p\mu_g} \\ p_1 & \dfrac{\mu_p r_g}{1-\mu_p\mu_g} & \dfrac{r_p}{1-\mu_p\mu_g} \end{array}$$

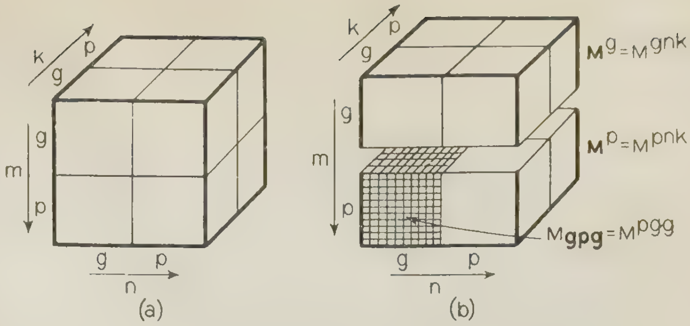

Fig. 10. Graphical representation of a triadic

(*a*)—The triadic M
(*b*)—Component matrices of the triadic

The impedance matrix of the outside network is

$$Z_2 = \begin{array}{c|cc} & g_2 & p_2 \\ \hline g_2 & R_g + L_g p + 1/C_g p & 0 \\ p_2 & 0 & R_p + L_p p + 1/C_p p \end{array} = \begin{array}{c|cc} & g_2 & p_2 \\ \hline g_2 & Z_g & 0 \\ p_2 & 0 & Z_p \end{array}$$

The transformation tensor of the interconnection is

$$C = \begin{array}{c|cc} & g & p \\ \hline g_1 & 1 & 0 \\ p_1 & 0 & 1 \\ g_2 & 1 & 0 \\ p_2 & 0 & 1 \end{array}$$

and the resultant impedance matrix of the interconnected system is $C_t \cdot (Z_1 + Z_2) \cdot C =$

$$Z = \begin{array}{c|cc} & g & p \\ \hline g & \dfrac{r_p}{1-\mu_p\mu_g} + Z_g & -\dfrac{\mu_g r_p}{1-\mu_p\mu_g} \\ p & -\dfrac{\mu_p r_g}{1-\mu_p\mu_g} & \dfrac{r_p}{1-\mu_p\mu_g} + Z_p \end{array}$$

The admittance matrix of the system is the inverse of Z, or

$$Y = \begin{array}{c|cc} & g & p \\ \hline g & \left(\dfrac{r_p}{1-\mu_p\mu_g} + Z_p\right) \Big/ Det & \dfrac{\mu_g r_p}{1-\mu_p\mu_g} \Big/ Det \\ p & \dfrac{\mu_p r_g}{1-\mu_p\mu_g} \Big/ Det & \left(\dfrac{r_g}{1-\mu_p\mu_g} + Z_g\right) \Big/ Det \end{array}$$

where

$$Det = \frac{(r_g + Z_g)(r_p + Z_p) - Z_p Z_g \mu_p \mu_g}{1 - \mu_p\mu_g}$$

For instance, the plate current due to a grid voltage is, by $Y \cdot \Delta e$,

$$\Delta i^p = \frac{\mu_p r_g}{(r_g + Z_g)(r_p + Z_p) - Z_p Z_g \mu_p \mu_g} \Delta e_g \tag{117}$$

This agrees with equation 470 in the book of McIlwain and Brainerd.

Modulation. Let Z_1 and Y be denoted as

$$Z_1 = \begin{array}{c|cc} & g_1 & p_1 \\ \hline g_1 & a & b \\ p_1 & c & d \end{array} \qquad Y = \begin{array}{c|cc} & g_1 & p_1 \\ \hline g_1 & e & f \\ p_1 & g & h \end{array}$$

The modulation triadic of the triode is given in equation 77 as

$$M_{1g} = \frac{1}{2}\times \begin{array}{c|cc} g_1 & g_1 & p_1 \\ \hline g_1 & \frac{-1}{r_g^2}\frac{\partial r_g}{\partial e_g} & \frac{-1}{r_g^2}\frac{\partial r_p}{\partial e_p} \\ p_1 & \frac{\partial G^{gp}}{\partial e_g} & \frac{\partial G^{gp}}{\partial e_p} \end{array}$$

$$M_{1p} = \frac{1}{2}\times \begin{array}{c|cc} p_1 & g_1 & p_1 \\ \hline g_1 & \frac{\partial G^{pg}}{\partial e_g} & \frac{\partial G^{pg}}{\partial e_p} \\ p_1 & -\frac{1}{r_p^2}\frac{\partial r_p}{\partial e_g} & -\frac{1}{r_p^2}\frac{\partial r_p}{\partial e_p} \end{array}$$

where the subscript of M_1 shows the middle unit vector of the triadic (also shown in the upper left hand corner of the matrix). Let

$$M_{1g} = \begin{array}{c|cc} g_1 & g_1 & p_1 \\ \hline g_1 & k & l \\ p_1 & m & n \end{array} \qquad M_{1p} = \begin{array}{c|cc} p_1 & g_1 & p_1 \\ \hline g_1 & p & q \\ p_1 & r & s \end{array}$$

These matrices have been given before.

In finding the new modulation triadic of the *combined* system, the following 3 matrices will be established first. (Since the effect of the transformation tensor C in the present simple problem is to change g_1 to g and p_1 to p in Z_1, M_p, and M_g, the simpler equation 116 without C will be used.)

Replacing $(\alpha)(\pm\beta)$ by (ω) and leaving the closed indices attached to each component of $Y^{(\alpha)}$, $Y^{(\beta)}$, and $Y^{(\omega)}$

$$Z_1\cdot Y^\beta = \begin{array}{c|cc} & g & p \\ \hline g & ae_\beta + bg_\beta & af_\beta + bh_\beta \\ p & ce_\beta + dg_\beta & cf_\beta + dh_\beta \end{array} = \begin{array}{c|cc} & g & p \\ \hline g & A & B \\ p & C & D \end{array} = P$$

$$(Z_1\cdot Y^\alpha) = \begin{array}{c|cc} & g & p \\ \hline g & ae_\alpha + bg_\alpha & ce_\alpha + dg_\alpha \\ p & af_\alpha + bh_\alpha & cf_\alpha + dh_\alpha \end{array} = \begin{array}{c|cc} & g & p \\ \hline g & E & F \\ p & G & H \end{array} = S$$

$$Y^\omega\cdot Z_1 = \begin{array}{c|cc} & g & p \\ \hline g & e_\omega a + f_\omega c & e_\omega b + f_\omega d \\ p & g_\omega a + h_\omega c & g_\omega b + h_\omega d \end{array} = \begin{array}{c|cc} & g & p \\ \hline g & K & L \\ p & M & N \end{array} = R$$

Now the value of M is to be found by equation 116; that is, by

$$M^{\omega\alpha\beta} = (Z_1\cdot Y^\alpha)_t\cdot[(Y^\omega\cdot Z_1)\cdot M_1]\cdot(Z_1\cdot Y^\beta)$$

or in the simplified form

$$M = S\cdot(R\cdot M_1)\cdot P$$

The center triadic $R\cdot M_1$ is found by dot multiplying the middle unit vector of M_1 with R. To avoid confusion each component of R should be multiplied separately by each component of M_{1g} and M_{1p}. For instance $(Lgp)\cdot(npgp)$ represents the product of p of the first term with g of the second term giving $g\cdot p$, which is zero, but $(Lgp)\cdot(rppg)$ produces $Lrpgg$, since $p\cdot p = 1$, leaving the oth[er] 3 unit vectors. It should be noted that the midd[le] unit vector of the triad $(rppg)$ is replaced by t[he] first unit vector of the other dyad while the oth[er] 2 unit vectors of the triad keep their position u[n]changed. Hence, multiplying each component [of] R with each component of M_1 results in a triadic

$$(Kgg + Lgp + Mpg + Npp)\cdot(kggg + lggp + mpgg + npgp \; pgpg + qgpp + rppg + sppp)] =$$
$$Kkggg + Klggp + Kmpgg + Knpgp$$
$$Lpggg + Lqggp + Lrpgg + Lspgp$$
$$Mkgpg + Mlgpp + Mmppg + Mnppp$$
$$Npgpg + Nqgpp + Nrppg + Nsppp$$

These components can be represented in 2 matric[es]. In the first matrix those components whose mid[dle] unit vector is g appear, and in the other those with [p] appear as

$$(R\cdot M_1)_g = \begin{array}{c|cc} g & g & p \\ \hline g & Kk + Lp & Kl + Lq \\ p & Km + Lr & Kn + Ls \end{array}$$

$$(R\cdot M_1)_p = \begin{array}{c|cc} p & g & p \\ \hline g & Mk + Np & Ml + Nq \\ p & Mm + Nr & Mn + Ns \end{array}$$

(The multiplication $R\cdot M_1$ may be performed [in] ways other than that shown.)

Each of these should be multiplied by S, as

$$S\cdot(R\cdot M_1)_g = \begin{array}{c|cc} g & g & p \\ \hline g & E(Kk+Lp)+F(Km+Lr) & E(Kl+Lq)+F(Kn+L \\ p & G(Kk+Lp)+H(Km+Lr) & G(Kl+Lq)+H(Kn+L \end{array}$$

and

$$S\cdot(R\cdot M_1)_p = \begin{array}{c|cc} p & g & p \\ \hline g & E(Mk+Np)+F(Mm+Nr) & E(Ml+Nq)+F(Mn+N \\ p & G(Mk+Np)+H(Mm+Nr) & G(Ml+Nq)+H(Mn+N \end{array}$$

The Modulation Triadic. The last 2 matric[es] should be multiplied by P, as $S\cdot(R\cdot M_1)_g\cdot P$, givi[ng] (see figure 10*b*) the matrix

$$M^g = \begin{array}{c|cc} g & g & p \\ \hline g & [E(Kk+Lp)+F(Km+Lr)]A + [E(Kl+Lq)+F(Kn+Ls)]C & [E(Kk+Lp)+F(Km+Lr)]B + [E(Kl+Lq)+F(Kn+Ls)]D \\ p & [G(Kk+Lp)+H(Km+Lr)]A + [G(Kl+Lq)+H(Kn+Ls)]C & [G(Kk+Lp)+H(Km+Lr)]B + [G(Kl+Lq)+H(Kn+Ls)]D \end{array}$$

With the aid of this matrix the product frequency components Δi^g are found to be

$$\Delta i^g = \Delta e\cdot M^g\cdot\Delta e$$

The other matrix of the triadic is $S\cdot(R\cdot M_1)_p\cdot P =$

$$M^p = \begin{array}{c|cc} p & g & p \\ \hline g & [E(Mk+Np)+F(Mm+Nr)]A + [E(Ml+Nq)+F(Mn+Ns)]C & [E(Mk+Np)+F(Mm+Nr)]B \; [E(Ml+Nq)+F(Mn+Ns)]D \\ p & [G(Mk+Np)+H(Mm+Nr)]A + [G(Ml+Nq)+H(Mn+Ns)]C & [G(Mk+Np)+H(Mm+Nr)]B \; [G(Ml+Nq)+H(Mn+Ns)]D \end{array} \quad (1$$

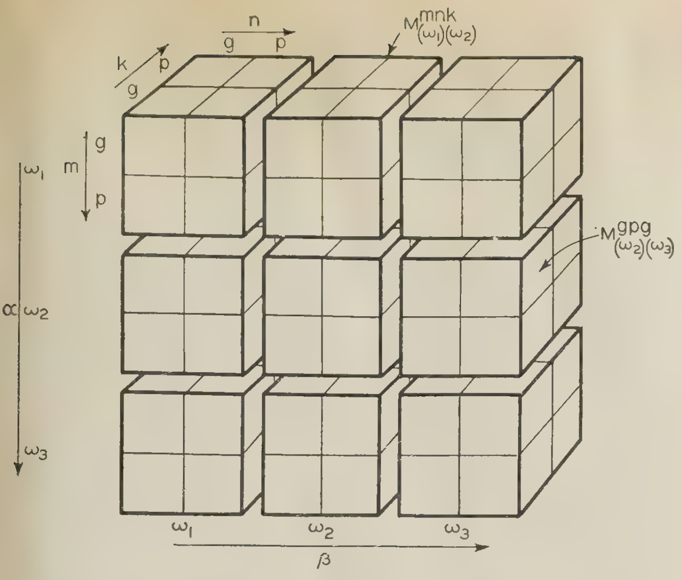

Fig. 11. Graphical representation of the compound polyadic $\Delta i^m_{(\alpha)(\beta)}$ as a set of vectors $I = i^m$ arranged in a square

With the aid of this matrix the product frequency components of Δi^p are found; that is,

$$\Delta \boldsymbol{i}^p = \Delta \boldsymbol{e} \cdot \boldsymbol{M}^p \cdot \Delta \boldsymbol{e} \tag{118}$$

The last 2 matrices $\boldsymbol{M}^g$ and $\boldsymbol{M}^p$ are components of the triadic $\boldsymbol{M}$ (figure 10*a*) giving all of the product frequency currents by

$$\Delta \boldsymbol{i}^{(\omega)} = \Delta \boldsymbol{e}_{(\alpha)} \cdot \boldsymbol{M}^{(\omega)(\alpha)(\beta)} \cdot \Delta \boldsymbol{e}_{(\beta)} \tag{119}$$

where $\omega = \alpha \pm \beta$.

Evaluating one of the 8 components of the triadic $\boldsymbol{M}$ (figure 10*a*) say $M'\boldsymbol{gpg}$ $(= M^{pgg})$, the component in the upper left-hand corner of matrix 117 (or in the shaded block of figure 10*b*):

$$M' = M(AEk + AFm + CEl + CFn) + N(AEp + AFr + CE_q + CFs) \tag{120}$$

But

$$M = -\frac{r_g\mu_p Z_{g\omega}}{D_\omega} \quad A = \frac{r_g Z'_{p\beta}}{D_\beta} \quad E = \frac{r_g Z'_{p\alpha}}{D_\alpha}$$

$$C = -\frac{r_g\mu_p Z_{p\beta}}{D_\beta} \quad F = -\frac{r_g\mu_p Z_{p\alpha}}{D_\alpha D} \quad N = \frac{r_p Z'_{g\omega}}{D_\omega} \tag{121}$$

where

$$D = (r_g + Z_g)(r_p + Z_p) - Z_p Z_g \mu_p \mu_g;\ Z' = r + Z;\ \omega = \alpha \pm \beta$$

Hence, one of the 8 components is

$$M^{(\omega)(\alpha)(\beta)} = \frac{r_g\mu_p Z_{g\omega}}{2D_\alpha D_\beta D_\omega}\left[Z'_{p\alpha}Z'_{p\beta}\frac{\partial r_g}{\partial e_g} + r_g^2\mu_p Z_{p\alpha}Z'_{p\beta}\frac{\partial G^{gp}}{\partial e_g} - \mu_p Z_{p\beta}Z'_{p\alpha}\frac{\partial r_g}{\partial e_p} - r_g^2\mu_p^2 Z_{p\alpha}Z_{p\beta}\frac{\partial G^{gp}}{\partial e_p}\right] + \frac{r_p Z'_{g\omega}}{2D_\alpha D_\beta D_\omega}\left[\frac{r_g^2}{r_p^2}\mu_p Z_{p\alpha}Z'_{p\beta}\frac{\partial r_p}{\partial e_g} + r_g^2 Z'_{p\alpha}Z'_{p\beta}\frac{\partial G^{pg}}{\partial e_g} - \frac{r_g^2}{r_p^2}\mu_p^2 Z_{p\alpha}Z_{p\beta}\frac{\partial r_p}{\partial e_p} - r_g^2\mu_p Z_{p\beta}Z'_{p\alpha}\frac{\partial G^{gp}}{\partial e_p}\right] \tag{122}$$

Each Z is calculated for the frequency of its subscript. If the grid voltage is the sum of n voltages of different frequencies, there are $2n^2$ different components $M^{(\omega)(\alpha)(\beta)}$, giving $2n^2$ different plate currents

$$\Delta i^{p(\alpha\pm\beta)} = M^{pgg(\alpha\pm\beta)(\alpha)(\beta)}\Delta e_{g(\alpha)}\Delta e_{g(\beta)}$$

by allowing α and β assume the frequency range 1, 2, ... n (or $\omega_1, \omega_2 \ldots \omega_n$). In the equation, p and g are fixed open indices.

When, similarly to equation 122, all 8 components of $\boldsymbol{M}$ are calculated, for each frequency of current in $\Delta \boldsymbol{i}^{(\omega)} = \Delta \boldsymbol{i}^{(\alpha\pm\beta)}$ (figure 11) there is a cube, such as shown in figure 10. Altogether there are $2n^2$

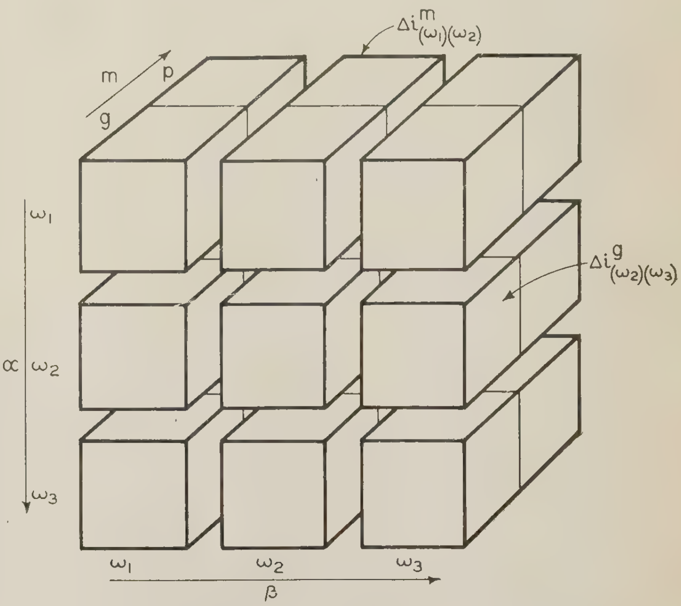

Fig. 12. Graphical representation of the compound polyadic $M^{mnk}_{(\alpha)(\beta)}$ as a set of triadics $M = M^{mnk}$ arranged in a square

cubes, of which half are shown in figure 12, each cube giving one particular product frequency current flowing in all the circuit. All of the currents given by a cube have the same frequency.

Correctness of Polyadic Equations

As a check on the correctness of the results obtained, the coefficient $M^{(\omega)(\alpha)(\beta)}$ will be calculated by the formulas given by McIlwain and Brainerd,[9] giving the product frequency plate currents for an applied set of grid voltages in their equation 468 as

$$i_p = e_g^2[\times\{(d_3(1 - a_1 z_a)^2 - d_1 a_3 z_a\}]$$

where d_1, a_1, d_3, and a_3 are defined in equations 469*a*, 469*b*, 469*c*, and 469*d* of their book.

Let their constants be changed to those of this paper; that is, let

$$\begin{array}{l|l|l|l} z = Z_p & \mu = \mu_p & m = \alpha & m + n = \omega = \alpha \pm \beta \\ z_a = Z_g & \nu = \mu_g & n = \beta & \text{etc.} \end{array} \tag{123}$$

Also let it be remembered that

$$\begin{array}{l|l|l} \mu_p = r_p G^{pg} & \dfrac{\partial G^{gp}}{\partial e_g} = -\dfrac{1}{r_g^2}\dfrac{\partial r_g}{\partial e_p} & \dfrac{\partial G^{pg}}{\partial e_p} = -\dfrac{1}{r_p^2}\dfrac{\partial r_p}{\partial e_g} \\ \mu_g = r_g G^{gp} & & \end{array} \tag{124}$$

One term in the denominator of their equation 469d is

$$r_g + v_m z_{am} = r_g + Z_{g\alpha}(1 - \mu_g \mu_p Z_{p\alpha}/Z'_{p\alpha}) = D_\alpha / Z'_{p\alpha} \quad (125)$$

hence, the denominator of 469d is

$$2D_\alpha D_\beta D_\omega / Z'_{p\alpha} Z'_{p\beta} Z'_{p\omega}.$$

The first term of 469d should be expanded thus:

$$r_g \frac{\partial r_g}{\partial E_g} [1 - v(d_{1m} z_m + d_{1n} z_n - v d_{1m} d_{1n} z_m z_n)] \quad (126)$$

and the expanded form of the first term should be combined with the third term, where μ_g in $\partial \mu_g / \partial e_g$ is replaced by $r_g G^{gp}$.

First consider those 4 terms in the numerator of their equation 469d that do not contain $d_{3(m+n)}$. After simplification

$$\frac{\mu_p Z_{g\omega}}{Z'_{p\omega}} \left(r_g \frac{\partial r_g}{\partial e_g} + \frac{\mu_p}{Z'_{p\alpha}} Z_{p\alpha} r_g^3 \frac{\partial G^{gp}}{\partial e_g} - r_g^3 \frac{\mu_p}{Z'_{p\alpha}} \frac{\mu_p}{Z'_{p\beta}} Z_{p\alpha} Z_{p\beta} \frac{\partial G^{gp}}{\partial e_p} - \frac{\mu_p}{Z'_{p\beta}} Z_{p\beta} r_g \frac{\partial r_g}{\partial e_p} \right) \quad (127)$$

If these terms are divided by the denominator just given, the result is:

$$\frac{r_g \mu_p Z_{g\omega}}{2D_\alpha D_\beta D_\omega} \left[Z'_{p\alpha} Z'_{p\beta} \frac{\partial r_g}{\partial e_g} + r_g^2 \mu_p Z_{p\alpha} Z'_{p\beta} \frac{\partial G_{gp}}{\partial e_g} - \mu_p Z_{p\beta} Z'_{p\alpha} \frac{\partial r_g}{\partial e_p} - r_g^2 \mu_p^2 Z_{p\alpha} Z_{p\beta} \frac{\partial G^{gp}}{\partial e_p} \right]. \quad (128)$$

Taking those terms of their equation 468 that contain $d_{3(m+n)}$

$$r_g^2 \left[-\mu_p^2 r_p \frac{\partial r_p}{\partial e_p} + \mu_p (r_p^2 - Z_{p\alpha} Z_{p\beta}) \frac{\partial \mu_p}{\partial e_p} + Z'_{p\alpha} Z'_{p\beta} \frac{\partial \mu_p}{\partial e_p} \right] \cdot \left[\frac{1}{2Z'_{p\omega} D_\alpha D_\beta} + \frac{\mu_g \mu_p Z_{p\omega} Z_{g\omega}}{2D_\alpha D_\beta D_\omega Z'_{p\omega}} \right] \quad (129)$$

If μ_p is replaced by $r_p G^{pg}$

$$\frac{r_g^2 Z'_{g\omega}}{2D_\alpha D_\beta D_\omega} \left\{ Z'_{p\alpha} Z'_{p\beta} G^{pg} \frac{\partial r_p}{\partial e_g} + Z_{p\alpha} Z_{p\beta} r_p \frac{\partial G^{pg}}{\partial e_g} + \mu_p (r_p^2 - Z_{p\alpha} Z_{p\beta}) r_p \frac{\partial G^{pg}}{\partial e_p} + [\mu_p (r_p^2 - Z_{p\alpha} Z_{p\beta}) G_{pg} - \mu_p^2 r_p] \frac{\partial r_p}{\partial e_p} \right\} \quad (130)$$

If $\partial G^{pg} / \partial e_p$ is replaced by $-(\partial r_p / \partial e_g)(1/r_p^2)$ and is simplified, the final result is

$$\frac{r_p Z'_{g\omega}}{2D_\alpha D_\beta D_\omega} \left[\frac{r_g^2}{r_p^2} \mu_p Z_{p\alpha} Z'_{p\beta} \frac{\partial r_p}{\partial e_g} + r_g^2 Z'_{p\alpha} Z'_{p\beta} \frac{\partial G^{pg}}{\partial e_g} - \frac{r_g^2}{r_p^2} \mu_p^2 Z_{p\alpha} Z_{p\beta} \frac{\partial r_p}{\partial e_p} - r_g^2 \mu_p Z_{p\beta} Z'_{p\alpha} \frac{\partial G^{pg}}{\partial e_p} \right] \quad (131)$$

Adding equations 128 and 131, the result for the coefficient of e_g^2, as given by McIlwain and Brainerd, is

$$M' = \frac{r_g \mu_p Z_{g\omega}}{2D_\alpha D_\beta D_\omega} \left[Z'_{p\alpha} Z'_{p\beta} \frac{\partial r_g}{\partial e_g} + r_g^2 \mu_p Z_{p\alpha} Z'_{p\beta} \frac{\partial G^{gp}}{\partial e_g} - \mu_p Z_{p\beta} Z'_{p\alpha} \frac{\partial r_g}{\partial e_p} - r_g^2 \mu_p^2 Z_{p\alpha} Z_{p\beta} \frac{\partial G^{gp}}{\partial e_p} \right] + \frac{r_p Z'_{g\omega}}{2D_\alpha D_\beta D_\omega} \left[\frac{r_g^2}{r_p^2} \mu_p Z_{p\alpha} Z'_{p\beta} \frac{\partial r_p}{\partial e_g} + r_g^2 Z'_{p\alpha} Z'_{p\beta} \frac{\partial G^{pg}}{\partial e_g} - \frac{r_g^2}{r_p^2} \mu_p^2 Z_{p\alpha} Z_{p\beta} \frac{\partial r_p}{\partial e_p} - r_g^2 \mu_p Z_{p\beta} Z'_{p\alpha} \frac{\partial G^{pg}}{\partial e_p} \right] \quad (132)$$

Equations 122 and 132 as derived by the 2 method are identical, both equations giving the sam magnitudes, frequencies, and signs for the produc frequency plate currents due to a set of voltages c various frequencies $\Delta e_{(\alpha)}$ applied in the grid circuit

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12. Theory of Multielectrode Vacuum Tubes, H. A. Pidgeon. *Be System Technical Journal*, v. 14, January 1935, p. 44–84.

13. Further Extensions of the Theory of Multielectrode Vacuu Tube Circuits, S. A. Levin and L. C. Peterson. *Bell System Technical Journa* v. 14, October 1935, p. 666–79.

A Session on Tensors at the 1937 Winter Convention

Under sponsorship of the AIEE committee on electrophysics, a session on tensors has been scheduled on the program of the AIEE winter convention to be held in New York, N. Y., January 25–29, 1937. Of the following 5 papers to be presented at that session, 4 already have been published in Electrical Engineering and the fifth is scheduled for the December issue:

1. The Tensor—A New Engineering Tool, A. Boyajian, August issue, pages 856–62.
2. Dyadic Algebra Applied to 3-Phase Circuits, A. Pen-Tung Sah, August issue, pages 876–82.
3. Tensor Algebra in Transformer Circuits, L. V. Bewley, November issue, pages 1214–19.
4. Tensor Analysis of Multielectrode-Tube Circuits, Gabriel Kron, November issue, pages 1220–42.
5. Complex Vectors in 3-Phase Circuits, A. Pen Tung Sah, Scheduled for December issue.

According to tentative plans of the committee, some leading mathematicians and physicists are to be invited to attend the session and discuss the papers.

Discussions

Of AIEE Papers—as Recommended for Publication by Technical Committees

On this and the following 32 pages appear all remaining discussions submitted for publication, and approved by the technical committees, on papers presented at the sessions on electrical machinery, transformers, measurements and selected subjects, electrophysics, and protective devices at the 1936 AIEE summer convention, Pasadena, Calif., June 22–26. Authors' closures, where they have been submitted, will be found at the end of the discussion on their respective papers.

Members anywhere are encouraged to submit written discussion of any paper published in Electrical Engineering, which discussion will be reviewed by the proper technical committee and considered for possible publication in a subsequent issue. Discussions of papers scheduled for presentation at an AIEE meeting or convention will be closed 2 weeks after presentation. Discussions should be (1) concise; (2) restricted to the subject of the paper or papers under consideration; and (3) typewritten and submitted in triplicate to C. S. Rich, secretary, technical program committee, AIEE headquarters, 33 West 39th Street, New York, N. Y.

Hydrogen Cooling of Rotating Machines

Discussion of a paper by C. M. Laffoon published in the June 1936 issue, pages 703–9, and presented for oral discussion at the electrical machinery session of the summer convention, Pasadena, Calif., June 24, 1936.

M. D. Ross (Westinghouse Electric & Manufacturing Company, East Pittsburgh, Pa.): The paper should be of interest to the central station industry, for it brings out the latest developments in frequency changers and large 3,600-rpm generators. The present trend in 3,600-rpm turbine-generators above 20,000-kw rating seems to be definitely toward hydrogen cooling, as most of the large units now on order are of this type.

One of the chief considerations in the design of the hydrogen-cooled units mentioned by the author has been the necessity of keeping the individual pieces of the units within the dimensions that can be handled by the railroads. Usually it is impossible to ship a machine that is wider than 12 feet when placed on the car and this limitation will affect the size of unit that can be built and tested at the factory. If it is necessary to go to larger frame diameters with larger units it may be necessary to stack the punchings and wind the stator in the field. This would be necessary if large 1,800-rpm turbine-generators were hydrogen cooled.

Another item of interest is the device used to measure the purity of the hydrogen gas in the machine. A small constant-speed motor drives a fan that samples gas continuously from the generator frame. Surrounding the fan runner is another unit with radial blades, which is mounted on ball bearings and is held from rotating by a spring. The torque between the 2-bladed elements is proportional to the density of the gas in the meter and the displacement of the outer element is also proportional. The density is registered on a scale, visible through a glass in the meter case. The mechanism also operates a switch to indicate too high a gas density. When driving out air or hydrogen with carbon dioxide gas the percentage of carbon dioxide can be estimated with sufficient accuracy by watching the meter reading, because pure carbon dioxide has a density of about 1.5 times that of air and the meter is graduated up to 1.5 air density.

The hydrogen-cooled turbine-generators described by the author are equipped with air-foil type propeller blowers mounted on the shaft. This type of blower has been used because centrifugal fans for outputs such as required here would have many undesirable mechanical features, whereas the construction of a satisfactory propeller type of blower is a much easier mechanical problem. Stresses in the blades can be kept low by the use of light alloy blades.

The 3,600-rpm hydrogen-cooled generators will be in operation early in 1937. Their installation will no doubt begin a new phase in power plant construction.

R. E. Hellmund (Westinghouse Electric & Manufacturing Company, East Pittsburgh, Pa.): The dissipation of heat from electrical machinery always has been one of the principal problems of the designer of machinery. Designs have progressed gradually from natural ventilation to forced ventilation and finally, in the larger machines, to enclosed cooling systems, first with air as a cooling medium and now with special gases such as hydrogen. In this last arrangement the heat extracted from the machine is transferred to radiators of a water-cooling system; therefore why the machine parts might not be cooled directly by water is not logically apparent. This matter has been carefully studied. The Westinghouse company, in co-operation with Von Kando, who carried on experiments along this line, has among other things designed a rotor arrangement for water cooling. The water was put in and taken out through glands at the center of the shaft and circulated through copper tubes arranged in proximity to the rotor conductors. One practical difficulty of this method in both rotating and stationary parts is that the cooling pipes must be in intimate contact with the parts to be cooled in order to be effective. The tubes are kept relatively cool by the water, but neighboring machine parts will be hot, which in turn leads to different expansion and contraction of those parts. This action, together with the possibility of vibration and consequent leaks in the conductors of the liquid, makes such an arrangement appear to be somewhat unsafe, especially if it is considered that even a single small leak may cause serious damage to a machine so large and expensive. In other words, direct cooling by water is a possible solution but one that does not seem to be justified in view of the extra risks involved; therefore, an indirect means, such as hydrogen, for transferring the machine heat to a water system seems to be preferable.

S. H. Mortensen (Allis-Chalmers Manufacturing Company, Milwaukee, Wis.): The paper shows that by substituting hydrogen for air as the ventilating medium in electrical machines more output per unit of active material, as well as a reduction in windage losses can be obtained. The output for which electrical machines can be built, being a function of rotor diameter, axial length, and speed, is limited by the characteristics of materials available and approaching coincidence between the operating speed of the rotor and its critical speeds. With existing materials it has been practicable to build machines of capacities thus far required for speeds of 1,800 rpm or less with air cooling; but where large 3,600-rpm machines are involved the maintenance of a reasonable factor of safety so far as mechanical stresses and critical speeds are concerned, make necessary the finding of some means for increasing machine output per unit of active material. In such cases hydrogen cooling makes practicable the construction of larger machines than could be built with air cooling. Further gains may be achieved by providing means for reducing the entrance temperature of the ventilating medium to values below present standards and by developing insulation materials permitting higher generator operating temperatures than those now accepted.

An approximate idea of gains in efficiency obtainable by ventilating with hydrogen instead of air may be obtained from table I of this discussion, where the respective losses are given for machines

rated 18,750 kva at synchronous speeds of 3,600, 1,500, 900, and 112½ rpm.

The efficiency comparisons are based upon the assumption that windage with hydrogen is $^1/_{10}$ that with air; however, table I does not take into consideration that the iron and copper losses probably would be higher in hydrogen-cooled machines than in air-cooled machines. The tabulation favors hydrogen cooling to this extent. The largest gain in output obtainable by substituting hydrogen for air would occur on machines whose output is limited by heating alone. In generators requiring a high short-circuit ratio, motors proportioned for high pull-out torque, and condensers designed for full lagging and leading capacity, the saving using hydrogen would be correspondingly less. Other factors—as example the difference in mechanical construction of an outdoor and an indoor type machine—may justify hydrogen cooling for the former, but not for the latter. The choice of air or hydrogen cooling for electrical machines, except where large 3,600 rpm generators are involved (which for mechanical reasons cannot be air cooled), then narrows down to an analysis of the advantages obtainable with hydrogen ventilation against its handicaps such as additional paraphernalia.

It is noted with interest that the copper damper winding used on the stator of the 9,375-kva hydrogen-cooled machine mentioned by the author permitted increased flux densities in the stator without appreciably increasing its hysteresis and stray losses. The writer obtained somewhat similar results on a certain machine by the insertion of nonmagnetic liners between the stator punchings and the yoke bore in combination with the use of nonmagnetic material in the yoke dovetails. Experience indicates, however, that only in very special cases is it necessary to resort to such refinements as very good efficiencies can be obtained by judicious application of nonmagnetic and nonconductive materials in suitable machine parts.

A further discussion of some of the author's experiences gained with hydrogen ventilation on machines with hydrogen sealing glands, such as the relations between gas pressures, temperature rise, and gas leakage, as well as corresponding duct velocities, would be interesting.

It would also be of interest to know which factors determined the operating pressure of the hydrogen in the machine of which the sealing glands are depicted in figure 5 of the paper, and what its value is. Were other fluids besides oil tested in conjunction with the sealing glands for the purpose of reducing hydrogen entrainment?

R. W. Wieseman (General Electric Company, Schenectady, N. Y.): The advantages, characteristics, and construction features of hydrogen-cooled synchronous machines have been described fully in the technical press. Hydrogen cooling for rotating machines was first proposed in the United States by W. R. Whitney of the General Electric research laboratory in 1921. Investigations made at that time indicated that turbine generators offered the greatest possibility for improvement when operated in hydrogen because of their high rotating speeds and large outputs per pound of active materials. One of the problems of applying hydrogen to those machines was that some form of sealing device was necessary to prevent the escape of gas where the shaft projects through the enclosing structure. Considerable development work was done to devise such seals, and the first machine to demonstrate hydrogen cooling, a 3,380-kva turbine generator, was tested in Schenectady in 1922. The first machine designed exclusively for hydrogen cooling, a 6,250-kva turbine generator, was built and tested in 1927. The condenser was selected to demonstrate hydrogen cooling in actual practice, and the first hydrogen-cooled synchronous condenser was built in 1927 and installed in 1928. The first hydrogen-cooled frequency changer was built in 1934 and installed in 1935. Up to the present time 14 synchronous condensers and 2 frequency changers have been installed in this country.

This group of machines represents a total of 53 machine years of dependable service. Ten of these machines are located out of doors without protection from weather conditions. Two condensers were operated 5 years without a single inspection of the main compartment. Maintenance expense is mostly the cost of periodic inspections and these inspections show a clean machine free from oil vapor and foreign matter. The bearing oil of a machine placed in operation in 1928 has never been changed, and a chemical analysis of this oil shows practically no deterioration. This machine also has its original collector-ring brushes. From actual experience with this group of machines we conclude that hydrogen cooling is fulfilling its obligation to the electrical industry by providing cleanliness, quietness, outdoor operation, reduced fire risk, increased efficiency, increased capacity, and increased life. Furthermore, the successful operation of these machines has renewed activity in hydrogen cooling for turbine generators which was initially considered the most advantageous application of hydrogen cooling for rotating machinery.

Synchronous Mechanical Rectifier-Inverter—II

Discussion and authors' closure of a paper by S. S. Seyfert, N. S. Hibshman, and D. C. Bomberger published in the May 1936 issue, pages 548–53, and presented for oral discussion at the electrical machinery session of the summer convention, Pasadena, Calif., June 24, 1936.

Vannevar Bush (Massachusetts Institute of Technology, Cambridge): The development of apparatus for the conversion of power between alternating and continuous current systems seems to move slowly. While the machinery in either separate field has become largely standardized, that in the intermediate field of conversion remains largely in a state of change.

The criteria by which any type of machine must be judged are first cost, efficiency, and reliability. In the conversion field the matter of reliability, which involves also maintenance, becomes of special importance. This arises because in conversion the circuits and apparatus employed are necessarily unusual and sometimes complex, unless intermediate conversion of power into mechanical form is utilized.

There have been many suggested mechanisms in the conversion field. Of these the combination of transformer and rotating mechanical rectifier has long had significant points of attractiveness from the standpoint of cost and at some voltage ranges of efficiency as well. The difficulty has been to insure reliability and low maintenance cost in the problem of commutation.

Many have attacked this problem; the writer did so many years ago with some degree of patience but with little success.

The striking fact about the work of the authors has been that this central problem has now been approached thoroughly and analytically. Their method of providing definitely for proper commutating conditions by supplying the correct amplitude and phase of the proper harmonic seems to give the answer.

The writer is delighted with this paper on the subject, for this carefully prepared theory has now been subjected to practical test, which has yielded confirmation on a machine of substantial capacity.

In a field such as that of conversion one would hardly expect any one form of equipment to become universal under all conditions of size, voltage, and operating specifications. Instead, it is to be anticipated that the field will be divided among

Table I—Comparison of Hydrogen Cooling With Air Cooling for Several 15,000-Kw 0.80-Power Factor Machines Having Different Synchronous Speeds

Rpm	3,600			1,500			900			112½		
Fraction of Full Load	1/2	3/4	1	1/2	3/4	1	1/2	3/4	1	1/2	3/4	1
Windage and ventilation loss in kilowatts	174	174	174	120	120	120	80	80	80	60	60	60
Other losses including bearing friction	154	195	251	166	213	275	120	177	255	263	351	473
Total losses with air cooling	328	369	425	286	333	395	200	257	335	323	411	533
Per cent efficiency with air cooling	95.81	96.82	97.25	96.32	97.12	97.43	97.40	97.76	97.81	95.87	96.47	96.5
Total losses with hydrogen cooling	172	213	268	178	225	287	128	185	263	269	357	479
Per cent efficiency with hydrogen cooling	97.75	98.14	98.24	97.67	98.04	98.12	98.32	98.38	98.28	96.54	96.93	96.9
Gain in efficiency, per cent	1.94	1.32	0.99	1.35	0.92	0.69	0.92	0.62	0.47	0.67	0.46	0.3

distinct types of apparatus. The immediate area of application of the converter described in the paper appears to lie in the range of moderate voltages, while a higher range becomes occupied by apparatus utilizing gaseous-conduction elements. For this lower voltage range and for applications where efficiency is of sufficient moment, this equipment certainly appears to have utility.

R. E. Hellmund (Westinghouse Electric & Manufacturing Company, East Pittsburgh, Pa.): The electrical industry finds itself in a rather peculiar situation with reference to rectifying equipment. Although the great amount of generation and motoring with a-c power is carried on essentially by 2 types of machines, namely, the synchronous and asynchronous, and the appreciable amount of generating and motoring with d-c can be handled by a single type of machine, a multiplicity of devices are being used in the relatively small activity of rectification. The reason for this is that none of these various types of devices is suitable for the wide range of applications involved. High-vacuum electronic devices can handle appreciable voltages at very small currents. The gas- or vapor-filled electronic devices with relatively low gas pressure are at present suitable for voltages up to possibly 25,000, but are limited in current-carrying capacity. Electronic devices with somewhat higher gas pressure, such as the so-called "rectigon" and "tungar," are suitable for certain applications but also are very limited in current and voltage range. In addition to these, there are certain devices being developed in Europe that use an open arc for rectification. There are pool-type rectifiers, vibrating mechanical rectifiers, rotating mechanical rectifiers, the old well-known synchronous converters, motor-generator sets, contact rectifiers, such as the copper-oxide type, and others. Aside from the capacity limitations, many of these rectifiers are limited from the viewpoint of economy in operation. The rectifier discussed in the paper is of the mechanical rotating type and the authors apparently have succeeded in solving the commutation problems of this type of machine in a satisfactory manner, at least within a certain range of current and voltage capacity; furthermore, they expect good efficiencies for relatively low operating voltages.

Although the many types of rectifiers enumerated show appreciable differences in working principles, many of them have the common characteristic of a substantially constant voltage drop over a wide range of loads, which materially influences their efficiencies at various operating voltages. The voltage drop in electronic devices is caused by the arc drop, and in some of the other devices, by contact resistances. If this voltage drop, indicated by D in figure 1 of this discussion, is an appreciable part of the total voltage E, it necessarily limits the maximum efficiency obtainable. If the devices have other losses, such as friction or core losses, as have rotating machines, or cathode-heating losses as in hot-cathode tubes, the total efficiency curve will be somewhat like curve G in figure 1, the maximum of which is of course below line H. As an example, the conventional mercury-arc rectifier has an arc drop of from 20 to 25 volts, which represents an appreciable loss for low operating voltages and makes difficult the competition of the mercury-arc rectifier with the conventional synchronous converter for the lower voltages, though it compares well with motor-generator sets now generally used for voltages below 500 volts. The advantage of the rectifier described in the paper as compared with the conventional mercury-arc rectifier is that the voltage drop of the various brushes should be considerably less than 25 volts, and in spite of the fact that there are other losses, such as brush friction and certain losses in the auxiliary machine, the arrangement should give satisfactory efficiencies for the lower voltages and may prove efficient for voltages as low as 220. Its

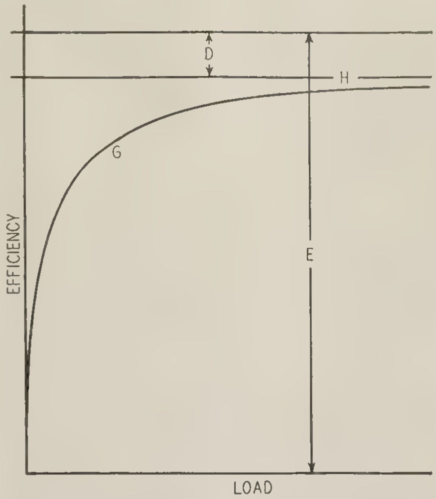

Fig. 1. Load-efficiency curve of any rectifier, based upon voltages

D—Voltage drop in arc E—Total impressed voltage
G—Total efficiency curve

closest competitor of practical importance might therefore be the so-called "ignitron," which is an electronic device having an arc drop of only 10 or 12 volts and no other appreciable losses, except the transformer losses which, however, are present in nearly all rectifying arrangements. It would seem that if the sum of the contact voltage drops in the device described in the paper can be kept low, full load efficiency slightly in excess of that of the igniter type of tube might possibly be obtained. However, because of the friction losses and the losses of the auxiliary machine, the low-load and most likely the all-day efficiency would be below that of the igniter type of tube in the majority of cases. A comparison of the first cost of the 2 devices is difficult, because neither of them is at present in appreciable quantity production. This is a condition that unfavorably affects practically all possibilities for rectification, particularly in many of the devices recently developed; most of them have not been manufactured in sufficient quantities to develop certain inherent possibilities of low cost and thus make them comparable with the older devices, such as the synchronous converter and motor-generator set, the manufacturing practices for which have been refined and improved over extended periods of time.

There is one operating difficulty that might develop in the device described in the paper, caused by the distribution of the current. It is always zero at certain portions of the rotating contact devices and maximum at others. This may lead to uneven wear and affect maintenance unfavorably. It is difficult to predict, however, whether this is of great practical importance.

S. S. Seyfert, N. S. Hibshman and D. C. Bomberger: The authors value the discussions offered by Vannevar Bush and R. E. Hellmund. This type of rectifier-inverter was first devised in anticipation of its possible application to d-c power transmission. The early experiments were made on a low-voltage scale and it was soon discovered that possibilities for unusually high efficiencies and low capital costs existed for such low-voltage conversions.

The contact voltage drop of the present apparatus is about 5 volts at full load. This is because the current must enter and leave the switching rings for each of the phases acting in series. This drop corresponds to the arc drop of the mercury rectifier, becoming less significant with increase of working voltage. Recent studies on collector-ring-contact drop have resulted in suggestions of technique whereby the losses due to such drops may be greatly reduced below the values encountered in the tests.

Some of the earlier applications of mechanical rectifiers to low-voltage electrolytic

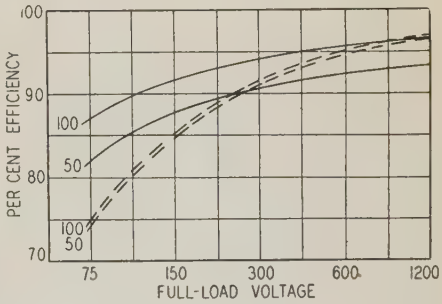

Fig. 2. Variation of rectifier efficiency with voltage change

—Synchronous mechanical rectifier
---Mercury-arc rectifier
Number on each curve indicates percentage of full load

work have been mentioned. It is in this field that the proposed device should show special economic advantage. If arranged for single-phase full-wave rectification, the brush-contact losses will be $^1/_4$ of those for 3-phase operation and when little or no reactance is found in the d-c circuit, the commutating duty of the harmonic set becomes very small.

A comparison of the approximate full-load efficiencies of the conventional mercury-arc rectifier and the apparatus discussed is given in figure 2 of this discussion. These efficiencies were computed for a series of operating voltages varying from 75 to 1,200. In each case nominal load

was assumed to be 60 kw. For the mercury rectifier the assumptions were a constant arc drop of 25 volts and an excitation and pumping loss of 550 watts. For the mechanical rectifier the brush losses were taken as inversely proportional to the voltage. The other machine losses were assumed to be constant and the reversing core loss was adjusted to the lower value, which a redesign of the harmonic generator will make possible. It may be noted that the 2 devices have the same full-load efficiency at 900 volts and the same half-load efficiency when operating at about 250 volts.

The paper neglects the advantage of the perfectly automatic reversibility of this converting means, which may be important in the case of some applications.

Stray Load Loss Tests on Induction Machines—II

Discussion and authors' closure of a paper by T. H. Morgan and Victor Siegfried published in the May 1936 issue, pages 493–7, and presented for oral discussion at the electrical machinery session of the summer convention, Pasadena, Calif., June 24, 1936.

R. E. Hellmund (Westinghouse Electric & Manufacturing Company, East Pittsburgh, Pa.): The new method described in the paper should, with proper care, give reasonably accurate results. The principal thing to be watched is the brush-friction losses of d-c machines, which are subject to erratic changes, as recognized by the authors. As will be noticed from figures 1 and 2 of the paper, the entire arrangement is somewhat complicated and therefore the authors' designation of the new arrangement as a "laboratory method" is quite fitting. The main value of this method consists in the checking of results obtained with methods that are perhaps theoretically less correct, but which are more suitable for ordinary test-floor operation. The authors have made one such check against the method of d-c excitation contained in the "Preliminary Report on a Proposed Test Code for Polyphase Induction Machines" prepared by the AIEE committee on electrical machinery in December 1934. Although the procedure outlined in the proposed code is somewhat vague, the method given in the paper under the heading "D-C Excitation Method" is more precise. It definitely suggests the use of the standstill test, with the torque measurements serving as a basis for the rotor loss and rotor resistance. It further allows for the loss attributable to the fundamental frequency leakage flux in the stator. It is assumed that curve D in figure 5 of the paper has been derived in this manner. This curve agrees reasonably closely with curve B derived from the laboratory method described in the paper.

Since this is only a single case, it should not be made the basis of a general conclusion, particularly in view of the fact that in the paper by C. J. Koch (reference 2 of the paper) greater discrepancies are indicated and in the discussion of the Koch paper by Schoenfeld and Henderson some decided discrepancies are brought out. Further checks seem to be highly desirable before the preliminary test code is accepted as the final standard. It furthermore seems advisable in giving the results of such checks to include some information regarding the machine design. It is, for instance, quite possible that with partially closed slots in both members and liberally designed tooth tips, the d-c excitation method may give reasonably close results, although this may not be the case with open stator slots and rotor slots entirely closed by a very thin bridge. The combination of the main and leakage fluxes in the bridge may cause such high saturation in the bridge that the closed rotor slots will be almost equivalent to open slots, in which case the load losses would be modified materially. It is appreciated that it is not always possible to give complete design data in such comparative tests, but at least some information should be given regarding the more essential features, such as the type of slot and tooth tip of the motor. Little progress can be made in determining the suitability of the d-c excitation method for various types of motors unless such data are made available.

Howard Maxwell (General Electric Company, Schenectady, N. Y.): When the "Preliminary Report on a Proposed Test Code for Polyphase Induction Machines" was prepared by the AIEE committee, it was realized that a single method of testing induction motors for efficiency could not well be selected for standardization, because the testing equipment is not available which would be suitable for use in making such a test in all the range of horsepower and speed covered. For this reason, several different methods of measuring the efficiency were included in this preliminary report on the test code.

In the paper the authors have proved the validity of the so-called belted load-back method described in a previous paper by Morgan and Narbutovskih ("Stray Load Tests on Induction Machines," Electrical Engineering, volume 53, February 1934, pages 286–90) and it seems that it would now be very appropriate for the belted load-back method to be included in the test code as an alternative method, along with the pump-back test described in section 80, as method C.

By any of the methods of making efficiency tests now included in the test code, with one possible exception, it is difficult for the purchaser of an induction motor to test its efficiency after he has it in his own plant. He usually relies on the manufacturer of the motor to test it for him at the factory.

The belted load-back test provides the purchaser with a relatively easy method by which he can at his own plant check the efficiency of his motors on at least one point of the load curve where 2 or more similar motors are available.

The exception just referred to is the segregated-loss method using the stray-load losses determined by the d-c excitation method. This is a test the purchaser can also make reasonably easily if he has a suitable source of d-c power available. It is very interesting to note that the test by this method shown in figure 5 made by the authors closely agreed with the results of the other methods used in their tests.

The manufacturers do not have equipment on hand to measures the output of the largest or highest speed motors so they must also rely on the segregated-loss method with stray-load loss determined by d-c excitation where they test the efficiency of such motors, so it is important, as well as interesting, that they were able to verify the validity of this d-c excitation method.

The authors claim a greater accuracy for the pump-back method they describe, because the losses are measured directly instead of being the difference of separate input and output measurements as is the case with the pump-back method listed in the test code. In principle this contention is correct but in practice this advantage is somewhat offset by the lesser inherent accuracy of the watt-meters on the low power factor of the loss supplying circuit.

T. H. Morgan and V. Siegfried: As pointed out by R. E. Hellmund, the d-c excitation method was checked against the precise determination for stray load loss for only a single case in this instance. It is also correct that one test does not justify a general conclusion as to the validity of this method, and that specifying the design of the machine is most important in interpreting the test results so obtained. The method as outlined in the proposed test code was followed in obtaining curve D in figure 5 of the paper, with the result that the loss caused by fundamental-frequency leakage flux in the stator was not included as part of the measured stray-load loss. Other measurements showed that the loss in this machine caused by this source was only 15 per cent of the total stray-load loss, which may account for the small discrepancy between curves B and D.

The d-c excitation method gives a measure of that portion of the stray-load loss which is due to tooth-frequency flux pulsations resulting from load current, and in cases where this component is predominant the test may give sufficiently accurate results. This accuracy may depend largely upon the design of the machine. The measurement of the fundamental-frequency loss component in the manner suggested in the paper provides for the inclusion of this part of the loss in the results from the d-c excitation method. There is no doubt as to the urgent need for more test information on various types of machines in order to answer satisfactorily the questions raised by Hellmund regarding the d-c excitation method.

The authors are in complete agreement with Howard Maxwell regarding the desirability of including the belted load-back method in the new test code. It gives a direct measurement of the losses of the machine under actual operating conditions. This test can be made readily without elaborate apparatus or special means for testing. The determination of the belt loss, which possibly might have offered some difficulty in the past, now is obtained readily with high accuracy by use of recently developed stroboscopic devices.

In loading-back methods where both machines are excited from the same power source, the measurement of stray-load

load loss falls in the same category as that of the no-load losses. Both measurements involve the use of wattmeters at low power factor, and it follows that accurate results for either depend upon the use of instruments giving high accuracy under these conditions. As pointed out in the paper under the heading "Measurements" the stray load loss is determined from the difference between 2 separate readings of the same wattmeter at nearly the same power factor, so that such error as does exist tends to be self-compensating. This is equally true for the belted load-back test.

Salient Pole Motors Out of Synchronism

Discussion and author's closure of a paper by A. H. Lauder published in the June 1936 issue, pages 636–49, and presented for oral discussion at the electrical machinery session of the summer convention, Pasadena, Calif., June 24, 1936.

L. V. Bewley (General Electric Company, Pittsfield, Mass.): Some rules for eliminating any number of rows and columns in the impedance matrix are given in appendix III of the paper. The occasion often arises for reducing the number of simultaneous equations in a problem, of which multiwinding transformer circuits and ground-wire theory furnish familiar examples. Thus in a recent paper ("Resolution of Surges Into Multivelocity Components," Electrical Engineering, volume 54, November 1935, pages 1199–1203) in order to eliminate the ground wires the writer followed the same procedure (without matrices) described by Lauder for reducing the number of equations. The procedure from which the rules are most easily seen is as follows: Let the set of simultaneous equations be split into 2 parts

$$e_\alpha = Z_{\alpha\beta}\, i^\beta + Z_{\alpha s}\, i^s \qquad (1)$$

$$e_r = Z_{r\beta}\, i^\beta + Z_{rs}\, i^s \qquad (2)$$

in which indices (α, β) range from 1 n and are to be eliminated, while indices (r, s) range from $n+1$. Solving equations 1 for i^β

$$i^\beta = \frac{A^{\beta\alpha}}{|Z_{\alpha\beta}|}(e_\alpha - Z_{\alpha s}\, i^s) \qquad (3)$$

in which $A^{\beta\alpha}$ is the cofactor of $Z_{\alpha\beta}$ in the determinant $|Z_{\alpha\beta}|$. Substituting equation 3 in equation 2 and putting $D_n = |Z_{\alpha\beta}|$

$$e_r = Z_{r\beta}\frac{A^{\beta\alpha}}{D_n}(e_\alpha - Z_{\alpha s}\, i^s) + Z_{rs}\, i^s$$

$$= Z_{r\beta}\frac{A^{\beta\alpha}}{D_n} e_\alpha + \left(\frac{Z_{rs}D_n - Z_{r\beta}A^{\beta\alpha}Z_{\alpha s}}{D_n}\right) i^s \qquad (4)$$

The numerators of the coefficients of e_α and i^s in foregoing equations are recognized as the expansions of "bordered determinants" (see McConnell's "Applications of the Absolute Calculus," page 19) so that equation 4 becomes

$$e_r = \frac{\begin{vmatrix} Z_{11} \ldots\ldots Z_{1n} & e_1 \\ \ldots\ldots\ldots\ldots & \\ Z_{n1} \ldots\ldots Z_{nn} & e_n \\ Z_{r1} \ldots\ldots Z_{rn} & 0 \end{vmatrix}}{D_n} + \frac{\begin{vmatrix} Z_{11} \ldots\ldots Z_{1n} & Z_{1s} \\ \ldots\ldots\ldots\ldots & \\ Z_{n1} \ldots\ldots Z_{nn} & Z_{ns} \\ Z_{r1} \ldots\ldots Z_{rn} & Z_{rs} \end{vmatrix}}{D_n}\, i^s \qquad (5)$$

and the reduced set of equations may be written

$$e_r' = Z_{rs}'\, i^s \qquad (6)$$

In the paper the voltages e_α are equal to zero for the amortisseur windings. If the field winding is assumed to be closed, the field axes also can be eliminated merely by reducing the impedance matrix, but if the field is excited, either the reduced matrix must still include the field axes, or a voltage-correction term must be added, as shown in equation 4 of this discussion, and as previously pointed out by Gabriel Kron ("The Application of Tensors to the Analysis of Rotating Electrical Machinery," *G. E. Review*, Dec. 1935). Ordinarily, however, the desirability of reducing the number of equations rests on the condition $e_\alpha = 0$ for all axes to be eliminated.

A. H. Lauder: L. V. Bewley's discussion of the paper is well worth while, for it deals with the elimination of co-ordinate axes. The author agrees with all that Bewley has said, and would like to emphasize the fact that Bewley's equations are in a more general form than the ones given in the paper. They would satisfy conditions where saturation or other nonlinear characteristics were involved.

One of the primary objects of the paper was to illustrate the performance of a motor when it operated out of synchronism. The author felt that the phenomenon could be clarified considerably by approaching the problem in the most elementary manner, that is, impressing alternating and direct voltages separately and combining the results. In doing this the treatment of eliminating co-ordinate axes was narrowed down to a few practical rules. Consequently the author is grateful to Bewley for supplying in such a thorough and concise manner the theory and proof that was lacking.

Analysis of Unsymmetrical Machines

Discussion of a paper by W. V. Lyon and Charles Kingsley, Jr., published in the May 1936 issue, pages 471–6, and presented for oral discussion at the electrical machinery session of the summer convention, Pasadena, Calif., June 24, 1936.

A. P. T. Sah (Massachusetts Institute of Technology, Cambridge): There have appeared recently in Electrical Engineering several papers on the analysis of unsymmetrical motors by either the revolving field theory or symmetrical components suitably chosen. The question has been asked whether the same problem can be solved with ease by the dyadic method as developed by the writer. The following outlines first the general solution of such a problem by dyadics and then illustrates the procedure by taking up in detail the capacitor motor already discussed by several previous writers.[1–3]

The fundamental equations for a motor without d-c excitation are:

$$\mathbf{V}_1 = \mathfrak{Z}_{11}\cdot\mathbf{I}_1 + \mathfrak{Z}_{12}\cdot\mathbf{I}_2 \qquad (1)$$

and

$$0 = \mathfrak{Z}_{21}\cdot\mathbf{I}_1 + \mathfrak{Z}_{22}\cdot\mathbf{I}_2 \qquad (2)$$

in which the symbols $\mathfrak{Z}$ with the same subscripts denote self-impedance dyadics and those with different subscripts the mutual impedance dyadics; $\mathbf{V}_1$ is the impressed vectorial voltage, and $\mathbf{I}_1$ and $\mathbf{I}_2$ are the vectorial stator and the vectorial rotor currents, respectively. Eliminating $\mathbf{I}_2$ between these 2 equations gives

$$\mathbf{V}_1 = \mathfrak{Z}\cdot\mathbf{I}_1 \qquad (3)$$

in which the impedance dyadic of the motor is

$$\mathfrak{Z} = \mathfrak{Z}_{11} - \mathfrak{Z}_{12}\cdot\mathfrak{Z}_{22}^{-1}\cdot\mathfrak{Z}_{21} \qquad (4)$$

Knowing this impedance $\mathfrak{Z}$, equation 3 can be transposed to give the vectorial stator current in terms of the known components of the impressed voltage. Symbolically this is

$$\mathbf{I}_1 = \mathfrak{Z}^{-1}\cdot\mathbf{V}_1 \qquad (5)$$

With $\mathbf{I}_1$ known, $\mathbf{I}_2$ can be evaluated from equation 2. The average steady state power output and hence the torque can then be calculated as the difference between the average steady state power input and the copper losses in the stator and the rotor. Thus,

$$\text{average power output} = \mathbf{V}_1\cdot\mathbf{I}_1 - \mathbf{I}_1\cdot\mathfrak{R}_1\cdot\mathbf{I}_1 - \mathbf{I}_2\cdot\mathfrak{R}_2\cdot\mathbf{I}_2 \qquad (6)$$

wherein $\mathfrak{R}_1$ and $\mathfrak{R}_2$ are the resistance dyadics of the stator and the rotor windings, respectively.

When a capacitor motor is to be analyzed by such a method, the symmetrical poly-

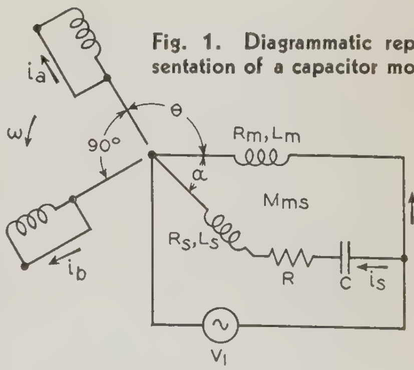

Fig. 1. Diagrammatic representation of a capacitor motor

phase winding on the rotor will be assumed to be replaced by a 2 phase winding, so that the impedance dyadics may be taken as uniplanar, having the same plane as the voltage and the current vectors. In other words, the problem involves only 2 dimen-

sions instead of 3. To find the impedance dyadic of the machine it is easiest to refer to the schematic diagram showing the relative positions of the different windings. In the accompanying figure 1 the axis of the main winding m leads that of the starting winding s by an angle α while the rotor winding a leads winding m by the angle $\theta = \omega t$, ω being mechanical angular velocity of rotor and t being time. From this diagram it may be seen that the self-impedance dyadics are

$$\mathbf{Z}_{11} = \begin{Vmatrix} Z_m & X \\ X & Z_s \end{Vmatrix} = \begin{Vmatrix} R_m + pL_m & p\,M_{ms} \\ p\,M_{ms} & R_s + R + pL_s + (1/pC) \end{Vmatrix} \tag{7}$$

$$\mathbf{Z}_{22} = Z_2 \begin{Vmatrix} 1 & 0 \\ 0 & 1 \end{Vmatrix} \tag{8}$$

in which $Z_2 = R_2 + pL_2$ while the mutual impedance dyadics are

$$\mathbf{Z}_{12} = Mp \begin{Vmatrix} \cos\theta, & -\sin\theta \\ N\cos(\theta+\alpha), & -N\sin(\theta+\alpha) \end{Vmatrix} \tag{9}$$

$$\mathbf{Z}_{21} = Mp \begin{Vmatrix} \cos\theta, & N\cos(\theta+\alpha) \\ -\sin\theta, & -N\sin(\theta+\alpha) \end{Vmatrix} \tag{10}$$

in which M denotes the mutual inductance between a rotor winding and the main winding when their axes coincide and N the ratio of the number of turns in the starting winding to that in the main winding. It may be stated that in equations 9 and 10 the usual assumption of a sinusoidally distributed flux for each winding is implied since the mutual inductance between a rotor winding and a stator winding is taken to vary as the cosine of angular displacement between their axes. Perhaps it should also be mentioned that $\mathbf{Z}_{12}$ and $\mathbf{Z}_{21}$ are not equal but the "transpose" of each other. With these impedance dyadics substituted into equation 4, there is obtained

$$\mathbf{Z} = \mathbf{Z}_{11} - M^2p \begin{Vmatrix} \cos\theta\dfrac{p}{Z_2}\cos\theta + \sin\theta\dfrac{p}{Z_2}\sin\theta & \cos\theta\dfrac{Np}{Z_2}\cos(\theta+\alpha) + \sin\theta\dfrac{Np}{Z_2}\sin(\theta+\alpha) \\ \cos(\theta+\alpha)\dfrac{Np}{Z_2}\cos\theta + \sin(\theta+\alpha)\dfrac{p}{Z_2}\sin\theta & \cos(\theta+\alpha)\dfrac{N^2p}{Z_2}\cos(\theta+\alpha) + \sin(\theta+\alpha)\dfrac{N^2p}{Z_2}\sin(\theta+\alpha) \end{Vmatrix} \tag{11}$$

Noting that the sines and cosines can be replaced by exponentials and that

$$(p/Z_2)\epsilon^{j\omega t} = \epsilon^{j\omega t}f$$

where

$$f = (p + j\omega)/[R_2 + (p + j\omega)L_2]$$

and

$$(p/Z_2)\epsilon^{-j\omega t} = \epsilon^{-j\omega t}f'$$

where

$$f' = (p - j\omega)/[R_2 + (p - j\omega)L_2]$$

equation 11 may be written in the following manner:

$$\mathbf{Z} = \begin{Vmatrix} Z_m - {}^1/_2M^2p(f+f') & X - {}^1/_2NM^2p(\epsilon^{j\alpha}f + \epsilon^{-j\alpha}f') \\ X - {}^1/_2NM^2p(\epsilon^{-j\alpha}f + \epsilon^{j\alpha}f') & Z_s - {}^1/_2N^2M^2p(f+f') \end{Vmatrix} \tag{12}$$

For the steady state solution, the complex impedance dyadic is to be obtained from equation 12 by replacing p by $j\omega_1$ as usual, that is,

$$\mathbf{Z} = \begin{Vmatrix} \check{A} & \check{B} \\ \check{C} & \check{D} \end{Vmatrix} \tag{12a}$$

From equation 5 the currents in windings m and s are respectively:

$$\check{I}_m = (\check{D} - \check{B})\check{V}_1/(\check{A}\check{D} - \check{B}\check{C}) \tag{13}$$

$$\check{I}_s = (\check{A} - \check{C})\check{V}_1/(\check{A}\check{D} - \check{B}\check{C}) \tag{14}$$

With the symbols properly interpreted these are the same as equations 122 and 123 given by Morrill (when α = 90 degrees), equations 19 and 20 given by Puchstein and Lloyd, or equations 48 and 49 given by Lyon and Kingsley. The evaluation of the rotor currents after the steady state has been reached shows that they are balanced but consist of 2 frequencies, one having a value of $(\omega_1 - \omega)$, i. e., 2π times the slip frequency S, and the other having a value of $(\omega_1 + \omega)$, i. e., $2\pi(2 - S)$ times the impressed frequency, when S is expressed as a fraction. The current at slip frequency when squared has a value

$$I_a^2 = \frac{(\omega_1 - \omega)^2M^2}{4[R_2^2 + (\omega_1 - \omega)^2L_2^2]} \times [I_m^2 + N^2I_s^2 + 2NI_mI_s\cos(\phi - \alpha)] \tag{15}$$

where ϕ is the phase difference between $\boldsymbol{I}_m$ and $\boldsymbol{I}_s$, and I_m and I_s denote the effective values of $\boldsymbol{I}_m$ and $\boldsymbol{I}_s$, respectively. The current at $(2 - S)$ frequency squared has a value

$$I_a'^2 = \frac{(\omega_1 + \omega)^2M^2}{4[R_2^2 + (\omega_1 + \omega)^2L_2^2]} \times [I_m^2 + N^2I_s^2 + 2NI_mI_s\cos(\phi + \alpha)] \tag{16}$$

The power output equation as given by equation 6 becomes

$$P_o = V_1(I_m\cos\phi_m + I_s\cos\phi_s) - R_mI_m^2 - (R + R_s)I_s^2 - 2R_2(I_a^2 + I_a'^2) \tag{17}$$

The average steady state torque in synchronous watts is

$$T = 2R_2[I_a^2/S - I_a'^2/(2 - S)] = (r/2S)[I_m^2 + N^2I_s^2 + 2NI_mI_s \times \cos(\phi - \alpha)] - [r'/2(2 - S)] \times [I_m^2 + N^2I_s^2 + 2NI_mI_s\cos(\phi + \alpha)] \tag{18}$$

wherein r and r' are effective rotor winding resistances referred to the main winding at frequencies obtained from $(\omega_1 - \omega)$ and $(\omega_1 + \omega)$. Equation 18 is the same as equation 109 of Morrill (when α = 90 degrees) equations 24 and 25 of Puchstein and Lloyd, or equation 50 of Lyon and Kingsley. When the rotor is blocked, $S = 1$ and $r = r'$; equation 17 simplifies to

$$T\text{ (blocked)} = 2rNI_mI_s\sin\phi\sin\alpha \tag{19}$$

which may be reduced to the equations already given by the other writers. It may be noted that with the 2-frequency rotor current known, one could invoke ordinary induction motor theory and write the average torque relation (equation 18) without going to the trouble of evaluating equation 17, since the current of slip frequency produces a torque in the forward direction and the current of $(2 - S)$ frequency produces a torque in the backward direction.

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2. CAPACITOR MOTORS WITH WINDINGS NOT IN QUADRATURE, A. F. Puchstein and T. C. Lloyd. ELEC. ENGG., v. 54, Nov. 1935, p. 1235–9.

3. ANALYSIS OF UNSYMMETRICAL MACHINES, W. V. Lyon and Charles Kingsley, Jr. ELEC. ENGG., v. 55, May 1936, p. 471–6.

J. F. H. Douglas (Marquette University, Milwaukee, Wis.): The paper provides an extension of the methods of symmetrical components to the theory of induction machines with unsymmetrical windings. The progress thus far made with this method leads to a hope for still further extension. Although the case of an air gap with varying reluctance in the direct and quadrature axis is specifically excepted, it is to be hoped that further improvements in the method may be secured for synchronous machines also.

The synchronous motor during starting has a symmetrical winding on the stator and an unsymmetrical winding upon the rotor, in addition to a wave of air-gap permeance which is different in the direct and in the quadrature axis. Although several papers have been presented on synchronous machines, using the method of symmetrical components, it is to be hoped that the authors will extend their investigations further along the line suggested.

Hydrogen Cooling—With Near-Critical Velocities

Discussion and author's closure of a paper by G. W. Penney published in the May 1936 issue, pages 530–4, and presented for oral discussion at the electrical machinery session of the summer convention, Pasadena, Calif., June 24, 1936.

R. H. Norris (General Electric Company, Schenectady, N. Y.): The paper is concerned with the heat transfer from the walls of a flat duct to hydrogen (or other gas) flowing through it at near-critical velocities,

both uniform and parabolic velocity distribution being considered.

The theory of this problem pertaining to laminar flow has been analyzed previously by M. A. Lévêque (*Annales des Mines*, series 12, volume 13, 1928, pages 201, 305, and 381) for uniform velocity distribution. Lévêque's analysis and results have been well summarized by T. B. Drew (American Institute of Chemical Engineers *Transactions*, volume 26, 1931, pages 52–5.) The author's analysis for this case is consistent with that of Lévêque but, since it is not carried so far, the analytic results as given in appendix II of the paper are not directly applicable to practical problems. This is explained later, and Lévêque's formulas, as stated in their more practical form, are given.

For parabolic velocity distribution, the author has presented numerical results for a single set of conditions (curve B in figure 3) without giving his theoretical analysis; but it is this case for which the analysis has not yet, to the writer's knowledge, appeared in any publication. Therefore, in its published form, the paper makes little contribution to the analytical aspect of the problem.

The paper is valuable, however, from the point of view of the specialist in heat-transfer analysis, because of its collection of new experimental data on the "local" heat transfer coefficients, and their variation from point to point in the duct as given in figures 3 and 4. These provide an indication of the accuracy to be expected from each of the 2 theoretical cases previously mentioned. These "local" coefficients are not directly useful for design purposes, however, for it is the performance of the duct as a whole that interests the designer.

From the point of view of the designer the results, both experimental and analytic, given in the paper seem inadequate for practical application. In the first place, the designer will not know definitely how to interpret or apply the results until certain questions, stated later in this discussion, are answered by the author. These questions pertain to conditions of flow of the inlet air, which may affect the performance by as much as 30 per cent, and to definitions that may affect the results by 15 per cent or more. In the second place, when the various possible answers to these questions are considered, it is found (as shown later) that they apparently lead to the following 2 dilemmas: (1) The results of major interest to the designer (the general formulas and the heat-transfer coefficients in figures 5 and 6 of the paper) apply only, for most of the velocity range considered, to a condition for which the entering air is quiet, which is not typical of practical applications; or, the test values apply to conditions with disturbed inlet air, whereas the theoretical values are for quiet inlet air. Since it is shown by figure 4 of the paper that the heat-transfer rate with disturbed air may be from 30 to 40 per cent greater than with quiet air over a large part of the duct, the apparently good agreement between test and theoretical values must then be due to coincidence or to some compensating factor not considered, so that unless such a factor is found and investigated, the reliability of the theoretical results for the general case will not be known. (2) The results in figures 5 and 6 of the paper (the test data, as well as the theoretical results are reliably applicable to design purposes only if combined with some new, presumably theoretical, equation for temperature difference or for total heat transfer, not given in the paper; or, the graphical results in figures 5 and 6 may be useful, but the general formulas are too incomplete for design purposes in the form published. As already mentioned, this last difficulty can be overcome for uniform velocity distribution by the use of Lévêque's formulas, as given at the end of this discussion.

In spite of the considerations just mentioned, the paper has some important significance, as regards the ratio of the heat-transfer coefficient for hydrogen to the coefficient for air at the same velocity. It shows that this ratio varies widely depending on the gas velocity and the tube spacing. Although the reader may not know definitely, how to apply the coefficients given in the paper to a practical problem of hydrogen cooling, the ratios between the coefficients for air and those for hydrogen probably are of about the correct magnitude, for regardless of the way in which they are applied, they are presumably applied in the same way for air as for hydrogen. Until the foregoing questions are answered, however, the writer would prefer to rely on Lévêque's theoretical formulas for the calculation of this ratio, for not only are all of Lévêque's results perfectly definite in meaning, but Penney's test results for local coefficients as given in figure 3, indicate that fair accuracy is obtainable from the assumptions used by Lévêque.

The major topics of this discussion having thus been outlined, the specific questions previously mentioned will now be stated and discussed. In this discussion all symbols have the meanings defined in the paper unless otherwise mentioned.

Question 1. Do the test results given in figures 5 and 6 of the paper apply when the entering air is quiet, or when it is disturbed?

In the critical region, which covers the range of velocities starting at the "critical velocity" (the critical velocity for figure 5 of the paper has been calculated by the writer to be 447 feet per minute for air and 3,110 feet per minute for hydrogen; for figure 6 the critical velocity is 1,250 feet per minute for air, and 8,720 feet per minute for hydrogen, based on a gas temperature of 40 degrees centigrade) and ranging upward to as much as 5 or 10 times this velocity, laminar conditions of flow can exist but are not stable; consequently, as indicated by figure 4 of the paper, a small disturbance of the flow near the inlet will be perpetuated, with an increase in the total rate of heat transfer of perhaps 30 per cent. Since the theoretical analysis is based only on pure, undisturbed laminar flow, and since in rotating electrical machinery the air entering the ducts generally is disturbed, and since most of the plotted data for air, and 2 of the 6 test values for hydrogen, lie in the critical region, the writer is led to the first dilemma previously stated.

The possibility that some of the plotted test data may correspond to truly turbulent, not laminar, flow conditions is indicated by the fact that the upper 2 values of K_v in curve B_1 of figure 5 of the paper have been found by the writer to agree reasonably well with the values predicted by a commonly used equation ("Heat Transmission," William Henry McAdams, McGraw-Hill Book Company, Inc., 1933, page 172, equation 12) for turbulent flow conditions. On the contrary, the existence of a region where the flow is neither truly turbulent nor truly laminar, is also indicated by the author's results. Although curve B of figure 4 of the paper is stated to apply to turbulent flow conditions, it shows a decrease in the local heat-transfer coefficient by about 50 per cent from inlet to exit, whereas it is known that for truly turbulent flow the coefficient should be constant except for relatively small effects of variations of properties of the gas with temperature.

Question 2. What is the definition of the mean temperature difference by which the "average" heat-transfer coefficient given in figure 5 or figure 6 should be multiplied in order to obtain the total heat transfer?

Of the 2 alternatives in common use—the arithmetic mean, and the logarithmic mean of the entrance and exit values—the latter often may be of about 15 or 20 per cent lower than the former, and there is reason to believe, as explained later, that the proper means in the present case is neither of these.

Question 3. Are the heat-transfer coefficients in figures 5 and 6, denoted here by $(K_v)_{\text{av.}}$ defined by the equation:

$$(K_v)_{\text{av.}} = \frac{1}{L}\sum_{y=0}^{y=L} K_v\,\Delta y$$

which is approximately equivalent to

$$(K_v)_{\text{av.}} = \frac{1}{L}\int_0^L K_v\,dy \qquad \textbf{(D1)}$$

where L is the length of the duct in the direction of flow and K_v is the "local" coefficient at the point y, or is this "average" value of K_v a "weighted" average in accordance with the variations in the local temperature difference, θ_D?

In order to appreciate the reason for these questions, consider the basic expression for the total heat transfer, W_L per unit width of the duct (valid in all cases)

$$W_L = 2\int_0^L K_v\theta_D\,dy \qquad \textbf{(D2)}$$

This follows directly from the definitions of θ_D and of K_v. In this discussion K_v always denotes merely the "local" coefficient, and $(K_v)_{\text{av.}}$ the "average" coefficient, the definition of the latter being the subject of question 3. Now, if the variation of θ_D with respect to y were only a small part of its initial value, equation D_2 may be replaced by

$$W_L = 2\,(\theta_D)_{am}\int_0^L K_v\,dy \qquad \textbf{(D3)}$$

where the subscript am indicates the arithmetic mean; or, if the variation of θ_D were large, but that of K_v were small, as is often the case with truly turbulent flow, equation ($D1$) might by

$$W_L = 2\,(\theta_D)_{lm}\int_0^L K_v dy \qquad \textbf{(D4)}$$

where subscript lm indicates the logarithmic mean. In the near-critical region both θ_D and K_v will generally vary greatly with y. For example, in figure 3 of the paper K_v is 3 or 4 times as great at the duct entrance as it is at the exit, and rough computations by the writer indicate that θ_D, for the conditions of figure 3 of the paper is 4 or 5 times as great at the entrance as it is at the exit; moreover, the data of figure 3 indicate that the variation of K_v and hence of θ_D from entrance to exit will be far from linear. Hence, neither equation $D3$ nor $D4$ of this discussion can be used with confidence, and the necessity for answers to questions 2 and 3 is evident.

The reasons for the second dilemma, to which these questions lead, will next be explained. Suppose that the answer to question 3 stated that equation $D1$ really is the definition of $(K_v)_{av}$ intended by the author. This interpretation is, in fact, indicated by the remarks following equation 8 of the paper. If this is so, then, since equation $D3$ and $D4$ of this discussion are not valid, neither the arithmetic nor the logarithmic means are applicable, and some new definition of mean temperature difference or a new equation for W_L is needed in order to be able to calculate the over-all performance. In the absence of tests data on the variation of θ_D with y, however, this definition or equation presumably would have to be derived from some theoretical assumptions, such as those used in appendix II of the paper, from which the variation of θ_D could be computed, as done, for example, in equation 7 of the paper. The writer thus is led to the first alternative of the second dilemma mentioned previously.

Next consider the alternative answer to question 3—that the variation in θ_D must be given weight in computing $(K_v)_{av}$. Since the method of computation is not explained, and does not seem to be immediately evident, led the writer to the second alternative of the second dilemma.

Lévêque's results (in the form given by Drew) which overcome this last difficulty, are as follows:

The theoretical total heat transfer per unit width of duct, under assumptions that agree with those used by the author in appendix II, is given by

$$W_L = c\gamma V_a (\theta_D)_{y=0} \left\{ 1 - \frac{8}{\pi^2} \sum_{i=1,3,5\,..}^{\infty} \frac{e^{-i^2 n}}{i^2} \right\} \quad \text{(D5)}$$

where

$$n = \frac{\pi^2 kL}{c\gamma V_a^2}$$

and e is the base of natural logarithms.

This result can be expressed conveniently in terms of a heat-transfer coefficient, if this coefficient is based not on the local coefficient K_v, but merely on the definition

$$h_{am} = \frac{W_L}{(\theta_D)_{am} \times 2L} \quad \text{(D6)}$$

where $2L$ is the surface area per unit width, $(\theta_D)am$ is the arithmetic mean of the inlet and outlet values of θ_D, and the subscript am of h_{am} merely indicates that this form of heat-transfer coefficient is to be used in connection with $(\theta_D)am$. The expression for h_{am} derived from equation $D5$ is then:

$$h_{am} = \frac{C\gamma Va}{L} \left\{ \frac{1 - \dfrac{8}{\pi^2} \displaystyle\sum_{i=1,3,5\,..}^{\infty} \frac{e^{-i^2 n}}{i^2}}{1 + \dfrac{8}{\pi^2} \displaystyle\sum_{i=1,3,5\,..}^{\infty} \frac{e^{-i^2 n}}{i^2}} \right\} \quad \text{(D7)}$$

The accuracy to be expected from equations $D5$ and $D7$ may be estimated from comparison of curve A (test values of K_v) of figure 3 of the paper and curve C (theoretical values of K_v based on the same assumptions as those of Lévêque). The presentation of this comparison of curves A and C seems to the writer to be the most valuable feature of the paper.

G. W. Penney: The work, a part of which is reported in this paper, was undertaken to obtain an experimental check of theories of heat transfer. In order to simplify the mathematics, the form of duct described in the paper was chosen. This does not represent a shape of much interest to the designer of electrical machines; furthermore, neither the logarithmic mean nor the arithmetic mean commonly used could represent accurately the conditions found. For these reasons a plot of heat-transfer constant based on the conventional logarithmic mean temperature was not considered to be of much value. Instead, an "average value," which was simply the arithmetic average of the various values of heat transfer along the duct length, was used. This gives a direct comparison between the mathematical and experimental results. If the curve of heat-transfer constant had been given in terms of the conventional logarithmic mean, it would have been strictly applicable only to this particular length of duct. Since no attempt was made to plot such a curve, it is not surprising that R. H. Norris finds himself in a dilemma when he attempts to use these values directly in machine design. Designers have a large amount of data giving heat transfer of air flowing in ducts of the various shapes found in machines so that the most useful information for the designer was considered to be the ratio heat transfer in hydrogen to heat transfer in air.

Norris also inquired about the condition of the air entering the duct. In figure 5 of the paper there is an error in printing. The arrow from the circle B_2 should point to the dotted line. The double curve given by the points B_1 show the range in values given by disturbed and undisturbed air entering the duct. The maximum disturbance tested being that given by the low speed fan used in the tests. Probably a greater increase in heat transfer could be obtained by greater turbulence, which could be obtained from a fan of higher speed. The other curves of figures 5 and 6 of the paper respresent lower values of Reynold's number where the turbulence given by a low speed fan gave only a slight increase in heat transfer since the turbulence disappears rapidly under these conditions. Points shown were obtained with both undisturbed and disturbed flow.

Circuit Breakers for Boulder Dam Line

Discussion and authors' closure of a pap by H. M. Wilcox and W. M. Leeds pub lished in the June 1936 issue, pages 626–3 and presented for oral discussion at the pro tective devices session of the summer con vention, Pasadena, Calif., June 26, 1936.

C. A. Powel (Westinghouse Electric Manufacturing Company, East Pittsburgh Pa.): The most striking feature of th circuit breakers described in the paper that the remarkable circuit-opening spee of 3 cycles has been obtained with relativel small modification to conventional design which shows that the principles applied t the original deionizing oil circuit breaker ar absolutely sound. Adherence to a conver tional design has the advantage that th breaker may be readily adapted to rapi reclosure service. There appears to be distinct trend toward such adherence Rapid reclosure may well create the majo demand for very high speed of operation, fo reasons of stability or maintenance o service.

The tremendous investment in the Bou der Dam lines fully justifies the operatin speeds specified, but the majority of trans mission lines do not have the financial im portance of these lines and the higher cost o the 3-cycle breaker probably dictates mor moderate speeds of from 5 to 8 cycles fo most applications.

To obtain high rupturing speeds th multiple break principle is used. When firs applied 15 or 20 years ago the multipl break idea did not appear to offer muc promise, because the quantity of gas liber ated by the arc with the contacts used a that time was not appreciably less tha that obtained with 2 breaks. That diff culty has been overcome by the great reduc tion in arc energy made possible by the ne rupturing principles and by the way i which electrostatic balance has been ob tained between the series contacts.

The use of a magnetic field in extinguish ing the arc has the merit that the higher th short circuit current, the stronger the mag netic field, and the quicker the arc will b ruptured. This inverse time characteristi is beneficial to the breaker performance.

S. W. Copley (Westinghouse Electric Manufacturing Company, San Francisc Calif.): One of the features of the breake described in the paper is of interest becaus it gives an explanation for the opinion hel by many engineers a few years ago that th "multibreak" type of circuit breaker do not accomplish as much as might be e pected of it. Tests indicated that increasi the number of breaks above the conver tional 2 ordinarily did not increase inte rupting ability, and such deficiency wa attributed to a belief that 1 or 2 brea accomplished the result.

In the breaker described in the paper multibreak design seemed absolutely nece ary because of the high speed requiremen therefore, the designers were forced to solv the problem of making each break do i share of work in interrupting the circui The solution was found in the static shiel

which distributes the voltage stress over the various breaks and prevents its concentration on any one. In the tank type of breaker the capacitances from contact to contact and from each contact to the tank are such as to cause unequal potential distribution unless shields are used. Apparently the system of shields described is effective.

The writer's understanding is that for lower operating speeds, such as 8 cycles, the 2-break arrangement would be considered adequate even at 287,000 volts, and the expensive multibreak shielded contacts could be avoided.

D. C. Prince (General Electric Company, Philadelphia, Pa.): The authors are to be congratulated on the solution of a very difficult problem. The interruption of 287-kv circuits is difficult even though unfettered by conventional tank configuration. J. Slepian pointed out ("Extinction of an A-C Arc," AIEE TRANSACTIONS, volume 47, October 1928, p. 1398–1407) a potential unbalance of the order of 9 to 1 in a tank-type breaker, so that the difficulty of securing substantially uniform distribution, not across 2, but across 10 breaks in a tank-type breaker will be appreciated.

The solution has involved the use of elaborate grading shields that further complicate the insulation problem, already serious at 287 kv, with its 650-kv 1-minute test. The authors have not said so, but it is presumed they have made a high-potential test with the crosshead contacts touching the bottoms of the stacks. Without such a test the stack insulation and grading are subjected to no voltage; with the breaker closed, all of those parts are at a common potential; with it fully open, the 2 groups of stacks assume their respective bushing potentials.

That the structure of the deionizing grid in this breaker approaches the explosion chamber is worthy of note. In the design described, the chamber is complete, although the volume of oil seems to be small for good explosion chamber practice. It would be interesting to know what pressure is generated in it and what mechanical factors of safety can be obtained in a structure requiring such high insulation levels.

The arrangement of the oil pockets suggests that a greater or lesser number may be called into play as current varies. It would be interesting to know whether the arc gets farther around under the urge of high magnetic fields and what happens when it reaches the end of the slot.

Now that this breaker has been introduced with 10 spring-driven oil-blast pistons to secure high speed operation at both low and high currents the spring-driven piston seems to be accepted. ("Circuit Breakers for Boulder Dam Line," D. C. Prince, ELECTRICAL ENGINEERING, volume 54, April 1934, p. 366–72.) Having used the spring-driver piston, it seems regrettable that so many other assisting expedients had to be added; but whether a single adequate piston or a multiplicity of small ones requiring assistance from supplemental means is used, is for the designer to decide.

A mechanical speed sufficient to give $1^1/_4$-cycle contact parting time is, by itself, no mean achievement. Since mechanical stresses increase usually with the square of the speed, as compared with the impulse breakers with their 2-cycle contact parting time, the problem in this breaker was $(2/1^1/_4)^2$ or over 2.5 times as difficult.

This extreme speed was made necessary by the fact that arcing time was, even with the small pistons, about $1^1/_4$ cycles, or double that of the corresponding impulse breaker. Also, a tendency of this time to increase at light currents is noted. The arcing times of table I of the paper are of some academic interest, but the proof of the transfer point seems somewhat obscure and the total arcing time certainly is the figure of greatest importance.

The demonstration of interrupting capacity of large breakers by special tests has been the subject of another paper before this session ("Special Tests on Impulse Circuit Breakers," W. F. Skeats, ELECTRICAL ENGINEERING, volume 55, June 1936, p. 710–17). The conclusions of that paper have been sharply questioned by J. Slepian in his discussion. Does not Slepian's reasoning apply with real force to the present paper in which the extrapolation depends upon the distribution of voltage among 10 breaks being the same under fault conditions as the electrostatic distribution? Van Sickle pointed out 2 years ago that this was not always the case ("Arc Extinction Phenomena in High Voltage Circuit Breakers—Studied with a Cathode Ray Oscillograph," AIEE TRANSACTIONS, volume 52, September–December 1933, p. 850–57).

Have the authors evidence that their breaker, in interrupting a short circuit, does not alter the voltage distribution? The single-break tests of table I of the paper go only to 1,500 amperes at 72 kv, whereas the testing equipment available goes to at least 5,800 amperes, as shown in the 5-break tests at a higher voltage. One wonders how much voltage unbalance these grids will stand at currents near the interrupting rating of the breaker. On this one point, the tests certainly do not extend anywhere near the interrupting capacity of the testing plant.

These breakers of the conventional tank type may well mark an all-time high in size, weight, and oil content, but size is not of itself an advantage. The porcelain-housed impulse breaker is believed to be inherently better adapted to obtain the voltage distribution required by the extreme insulation level and to the speed requirement of this application.

R. E. Hellmund (Westinghouse Electric & Manufacturing Company, East Pittsburgh, Pa.): The paper is so complete and clear in its description of the new breakers and in pointing out the necessity for the various features that little can be added along that line. With the clear and logical description of the successfully completed development everything looks very simple and papers of this character possibly do not fully portray the enormous amount of research activities and testing work required to determine the best and simplest final designs and to demonstrate the adequacy and the liberal margin of safety shown in the paper.

The last sentences of the paper not dealing directly with this particular design but making reference to the possible alternative of using a porcelain-clad structure, may be worth amplifying. Even though the last sentence adequately expresses an attitude with which few would take issue, something more might be said in order to avoid any misunderstanding. In view of a marked trend on the European continent toward porcelain-clad structures and the installation of several porcelain-clad breakers in this country, it is only natural that those not familiar with all of the facts should wonder about the whys and wherefores, especially as they apply to Europe.

Large power concentrations occurred in America much earlier than in Europe, making it necessary to carry on extensive rupture tests on power systems, and to install laboratories capable of testing equipment of large capacity several years before they were considered necessary in Europe. As a consequence, there was a certain period during which fully adequate tank-type breakers were available here and most European structures were quite inferior in rupturing capacity. It was then that the increasing power concentrations in Europe brought about some bad oil breaker explosions. These created a demand for breakers not using oil. The air-blast breaker of the Algemeine Elektricitäts Gesellschafts was, so far as the writer knows the first attempt to satisfy this demand, and at the time it attracted sufficient attention to cause other manufacturers to undertake similar developments. Siemens-Schuckert developed their so-called expansion (The name "expansion breaker" was chosen because, according to the theory of the Siemens engineers, the adiabatic expansion of the gases generated by the arc which is brought about by their specific construction plays an important part in extinguishing the arc.) breaker using water with an antifreeze liquid for voltages up to about 40,000. A porcelain-clad structure with reduced oil volume was developed by them for higher voltages. Many other manufacturers followed that example, and there are a great many porcelain-clad breakers giving satisfactory service and representing a fair solution of the problems as they exist in Europe.

This statement is made in spite of the fact that at the start various troubles were reported with those breakers. The air-blast breakers, for instance, presented some problems in insulating the enclosed spaces unless the air was carefully dried; in other words, the breakers required air conditioning in addition to a separate source of power and tanks for compressed air. There was also some criticism of objectional noises until mufflers or similar means were provided to reduce this. The operation now is satisfactory, but a breaker designed with all the adjuncts mentioned and with their operation dependent upon them does not seem to represent a solution to be adopted hastily. The complications of such breakers might be justifiable where, as customary in Europe, breakers of ratings up to 100,000 volts or more are frequently installed inside expensive station buildings, but they certainly do not seem to be the best design for American conditions, under which high-voltage stations are universally of the outdoor type.

The Siemens expansion breakers for lower voltages are not suitable for unprotected outdoor mounting, and although satisfactory in Europe, where it is customary to house the lower voltage equipments, they would not be suitable for the American market. The porcelain-clad expansion oil breakers for voltages above 40,000 can be

used outdoors, but in their present form would not satisfy modern requirements. Although their arc duration is short, their operating mechanism is not designed for high-speed operation. Furthermore, it would be difficult to make some of the porcelain columns of sufficient strength to withstand the heavy mechanical stresses incident to high-speed operation. In some European countries there has been no apparent demand for high-speed breakers so far, possibly because of the general use of the Peterson coil. Incidentally, everything else being equal, the Peterson coil should reduce materially the number of breaker operations and thus minimize oil contamination and favor the use of a very small volume of oil. With grounded systems and more frequent breaker operations, a small volume of oil would mean more frequent renewal of oil.

All of this, together with certain problems in connection with securing oiltight joints between metal and porcelain parts, the high cost of arranging for separate current and voltage transformers or voltage-tapping devices supports the opinion expressed that at the present time a complete swing away from the simplicity, ruggedness, and reliability of the steel-tank design is not justified.

H. M. Wilcox and **W. M. Leeds:** The authors appreciate the constructive discussion offered, particularly that of D. C. Prince, who has discussed several interesting points. His congratulations upon the problem of securing a satisfactory distribution of voltage among the 10 breaks perhaps accord this particular feature more prominence than is rightly due it when compared with other problems arising in the course of this development. It is true, as pointed out, that the unbalance between 2 breaks in a steel tank may be as great as 9 to 1 under certain conditions, but this inherent unbalance may be determined by comparatively simple means and, once known, can be corrected by paralleling each break with capacitance in sufficient quantities to become the controlling factor in determining the voltage across that break.

The authors are not entirely in agreement with Prince's statement that insulation of the stack and static shield are not subjected to voltage on insulation test unless the cross bar contacts touch the bottom of the stacks. The tests for voltage distribution given in the paper were made with the cross bar in this position, but other tests for voltage distribution with the contacts fully open, not referred to in the paper, indicate a very definite voltage gradient across the 10 contact breaks and the 2 disconnect breaks, this gradient existing during the high-voltage insulation test. Since this same gradient exists at all times during which the breaker will be called upon to withstand service voltage in the open position, it appears that the standard test for insulation in the open position of the cross bar meets all requirements. Adequacy of the stack and shield insulation under short circuit conditions has been demonstrated amply by the interrupting tests at 264 kv across half of a pole unit.

Whether the electrostatic distribution among the several breaks is disturbed during the interruption of fault currents has been questioned. In the references cited (Van Sickle, and Davies and Flurscheim) an analysis of cathode-ray oscillograms obtained on certain oil circuit breaker tests revealed the presence of appreciable leakage currents across the contact gap immediately after the final current zero of the arcing period; sufficient in some cases to modify the rate of rise of recovery voltage. However, such influence as this slight conductance had on the voltage distribution seemed to be in the direction of making it more uniform instead of less. Similar oscillograms of short circuit interruptions with the Boulder Dam grids show positive arc rupture without the presence of such leakage currents, indicating that the conductance across the contact gaps is negligible compared with that of the shield, and that the latter becomes effective in controlling voltage distribution immediately upon the cessation of current through the arc. This is the principle upon which the breaker was designed, and its successful interrupting performance at high voltages per inch of contact separation indicate that it is functioning as designed.

Some of the discussion suggests that the breaker described in this paper approaches the explosion chamber as well as the spring-driven-piston impulse device. Apparently Prince misinterprets the purpose of the small dashpot plungers in the grid design. As stated in the paper, their principal function is to assist in transferring to the arcing horns arcs of low current value generating only weak magnetic fields. Considering their small size and the negligible energy required to operate them, these pistons obviously could have no direct arc-rupturing effect. It is agreed that the oil content of the grid is small for explosion chamber practice, but its excellent performance in terms of volts interrupted per inch of contact separation suggests that its principles of operation may be quite far removed from those of the explosion chamber. So far as the mechanical strength of the grid stacks is concerned, an adequate factor of safety has been demonstrated directly by interrupting currents at reduced voltage up to values more than twice the current interrupting rating of the breaker. Since accurate knowledge of the internal pressure is not essential in designing this type of device, no attempt was made to determine the exact magnitude of pressure for different test conditions.

The conclusion that a greater or smaller number of oil pockets in the interrupting slot may be called into play as the current varies is not entirely substantiated by testing experience. It is true that field strength varies with current up to the point of saturation, but examination of the arcing horns after many interrupting tests at varying currents indicates that a surprisingly large proportion of the arc trails ceased on a relatively small section of the horn located from $^1/_2$ to $^2/_3$ of the way around the slot. The reason is that the slot restrictions are graded so that progress of the arc becomes increasingly difficult as it moves around the slot. The heavier currents generate a stronger driving field, but the stronger field must move an arc of greater cross section in the restricted passage and the 2 tend to counterbalance each other. It appears that the element of time (waiting for a current zero) is the more important factor in determining the distance an arc will travel on the horns.

Prince's mathematical determination of the difficulties involved in securing $1^1/_4$-cycle contact parting time in the deionizin[g] grid breaker as compared with the 2-cyc[le] parting time of the impulse breaker seem[s] to have been made without due conside[r]ation of the comparative masses involve[d]. In contrast to the problem of getting larg[e] oil-driving pistons in motion before separat[]ing the contacts, in the grid design it i[s] necessary only to accelerate the light con[]tact members themselves. The sma[ll] plungers add practically nothing to the loa[d] for they merely move into the grids a quan[]tity of oil equal to that displaced by th[e] contacts. In fact, the over-all stored energ[y] required to give $1^1/_4$-cycle contact partin[g] time in a 3-pole grid breaker is only 15,00[0] pound-inches as compared with a reporte[d] stored energy of many times that amoun[t] necessary to secure 2-cycle parting time i[n] the impulse breaker of the same rating.

A Faster Carrier Pilot Relay System

Discussion of a paper by O. C. Traver an[d] E. H. Bancker, published in the June 193[6] issue, pages 688–96, and presented for ora[l] discussion at the protective devices sessio[n] of the summer convention, Pasadena, Calif. June 26, 1936.

R. M. Smith (Westinghouse Electric & Manufacturing Company, Newark, N. J.) **E. L. Harder,** and **W. A. Lewis** (Westing[]house Electric & Manufacturing Company East Pittsburgh, Pa.): Field experienc[e] with carrier pilot relaying installations together with further study and develop[]ment in this field, has brought into genera[l] acceptance certain desirable features for [a] relay system of this type. The writers ar[e] in general accord with the features outline[d] by the authors. The impedance type o[f] fault-detecting elements have been used fo[r] several years in high-speed distance relays and the same principle of fault detecting ha[s] been used with carrier-pilot systems. Th[e] authors doubtless are now adopting thi[s] type of fault-detecting element because dis[]crimination between faults and loads is pos[]sible over a much greater variety of condi[]tions.

Additional features that are very desir[]able, and which were not mentioned by th[e] authors, are:

1. The relay protection should be worked ou[t] completely as a unit, including provision for back-u[p] protection and operation of the relay equipmen[t] when the carrier current equipment is out of servic[e] for any reason. Proper fault discrimination and a[s] many of the features of the carrier protection a[s] possible should be retained when the carrier equip[]ment has been removed from service.

2. Blocking during out-of-synchronism period[s] and blocking during external faults should b[e] accomplished by independent devices so that out[-]of-synchronism blocking normally can be applie[d] to back-up protection, if desired, as well as to th[e] carrier protection; also so that relay circuits no[t] requiring out-of-synchronism blocking never ar[e] interrupted by the out-of-synchronism action.

With the usual high speed distance pro[]tection, phase faults in the center 80 per cen[t] or more of a line section are cleared a[s] rapidly or possibly slightly more rapidl[y] than is possible with carrier current relaying Only in the so-called "end zones" does car[]rier relaying offer speed advantages an[d]

then only for the breaker remote from the fault. By adding carrier current to the second element of the distance protection maximum speed can be obtained over the entire section and with the carrier equipment out of service for inspection or maintenance, the fully selective high-speed distance protection can be maintained.

Figure 1 of the paper indicates that a single ground fault detector is used. On many systems ground current of magnitude equal to the relay setting may flow through a line section to a remote ground fault beyond. Generally identical ground relay settings at the 2 ends of a line are impossible to obtain. Charging current also causes some current difference at the 2 ends, and current transformer errors may add to the difference; consequently, there is a possibility of the ground current relay picking up at only one end of the line and resulting in undesired tripping. A similar condition exists for the phase relays, and becomes important if one set of the fault detectors is omitted with the out-of-step feature as indicated in the paper. The lower voltage at that end of a good line nearest to a fault favors the fault detector at the blocking end, provided it is of the impedance type; however, when power feeds through a short line to a remote fault the voltage difference between the 2 ends of the short line may be small, thus tending to cause faulty operation unless the fault detectors are accurately set to discriminate under this condition. This emphasizes the desirability of distance type fault detectors and further indicates that improved margins of safety are obtainable with elements capable of accurate distance measurement.

The directional-element time curve, figure 3 of the paper, does not give the time for single line-to-ground faults. For that type of fault, there may be load power of considerable magnitude in the reverse direction of the fault power. This would result in a greatly reduced torque in the relay, which would respond to the difference between load and fault power. The operating time on ground faults therefore is important and would be of general interest. Likewise, no oscillograms have been given for the line-to-ground faults. Since the majority of faults are of this type, this appears an important omission.

The 3-phase fault of figure 8 of the paper starts out as a single line-to-line fault. This would eliminate the restraint and provide a large operating torque to start the directional element during the interval from the instant of fault on 2 lines (indicated by voltage reduction) until the third phase is involved (indicated by further voltage reduction and current rise in the phase shown on the oscillogram). This does not therefore appear to be representative of a 3-phase fault. The oscillogram indicates zero voltage at the far end of the line under that condition. That zero voltage further indicates that the torque was contributed largely during the interval while the fault was single phase. In figure 9 likewise the fault starts single phase. Although the single line-to-line fault later involving a third phase frequently occurs it appears somewhat misleading to indicate the relay time so obtained to be the time for a 3-phase fault.

The tripping time is stated to be the sum of directional element time plus the dropout time of the receiver relay. This implies that the directional element operates as rapidly before restraint is removed as afterward. If so, why is the restraint removed?

The action of the relay system for a particular case of simultaneous fault, namely, faults on 2-line sections which are in series between 2 power sources is mentioned. This case is considered with and without a source between the affected line sections. There are numerous other types of simultaneous faults such as on parallel lines and between circuits of double circuit lines, that would need to be investigated before a relay scheme could be acclaimed as perfect for simultaneous faults.

Other cases including parallel lines and loop feeds with intermediate load points involve more complex relations among the relay quantities. Careful analysis indicates that faulty operation is possible under such conditions.

Philip Sporn (American Gas and Electric Company, New York, N. Y.): The system described is almost identical with that in service on the systems of several subsidiaries of the American Gas and Electric Company. About the only important difference between the scheme the authors describe and the one on the American Gas and Electric system is that the latter system has been developed so that it is inherently somewhat faster in operation than the system described in the paper. A description of the scheme employed on this system was published in the October 12, 1935, issue of *Electrical World* under the title "One Cycle Carrier Relaying Accomplished."

The similarity of the 2 schemes is shown by the fact that in both of them 3 sets of circuit-opening contacts and one set of circuit-closing contacts are employed on the instantaneous fault detectors to accomplish the following:

1. Removal of voltage restraint from the power directional relay.
2. Removal of the negative bias on the transmitter grid.
3. Transferring the energization of the receiver relay from the local battery circuit to the carrier circuit.
4. Applying negative polarity to the contacts of the receiver relay.

In both schemes, the de-energizing of the receiver relay in conjunction with the energizing of the fault detector causes tripping of the oil circuit breaker.

The only difference between the 2 schemes is in the functions of the power directional relay. In the scheme described by the authors, the contacts of the power directional relay control the negative bias to the transmitter grid and the contacts of the directional relay normally are held open by voltage restraint; thus, for an internal fault the normally-open contacts of the directional relay will have to close to stop transmission of carrier to allow the receiver relay contacts to close and trip the circuit breaker. In the scheme developed for the American Gas and Electric systems the contacts of the power directional relay control the plate voltage of the transmitter and the contacts of the power directional relay are normally held closed by voltage restraint. Then, in case of an internal fault the normally closed contacts of the directional relay must open to stop transmission of carrier, which allows the receiver contacts to close and trip the circuit breaker. It is obvious, of course, that inherently a power directional relay will open its contacts faster than it will close them; therefore, a scheme employing circuit-opening contacts on a power directional relay will be faster than a scheme employing circuit-closing contacts.

In the scheme described by the authors 2 sets of fault detector relays are employed with the primary purpose of preventing false operation when the short circuit current on the line section is near the pick-up value of the fault detector relays. The possibility of this condition is no greater than the possibility of a potential fuse blowing and causing a false operation from the impedance fault detectors on load current. It is decidedly preferable to use only one set of fault detector relays, both from the standpoint of simplicity and reliability of operation.

A further complication in the scheme the authors describe is that introduced by the out-of-step scheme of protection used. This not only adds complication to the scheme but also delays the tripping time in the case of an internal 3-phase fault. The lockout relay employed in the writer's scheme does not result in the disadvantages mentioned previously, but adequately provides for out-of-step conditions and provides faster operation in event of a 3-phase internal fault.

There is a further point of decided practical importance: the scheme of assembling all units with their accessories in one case. The writer knows from experience that it is not possible, if this is done, to design a

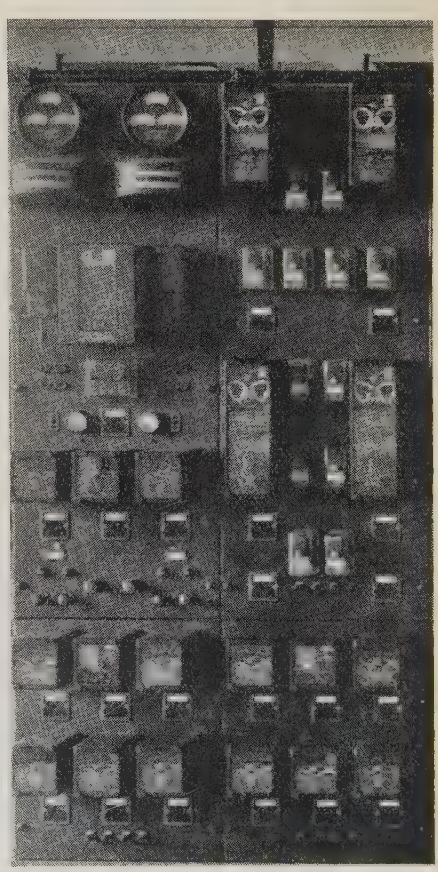


Fig. 1. Front view of typical carrier current relay panels in use on the American Gas and Electric Company system

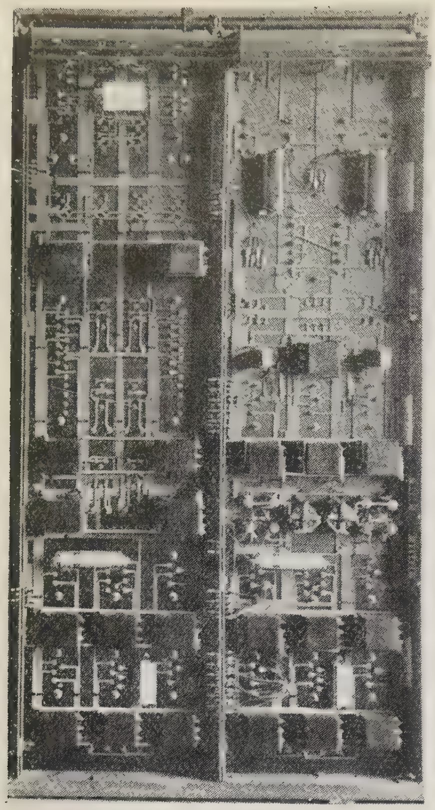


Fig. 2. Rear view of the same panels shown in figure 1

simple, practical, and flexible scheme for testing the various component relays. It is impossible to keep any relay scheme, and particularly a carrier current relay scheme, operating dependably unless simple and practical provisions are made for testing the individual relays periodically. There is a further objection to the location of all components in one case: the carrier relay arrangement must be such that it can be worked in with other existing schemes of relay protection. This must be done, if it is to be done economically, by using the existing panels. It is almost impossible to do this with the carrier current relays arranged in a single case as shown in figure 5 of the paper. The explanation in the paper is that by putting all components in one case, back-of-board wiring can be almost eliminated, and complicated back-of-board wiring as shown in figure 7 of the paper can be simplified greatly. In fact, if the wiring between components is made in one case it is almost impossible to introduce test facilities. Further, in the back-of-board arrangement shown in figure 7 as a typical arrangement, the authors may be suspected of being somewhat disingenuous. They have taken a photograph of a man in a discarded suit of old working clothes and compared it with a sketch of another in his best and then have tried to prove from this that the first individual is a hobo. Figure 1 of this discussion shows more correctly what the problem is in fitting in carrier current relaying with existing relays. This shows a section of an actual board that is 2 panels wide. In addition to the relays previously installed, which become back-up relays, carrier current relaying has been provided for 2 132-kv circuits, using separate components for the carrier scheme. Often the carrier may be added without even changing existing panels.

Figure 2 of this discussion shows the back-of-board wiring for these same 2 panels. Even with all the carrier relays in one case, the writer doubts that the wiring would be any simpler than that shown here, and especially if an attempt is made to co-ordinate the carrier and the back-up protective relays and to provide adequate testing facilities.

The authors deserve a great deal of credit for coming to the idea of fast carrier relaying. The idea is novel and the more encouragement it can get the faster will it be applied. When the application of it becomes more extensive, as it surely will with time and with a reduction in cost, the major contribution that fast carrier relaying has made to the transmission art will be fully realized.

Performance of Distance Relays

Discussion and author's closure of a paper by Giuseppe Calabrese published in the June 1936 issue, pages 660–72, and presented for oral discussion at the protective devices session of the summer convention, Pasadena, Calif., June 26, 1936.

W. A. Lewis (Westinghouse Electric & Manufacturing Company, East Pittsburgh Pa.): The first analysis of the performance of distance relays by the method of symmetrical components, so far as the writer is aware, is contained in a paper by the writer and L. S. C. Tippett entitled "Fundamental Basis for Distance Relaying," presented in 1931. This paper was never published in the TRANSACTIONS, so that the mathematical work is not generally available, but an abstract appeared in ELECTRICAL ENGINEERING (volume 50, June 1931, p. 420–22). In this paper a study of both phase and ground faults was made on the same circuit to which the relays are connected. The results for phase faults are in agreement with those obtained by the author for faults between the relay and the transformer bank, and for ground faults they form the basis for the ground distance relay described by R. M. Smith and S. L. Goldsborough ("A New Distance Ground Relay," volume 55, June 1936, p. 697–703).

There is one connection not analyzed by Calabrese, which appears to be quite important for the present discussion. In this connection both the potential and current coils of the distance elements are connected in star. An analysis by a method similar to that described would show that for a 3-phase, a line-to-line, or a line-to-line-to-ground fault beyond the transformer bank, one of the relays would give a constant impedance indication equal to Z_f and the other relays would give a greater impedance indication. Thus a distance relay connected with line currents and line-to-ground voltages on one side of a star-delta bank would give the proper indication for relaying phase faults on the other side.

The author has pointed out that the converse relation is approximately true, namely, that relays connected with delta-connected current and voltage coils will give some protection for ground faults beyond the transformer bank. Because of the effects of zero-sequence current in case the fault is on the star side, the latter relation is not exact when the transformer bank is star-connected on the faulted side.

Connecting distance relays with star-connected potential coils is not customary, since this connection normally would be applied for ground fault protection, and it has been shown that when the zero-sequence impedance is different from the positive-sequence impedance, distance relays will not give a constant indication for ground faults in a given location, the indication being

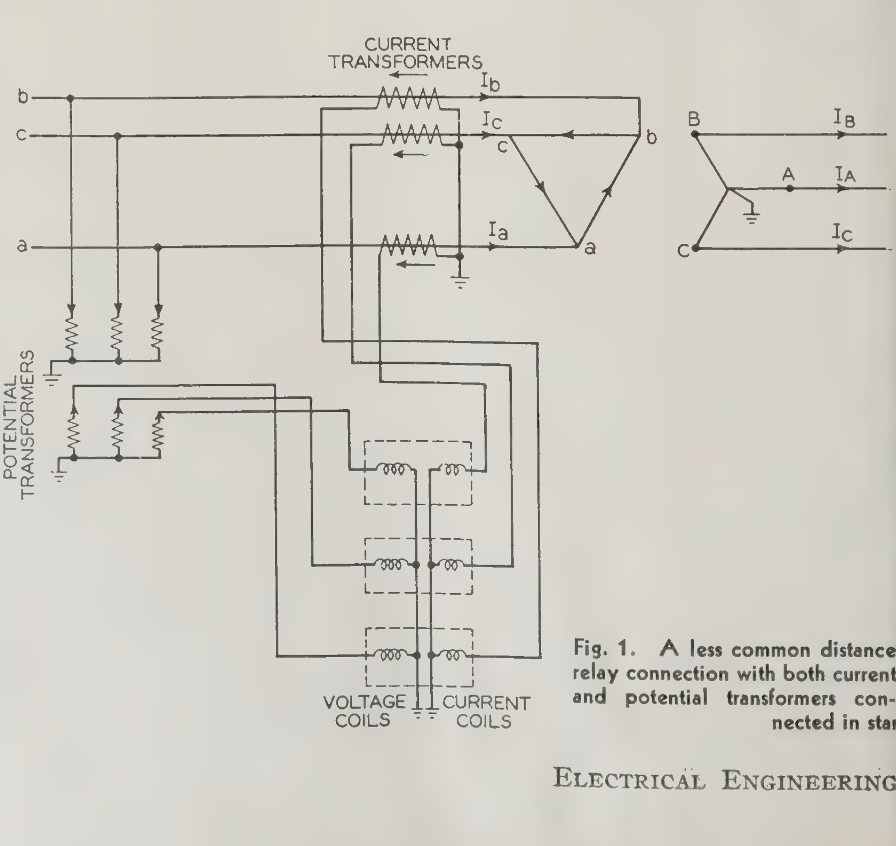


Fig. 1. A less common distance relay connection with both current and potential transformers connected in star

affected by system conditions. The reactance relay described by Smith and Goldsborough overcomes this objection by introducing current compensation to correct for the difference between positive and zero-sequence impedance, and therefore offers a means of ground fault protection between the relay and the transformer bank. The compensation consists entirely of introducing into the relay current coil a fraction of the residual or zero-sequence current in addition to the line current. For phase faults beyond the transformer bank, there is no zero-sequence current at the relay location, and the compensation is ineffective. Under such conditions the compensated relay gives the same indication as an uncompensated relay having star-connected current and potential circuits, and thus will operate properly for phase faults beyond the transformer bank.

Whenever protection for phase faults beyond a star-delta transformer bank is desired, the reactance ground relays described by Smith and Goldsborough may be used, protecting for ground faults between the relay and the transformer bank and for phase faults beyond. In the tests just mentioned this arrangement was tried for faults beyond the transformer bank, including arcing faults of moderately high resistance, and the correct operation was obtained in every case.

Giuseppe Calabrese: In making investigations involving long mathematical developments, mistakes may be made, no matter how careful one tries to be; therefore, a statement that the results have been checked is always welcome.

The connection with potential and current transformers connected in star was not analyzed because the other 2 connections, namely, with potential transformers in delta and current transformers in either delta or star are those generally used; however, the analysis and formulas of the paper may be extended easily to include the case for potential and current transformers connected in star. The 3 relays would be connected as shown in figure 1 of this discussion. The primary impedances indicated by them would be

$$
\begin{aligned}
Z_a &= \frac{E_a}{I_a} = \frac{E_{a0} + E_{a1} + E_{a2}}{I_{a0} + I_{a1} + I_{a2}} \\
Z_b &= \frac{E_b}{I_b} = \frac{E_{a0} + \alpha^2 E_{a1} + \alpha E_{a2}}{I_{a0} + \alpha^2 E_{a1} + \alpha I_{a2}} \qquad (1) \\
Z_c &= \frac{E_c}{I_c} = \frac{E_{a0} + \alpha E_{a1} + \alpha^2 E_{a2}}{I_{a0} + \alpha I_{a1} + \alpha^2 I_{a2}}
\end{aligned}
$$

where E_{a0}, E_{a1}, E_{a2}, I_{a0}, I_{a1}, I_{a2} are the sequence components of the primary voltages and currents at the relay location.

For faults past the power bank, equations 7 and 8 of appendix I of the paper may be substituted in equations 1 of this discussion.

$$
\begin{aligned}
Z_a &= \frac{jE_{A1} - jE_{A2}}{jI_{A1} - jI_{A2}} = \frac{E_{A1} - E_{A2}}{I_{A1} - I_{A2}} \\
Z_b &= \frac{j\alpha^2 E_{A1} - j\alpha E_{A2}}{j\alpha^2 I_{A1} - j\alpha I_{A2}} = \frac{E_{A1} - \alpha^2 E_{A2}}{I_{A1} - \alpha^2 I_{A2}} \\
Z_c &= \frac{j\alpha E_{A1} - j\alpha^2 E_{A2}}{j\alpha I_{A1} - j\alpha^2 I_{A2}} = \frac{E_{A1} - \alpha E_{A2}}{I_{A1} - \alpha I_{A2}}
\end{aligned}
$$

Using the same convention adopted in the paper of indicating with the symbols E_{a0}, E_{a1}, E_{a2}, I_{a0}, I_{a1}, I_{a2} the sequence components of the voltages and currents at the relay location as viewed from the fault, the above formulas may be rewritten

$$
\begin{aligned}
Z_a &= \frac{E_{a1} - E_{a2}}{I_{a1} - I_{a2}} \\
Z_b &= \frac{E_{a1} - \alpha^2 E_{a2}}{I_{a1} - \alpha^2 I_{a2}} \qquad (2) \\
Z_c &= \frac{E_{a1} - \alpha E_{a2}}{I_{a1} - \alpha I_{a2}}
\end{aligned}
$$

With this convention, equations 1 of this discussion apply with faults between the relays and the power bank, equations 2 apply with faults past the bank.

With a fault not involving ground between the relays and the power bank, $E_{a0} = I_{a0} = 0$ and equations 1 of this discussion become

$$
\begin{aligned}
Z_a &= \frac{E_{a1} + E_{a2}}{I_{a1} + I_{a2}} \\
Z_b &= \frac{E_{a1} + \alpha^2 E_{a2}}{I_{a1} + \alpha^2 I_{a2}} \qquad (3) \\
Z_c &= \frac{E_{a1} + \alpha E_{a1}}{I_{a1} + \alpha I_{a2}}
\end{aligned}
$$

Reference to table II of the paper shows that equations 2 and 3 of this discussion are identical respectively with the formulas of the third and fourth column. More exactly, $Z_a = Z_{bc}$, $Z_b = Z_{ca}$, $Z_c = Z_{ab}$. The formulas for the relay primary impedance indications with the potential and current transformers connected in star, as shown in figure 1 of this discussion, and faults past the power bank are the same as those obtained with the potential and current transformers connected in delta, as in figure 2 of the paper, and faults between the relays and the bank. Conversely, with the potential and current transformers connected in star, the primary impedance indicated by the relays with faults not involving ground between the relays location and the power bank are the same as those obtained with the transformers connected in delta and faults past the bank. Thus reference to the fourth column of table III of the paper shows that with the transformers connected in star and faults past the power bank, there is at least one relay indicating a primary impedance equal to the impedance Z_f to positive-sequence currents of the circuit interposed between the relays and the fault. In applying the formulas of the fourth column of table III to this case, $(Z_0 + Z_{f0})$ should be replaced with Z_{f0}.

The foregoing conclusion, in accordance with W. A. Lewis' statement, applies also to the compensated relay proposed by Smith and Goldsborough when properly modified to cause it to operate for faults past the power bank.

The conclusion of the correspondence between the relay primary indication with the potential and current transformers connected in star and in delta may be extended to include faults involving ground between the relays and the power bank by preventing zero-sequence currents from flowing in the potential and current coils of the relays supplied by the star-connected potential and current transformers.

Where this is done, formulas 3 of this discussion apply in all cases for faults between the relays and the power bank, regardless of whether they involve ground or not. The formulas of the fifth column of table III of the paper apply also provided that Z_{f0} is replaced with $(Z_0 + Z_{f0})$.

The zero-sequence current I_{a0} may be prevented from flowing in the current coils of the relays by filtering it in the manner suggested by Lewis and Tippett in the paper referred to by Lewis; namely, by leaving ungrounded the common connection of the current coils of the impedance elements of the relays and by connecting a zero-sequence network across the 3 secondaries of the current transformers.

Zero-sequence currents may be prevented from flowing in the potential coils simply by leaving ungrounded the common connection of 3 potential coils of the impedance elements.

The last arrangement gives no definite protection for line-to-ground faults between the relays and the power bank, which must be cared for in some other manner. The arrangement may be used if a change in the balance points for faults between the relays and the bank is not objectionable and definite balance points for faults past the transformer bank are desired.

A New Distance Ground Relay

Discussion and authors' closure of a paper by S. L. Goldsborough and R. M. Smith published in the June 1936 issue, pages 697–703, and presented for oral discussion at the protective devices session of the summer convention, Pasadena, Calif., June 26, 1936.

W. A. Lewis: See discussion, page 1254.

A. R. van C. Warrington (General Electric Company, Philadelphia, Pa.): Full agreement is accorded the authors that prompt clearing of a single-phase ground fault before it involves another phase is a desirable improvement in transmission line protection. High-speed reactance relays are well suited to this application when connected as shown in figure 7 of the paper.

In the fourth paragraph, under "general considerations," the authors condemn potential switching schemes because "they require selector and transfer relays to change voltage connections." The writer agrees that auxiliary devices should be reduced to a minimum, and should like to point out that protection against both interphase and ground faults is obtained by the method shown in figure 9 of a previous paper ("Control of Distance Relay Potential Connections," A. R. van C. Warrington. ELECTRICAL ENGINEERING, v. 53, January 1936, p. 206–16) with only one circuit-closing contact per phase, whereas in the scheme now described in the author's paper, there are 3 circuit-closing and 2 circuit-opening contacts per phase, which still give no protection for interphase faults.

Employing a potential switching scheme based on figures 5 and 10 of the 1934 paper,

54 reactance relays have correctly cleared the 34 faults of all kinds that have occurred since the installation was made early in 1935.

In the last paragraph under "general considerations" the authors have alleged a lack of compensation for mutual induction in the schemes described in the 1934 paper. Their attention is invited to figures 8, 9, and 10 of that paper, which show 3 different methods of residual compensation, one of which (figure 9) corresponds closely to their figure 7, and to the last 3 pages of the 1934 paper, which goes into the subject thoroughly, including the authors' "unpredictable effects of double ground faults."

Roger Dubusc (nonmember; Compagnie pour la Fabrication des Compteurs et Matériel d'Usines à Gaz, Montrouge, France): The method of correct distance relaying for phase-to-ground faults described in this paper has been employed satisfactorily in France since 1931, but without the compensation of mutual inductance of a parallel line. (French patent number 686,430, December 12, 1929 and its addition number 38,612, May 31, 1930.) In a paper entitled "Un nouveau relais de distance pour la protection des lignes aériennes à haute tension" published in the *Revue Générale de l'Electricité* (volume 31, February 20 and 27, 1932, p. 251–9 and 282–92), the writer and his assistant described the way to realize the proper connections for ground faults. Let

Z_d be the positive phase sequence impedance between the relay and the fault

Z_i the negative phase sequence impedance between the relay and the fault

Z_0 the zero phase sequence impedance between the relay and the fault

I_1 the current in phase 1

I_2 the current in phase 2

I_3 the current in phase 3

I_d the positive phase sequence current

I_i the negative phase sequence current

I_0 the zero phase sequence current $= \dfrac{I_T}{3}$

I_T the current in the ground

U_1 the star voltage of the phase 1 (supposed to be the faulty phase)

Then

$$I_1 + I_2 + I_3 + I_T = 0$$
$$U_1 = Z_dI_d + Z_iI_i + Z_0I_0$$
$$I_1 = I_d + I_i + I_0$$

In the lines

$$Z_d = Z_i$$

hence

$$U_1 = Z_d(I_d + I_i) + Z_0\frac{I_T}{3}$$

$$I_d + I_i = I_1 - \frac{I_T}{3}$$

$$U_1 = Z_dI_1 - \frac{Z_dI_T}{3} + \frac{Z_0I_T}{3} = Z_dI_1 + \frac{Z_0 - Z_d}{3} I_T$$

$$Z_d = \frac{U_1}{I_1 - \dfrac{Z_0 - Z_d}{3Z_d}I_T}$$

The coefficient $\dfrac{Z_0 - Z_d}{3Z_d} = k$ may be called the coefficient of ground impedance.

For the current circuits of French relays 2 windings are used, one energized from the current I_1 and the other energized from the current I_T, with taps for adjusting the coefficient k from 0.75 to 1.5.

The writer is pleased to know that this method has been extended to the compensation of mutual inductance in parallel lines.

S. L. Goldsborough and R. M. Smith: The authors should like to point out that figure 7 of A. R. van C. Warrington's paper, "Control of Distance Relay Potential Connections" shows a method of using 3 distance relays for both phase-to-phase and phase-to-ground faults. The diagram indicates clearly that 2 relays may operate on a phase-to-phase-to-ground fault, but Lewis and Tippett show in their paper, "Fundamental Basis for Distance Relaying" (reference 2 of the paper) that one or the other of the relays may over-trip for this type of fault, depending on the ratio of the line-to-line to the line-to-ground resistance at the point of fault. As pointed out in the paper, actual field tests have checked the mathematical analysis and have shown that to prevent incorrect relay operation neither relay should be allowed to trip on this type of fault. Figure 9 of the paper shows such a connection and it should not be compared with figure 7 of Warrington's paper.

Apparently Warrington has misunderstood the use of the term "mutual induction." It refers to the zero sequence coupling of 2 adjacent transmission lines causing an induced voltage in one by zero sequence current in the other or in the common earth return. The figures mentioned by him do not contain any means for compensating for this effect.

Special Tests on Impulse Circuit Breakers

Discussion and author's closure of a paper by W. F. Skeats published in the June 1936 issue, pages 710–17, and presented for oral discussion at the protective devices session of the summer convention, Pasadena, Calif. June 26, 1936.

Joseph Slepian (Westinghouse Electric & Manufacturing Company, East Pittsburgh, Pa.): The object sought in the proposed method of testing oil circuit breakers is highly desirable, for it would seem to permit the testing of circuit breakers with only a fraction of the high power laboratory equipment now used by manufacturers; however, the principle of the method depends upon a theory of happenings at zero current, which is under considerable question. The method probably should be thoroughly checked by experiment to see whether it gives the same limits as conventional methods within the capacity of available high-power laboratories before credence is given to the application of the method to power magnitudes beyond the range of the laboratory. No such thorough check is given in this paper, so that some incredulity on the part of the writer may be pardoned.

A similar method has been proposed and applied to the testing of rectifier tubes under the name of "cheater" circuits ("The Phantom Tester," D. D. Knowles and C. E. Heller. *Electronics*, volume 6, September 1933 p. 248–9). The writer's experience, however, is that "cheater" circuits do not predict correctly the behavior of tubes in practical full power circuits.

The writer doubts strongly that the proposed method is fundamentally sound, but will limit his discussion to examining the oscillograms shown in the paper, which he believes are all of the type the author himself rejects as inconclusive.

The various possible happenings at zero current are analyzed, in applying his method, in paragraphs 1 to 5 on page 714 (Electrical Engineering for June 1936). Only when the conditions of paragraph 5 are realized is the test conclusive. The cathode-ray oscillogram of figure 11 of the paper is judged to be inconclusive and to come under paragraph 3, and the writer agrees with this. The spark gap is broken down ahead of the last current zero, and current flows from the high-voltage transformer. That current adds to the current in the breaker under test and subtracts from the current in the auxiliary breaker. The current in the auxiliary breaker therefore reaches zero first and the arc there goes out at −500 microseconds in figure 11. Remaining is the current from the high-voltage transformer in the test breaker which decreases from about 30 amperes to zero in 500 microseconds—but now the oscillograms of figures 7 and 8 of the paper are exactly of the same type!

In figure 7 the current in the auxiliary breaker goes out at $t = 0$, leaving a discernible current in the test breaker that decreases from about 6 amperes to zero in 100 microseconds, which is the same rate of decrease of current as in figure 11 of the paper. The actual current zero occurs at $t = 100$ microseconds, in agreement with the voltage zero; furthermore, a close examination shows a distinct reversal of current of a few amperes, in contradiction to the statement near the end of page 713 of the paper.

Figure 8 is entirely similar: The current in the auxiliary breaker here reached zero about 90 microseconds before the current in the test breaker, leaving a current of about five amperes at $t = 0$. Again a reversal of current at the current zero $t = 90$ is discernible.

In figure 9 the current in the auxiliary breaker reached zero only 40 microseconds before the current zero in the test breaker. The current from the high-voltage transformer then was so small that the capacity of the circuit determined the character of the voltage prior to zero current.

The test of figures 7, 8, and 9 are all in the same class as that of figure 11, and therefore are not tests that can be accepted as conclusive according to the criteria given in the paper.

The tests of figures 7, 8, and 9 are favorable indications of the performance of the

tested breaker, but they do not agree with the theory proposed for the breaker. Putting in the correct time of zero current, the rates of voltage rise appear to be 5,750, and 12,600 volts per microsecond in figures 7 and 8 instead of the 1,750 and 2,600 volts per microsecond as given in the paper. In figure 9 the arc space within not over 10 microseconds after $t = 0$ was bearing 150 kv, or a rise of 110 kv in 10^{-5} seconds or 11,000 volts per microsecond. Those rates of voltage rise are beyond the breaker limits as predicted by the advocated theory, and the arc should have reignited. The results are more compatible with a theory that ascribes a dielectric strength to the arc space even before zero current, and a leakage or conductance that decreases continuously through current zero.

The breakdowns such as described on page 713 and in figure 13 are observed also in other types of oil breakers and cannot be used as an argument that this breaker operates in a fundamentally different manner from other breakers as is stated in the paper. The argument that this breaker is different because of the absence of decay of currents of a few amperes is disproved by the actual appearance of decaying currents of that magnitude in figures 7, 8, and 9.

H. P. St. Clair (American Gas and Electric Company, New York, N. Y.): The importance of the work described in this paper may not be generally appreciated. If full substantiation can be given to this method by which full restored voltage is applied to the breaker with sufficient rapidity after interruption of full rated current at reduced voltage to simulate the actual interruption of full rated current at full voltage, it means that on breakers of this type, at least, laboratory tests at many times the rating of the laboratory can be carried out.

More work should be done in developing the method and, if at all possible, check tests should be made at full capacity in the field. Although such special tests were possible on the impulse breaker only because of its special characteristics, the apparently successful simulation of full capacity tests represents a real pioneering advance in testing technique.

Having made some study of the problems of testing oil circuit breakers, the writer is aware of the present high investment in testing facilities. With the deviation between testing plant capacities and breaker ratings continuing to increase, the industry is faced with either the heavy expense of greatly enlarged testing plants or with an increasing reliance upon so-called synthetic tests. Too great importance, therefore, cannot be given to the author's contribution, which offers great promise of solving this dilemma. A great deal of credit is due for what has been accomplished, and every encouragement should be given to its continuation.

W. F. Skeats: J. Slepian seems to overemphasize the dependence of the principle of this testing scheme upon a theory of the happenings at current zero which, as he states, is questionable. It would seem that when the breaker has demonstrated its ability to withstand a given voltage established across its contacts with a given speed after the passage of a definite current, it has demonstrated its ability to clear a circuit in which those conditions are duplicated. Such items as the establishment of dielectric strength before current zero, for example, do not seem to be involved, and any question that may exist appears irrelevant.

The value of an experimental check such as Slepian mentions is not denied. The author regrets that with the short time available for these tests such a check was not worked out.

Despite Slepian's statement, there are marked differences between figure 11 of the paper and figures 7, 8, and 9. In figure 11, the breakdown of the gap can be placed with fair accuracy from the magnetic oscillogram and occurs more than 0.001 second before the zero of current, whereas in the other cases the breakdown occurs, as shown by the sharp discontinuity in the cathode-ray oscillogram of voltage, within about 100 microseconds after current zero; furthermore the lack of conclusiveness of figure 11 may be considered a matter of degree. This oscillogram does not prove conclusively that the breaker is capable of interrupting the effective current indicated by the magnetic oscillogram followed at 200 microseconds after current zero by a voltage of 250 kv, because of the abnormal slowness with which the last 30 or 40 amperes decayed; but it does prove conclusively and conservatively, that the breaker will interrupt such a current followed at 700 microseconds after current zero (which arises from taking the time of current zero in the auxiliary breaker) by a voltage of 250 kv. This, however, corresponds to a recovery rate of only about 350 volts per microsecond, which is very much lower than the rates established in other tests and hence was discarded without further comment. Assuming that the currents Slepian mentions exist in the other figures, the claims with reference to those figures are made on the latter basis, so that those claims are still felt to be established not only conclusively but also conservatively.

The author has been quite unable to confirm the "distinct reversal" in current that Slepian claims to observe.

Contrary to his statement, the rates of voltage rise he cites are within the breaker limits as predicted by the theory. D. C. Prince's paper on this breaker shows a limiting recovery rate of 3,500 volts per microsecond for a double cross-blast port operating with an oil pressure of 100 pounds per square inch. Dividing this value by 0.30, the maximum fraction of the total voltage appearing across any one break, a figure of 11,700 volts per microsecond results. If one may assume an error of 2 microseconds in Slepian's estimate of the time of current zero in figure 8, this value exceeds all the figures that he presents.

The author's attitude on the value of a scheme such as that described in the paper is much the same as that which H. P. St. Clair expresses and may be stated as follows: The method of testing high power circuit breakers now most widely used in the factory consists of making tests at full voltage and comparatively light current on the one hand and at full current or more and a reduced voltage on the other. For certain types of breakers this has a good deal of theoretical justification, and when supplemented by occasional field tests is acceptable as an alternative to the large investment required in order to make full power factory tests. When it becomes necessary to extend this method to 4 or 5 times the capacity of the testing station, however, it is felt that any system that increases this capacity at reasonable cost is a step forward, in spite of some shortcomings from the standpoint of the ideal. It is in this spirit that the paper was prepared, with the shortcomings as the author sees them listed and discussed therein.

Some Applications of Instrument Transformers

Discussion and author's closure of a paper by Otto A. Knopp published in the May 1936 issue, pages 480–9, and presented for oral discussion at the measurements and selected subjects session of the summer convention, Pasadena, Calif., June 25, 1936.

F. B. Silsbee (National Bureau of Standards, Washington, D. C.): This paper gives an excellent general survey of the long and valuable development which has been carried out in the field of precise a-c measurements. It constitutes an interesting example of the pendulum-like manner in which a given line of human thought often progresses. In the early days of electrical measurements it was expected that each college or central station laboratory would form a self-contained unit and set up its own electrical standards independently of any other laboratory. The silver voltameter and the mercury ohm were devised primarily to permit such independent operation while still giving national and international consistency. Because of the great labor involved in these fundamental measurements and the risk of introducing errors that might accumulate so as to produce significant discrepancies, this practice and point of view gradually have been abandoned in favor of the present system in which the units are maintained by large national laboratories, and their values are disseminated by the periodic testing of apparatus and instruments shipped to the central laboratories for test.

It happens that the values of ratio and phase angle embodied in instrument transformers are physically dimensionless quantities, the "units" for which are consequently universally available. However, the habit of obtaining standards from the central laboratories has persisted even in this field, and most central stations base their ratio measurements on standard transformers that have been tested at some central laboratory. A major reason for this practice, in addition to its parallelism with the procedure used for other quantities, lies in the fact that the comparison of 2 transformers of nominally equal ratio is in general much easier than the direct determination for the ratio of a single transformer. It has remained for the inventive genius of the author, stimulated perhaps by the great physical distance between San Francisco and Washington, to show that at least so far as instrument transformer ratios are concerned it is entirely practicable for an individual laboratory still to be self-contained and to obtain results that are con-

sistent not only throughout its own system but universally.

Regrettably space limitations have prevented a more detailed description of the individual pieces of apparatus. In particular it would be of interest to learn more about the detector-wattmeter, the current and voltage ratings and the resistances of the various ranges, and a more quantitative statement as to the errors to be expected from mutual inductance if a torsion head is not used.

Stanley Green (Duncan Electric Manufacturing Company, Lafayette, Ind.): The content of the paper should be stimulating in taking full advantage of the possibilities of current and potential transformers in the laboratory in effecting simplification and saving both in apparatus and labor. The idea, as expressed in figure 3 of the paper and in the author's discussion of it, that multirange transformers are in reality "turn potentiometers" is a novel one, although it has been known, perhaps unconsciously, by engineers for a long time.

The stability of turn ratio on a properly designed magnetic core probably is greater than the stability of nearly any other quantity in any form of laboratory apparatus. The effect of magnetic turns is influenced by changes in permeability of the magnetic material in the core, of course, but only secondarily, for magnetizing current is the only factor changed thereby and this current does not enter directly into the quantitative measurements. Moreover, experiments conducted during a long period of time by many independent observers indicate that the permeability of modern core materials with respect to aging is exceedingly stable; in fact, the permeability of iron at the flux densities of usual operation increases slightly instead of decreasing over long periods of time.

In addition, the actual position of the turns on the core of a transformer affects the resultant value of its inductive effect not as a primary factor, but indirectly. With a properly constructed transformer the physical or dimensional relations of the turns on the core of a given transformer should remain constant.

With these fundamental advantages, perhaps it is questionable whether engineers have given enough attention to applications of current and potential transformers for laboratory requirements and standardization purposes.

The company with which the writer is associated has been making some efforts to utilize a few standards and convert to all necessary current and voltage ranges by means of instrument transformers; this has been done with special benefit on stroboscopic equipment for calibrating watt-hour meters. By this means a considerable saving of laboratory man-hours has been attained, together with superior accuracy and less uncertainty of results.

There is enough material in the paper to warrant considerable expansion.

C. T. Weller (General Electric Company, Schenectady, N. Y.): The author is to be commended for his ingenious schemes and devices, among which the d-c transformer should be mentioned. The writer remembers testing some early transformers of that type. As he states, the a-c transformer is of greater practical importance; therefore, he has concentrated his later efforts upon it.

Electrical standardization is based upon standard cells and standard resistors. Of these, only the resistors can be designed in forms suitable for direct use in standardizing instrument transformers; consequently, methods incorporating resistors for this purpose were chosen. The "shunt method" is used for current transformers, and the "potentiometer method" is used for potential transformers. These methods have been generally adopted by standardizing laboratories, the usual ranges being up to about 2,500 amperes and 30,000 volts, respectively. For higher currents special transformers of the general type described by the author are used sometimes. For higher voltages various substitutes for resistors are used because of the rapidly increasing cost; however, resistors were considered to be sufficiently superior to justify the cost, and the resulting equipment has been described in a paper entitled "132-Kv Shielded Potentiometer for Determining the Accuracy of Potential Transformers" (AIEE Transactions, volume 48, July 1929, pages 790–801). While some progress has since been made with substitutes for this voltage range the writer is not convinced that any of them are on a par with resistors. The use of transformers with multiple windings is not considered to be a direct method of standardization.

In connection with our development work the writer has used several special transformers for interchecking. The most valuable current transformer has 6,000 ampere-turns; the various windings are equally distributed around a core of the ring type. Practically all standard current ratings can be handled without changing the ampere-turns, and compensating expedients are thus unnecessary. The percentage of ampere-turns required for excitation of a given core (and the corresponding error) decreases as the total ampere-turns increase.

The one-to-one-ratio method for verifying the accuracy of transformers does not assure that the other primary windings have the same effective distribution as the test primary winding. The author has recognized this difficulty, but apparently has not stated whether the accuracy of the other ratios has been checked by standard methods, and if so, what results were obtained. The extension of the method becomes particularly difficult as the ratio of the potential transformer increases. The charging current constitutes an appreciable part of the exciting current at the higher voltages. It would be interesting to learn just how the author proposes to overcome the effect of charging current.

I. M. Stein (Leeds & Northrup Company, Philadelphia, Pa.): This paper is in the nature of a review, treating the various phases of the subject in general terms only; accordingly, this discussion also will be in somewhat general terms. Although the paper is a review, it is limited almost exclusively to the work of the author and omits some other important references, particularly the National Bureau of Standards Research Paper No. 580 by F. B. Silsbee and his associates.

Although multirange instrument transformers, torsion-head wattmeters, and methods for using a wattmeter without introducing error by mutual inductance between the windings were all known before the author undertook his development work, it is none the less true that the author's refinements in the design of multirange instrument transformers are creditable contributions.

Probably the outstanding application for the multirange current transformer is for very large currents, that is, currents of 2,500 amperes or more. Although the author does not refer to currents above 3,000 amperes, the National Bureau of Standards has designed and is using similar apparatus for currents up to 12,500 amperes. When using the orthodox resistance-inductance method for such large currents, the design of the heavy current shunts becomes expensive and difficult, primarily because of the residual inductance of the shunts. The Bureau of Standards does not depend upon the autocalibration or "one-to-one" calibration of the turn-ratio apparatus as recommended by the author, but determines the ratio and phase-angle characteristics of the turn-ratio transformer on one or more of the lower ranges using standard resistance and inductance apparatus.

Whether the turn ratio method is advantageous in checking high-range potential transformers is questionable, for the author's work has not been extended to very high voltages. The high-voltage noninductive resistors used in the orthodox methods have several other applications in a high-voltage a-c laboratory. The measurement of dielectric loss is an important example.

For routine calibration of both current and potential transformers in meter laboratories the use of several standard multirange transformers that have been standardized by the Bureau of Standards or some other authorized laboratory would appear to offer advantages over the single expensive transformer arrangement proposed by the author. Such measurements are made primarily in connection with the purchase and sale of electrical energy and, hence, are subject to review by the public or representatives of the public, such as public service commissions. Under these circumstances, it is desirable to have the standards in as simple a form as possible and provided with certificates showing the results obtained by direct comparison with the primary legal standards; moreover, standard high-grade multirange instrument transformers are available today at moderate prices so that duplicates may be provided without involving a prohibitive cost, and checks against duplicate standards are always reassuring.

The author stated that the average meter accuracy on the system of the company with which he is associated has improved noticeably after the use of his testing methods. The writer should like to ask whether this improvement is clearly the result of the use of these methods. For at least 15 years methods and apparatus have been available for measuring the ratio and phase angle of instrument transformers to within tolerances much closer than necessary, in view of the limitations in the characteristics of the watt-hour meters with which the transformers were associated in service, and in view of the limitations in the characteristics

of the instrument transformers themselves. However, during the past decade the manufacturers of instrument transformers and watt-hour meters have made marked improvements in products that were already splendid, with the result that the errors in both instrument transformers and watt-hour meters have not only been reduced to very small values, but also have been made practically constant over a wide working range. Such improvements in instrument transformers and watt-hour meters are essential to improved meter accuracy under varying load conditions, and the writer should like to ask whether the improvement in meter accuracy noted by the author may not be due in a large part to a greater use of the more refined types of instrument transformers and meters.

With the terminology of the paper the writer must disagree strongly, particularly with regard to the name "turn potentiometer" for devices of the type described. Although it is true that on paper the electrical network employed in these transformer devices is analogous to that in a simple potentiometer, the analogy may not be carried farther. In the potentiometer the secondary instrument is a null or zero-deflection device, whereas in the transformer devices the secondary instrument always deflects, and the noll or balance part of the apparatus is in the magnetic circuit and is not shown in diagrams *b* and *c* of figure 3 of the paper. Even though the analogy were more complete, it is questionable whether it would be desirable to attach the name "potentiometer" to these devices, because if that name were applied to a great variety of devices differing widely in principle, the name would cease to be a distinctive for any circuit or principle. The writer must disagree with the statement that a turn is a more stable quantity than a resistance. With regard to the author's use of the term "absolute standard," the devices described are not absolute standards in the true sense, that is, in the sense that their values may be determined from their dimensions. In the opening abstract of the paper the author refers to "primary standards," whereas obviously he is referring to secondary-standard or working-standard deflection instruments. The writer takes exception also to the use of the terms "amperage" and "wattage" in the opening abstract instead of the terms current and power.

E. C. Wentz (Westinghouse Electric & Manufacturing Company, Sharon, Pa.): From the point of view of the instrument transformer engineer, the paper is particularly interesting in its demonstration of the principle that instrument transformers are able to accomplish more than their nominal function of transformation. Results are obtained by ingenious methods that would not be possible without instrument transformers.

The construction of the multitap standard transformer also is interesting. There is no doubt that the method employed by the author, that of winding the coils for equivalent resistances and reactances, is the best and most certain method of obtaining his object—equal performance on all ratios. However, it seems questionable whether the auxiliary autotransformer used in conjunction with the main current transformer to give additional ratios, could not practically be replaced with a tap or extension in the secondary winding. It is also difficult to see how the various sections of the primary winding of the voltage transformer can all be wound equally coupled to the secondary. The practical solution of this problem would be to make the reactance sufficiently low that slight differences would be unimportant.

C. O. Werres (General Electric Company, Schenectady, N. Y.): The paper brings together a record of the author's contributions to the art of measurements covering a period of about 25 years, with particular reference to meter and instrument transformer testing and calibrating methods.

The same space of 25 years has witnessed the development of instrument transformer standardizing equipment and technique commensurate with its importance as a vital part in the billing of the nation's electric power consumption, which last year was some 2 billion dollars' worth.

In 1909 L. T. Robinson presented before the Institute at Frontenac, N. Y., a paper entitled, "Electrical Measurements on Circuits Requiring Current and Potential Transformers" (AIEE TRANSACTIONS, volume 28, part 2, 1909, pages 1005–39) and the record of the discussion is refreshing to read, particularly the contributions of C. H. Sharp and M. G. Lloyd. That paper described primary equipment for testing current transformers of ratings up to 2,000 amperes and potential transformers up to 33,000 volts. The intervening years have witnessed an increase in ratings, until today the ratings of the same types of equipment have been extended to 4,000 amperes and 150,000 volts, thus covering the range needed for direct primary measurement. The increase in voltage range to 150,000 volts was described by C. T. Weller in a paper presented in 1929 "132-Kv Shielded Potentiometer" (AIEE TRANSACTIONS, volume 48, July 1929, pages 790–801). The accuracy obtainable by the use of such equipment permits certifying results correct to within 0.1 per cent on ratio and 3 minutes on phase angle. This accuracy is common to all laboratories having similar equipment.

Naturally such equipment is large and expensive and the technique involved is of the highest order. This is a necessary part of the manufacturer's equipment and must be of the best. For the user there is a need for secondary intercomparison methods and such methods have been developed. Prominent among these is the familiar Silsbee testing set, a contribution of F. B. Silsbee of the National Bureau of Standards. Such testing sets, using primary transformers certified by primary standardizing equipment, provide a means for comparison that covers a wide field of usefulness. The technique involved is not complex; yet it involves a knowledge of the problem, particularly with respect to sources of error. The accuracy of comparison is stated to be 0.1 per cent on ratio and 5 minutes on phase angle. To these errors must be added the departure of the standard transformer from the true value, but if this is within 0.1 per cent on ratio and 3 minutes on phase angle as certified with the best primary outfits, the over-all results are very good.

The author's developments are useful for more moderate currents up to 3,000 amperes and voltages to 13,200 volts. Primary calibration is obtained from the counted-turn ratio of a transformer of ample design with taps, which is Knopp's way of covering this part of the field with lower cost. Extensions beyond this field need the more substantial primary calibrating equipments.

O. A. Knopp: Most of the discussers have expressed a desire for more detailed information on the subject matter. The author regrets that space limitations did not permit supplying more technical information or making comparisons with other methods and developments. Some of the information is supplied in answering the questions asked.

F. B. Silsbee's remarks are appreciated, coming as they do, from a man connected with the National Bureau of Standards, whose special scientific work includes the subject discussed in the paper. He might have felt that the author was encroaching on the legitimate field of action of the Bureau of Standards. The author did not have in mind that public utility laboratories should do the work of the Bureau of Standards; however, it is felt that if it is possible to do some of the work now being done at the Bureau of Standards by simplification of methods with equipment available in most utility laboratories, it should be performed by them in order to facilitate their work and enable them to make quick decisions.

To answer Silsbee's questions the following technical data are given on the most commonly used type of detector wattmeter, similar to the one described in the paper: The zero graduation is at the center of scale; the range is 15 watts on the torsion-head scale and 30 watts on the indicating scale, with current coils in series using the 150-volt potential range. The resistance and inductance characteristics are as follows:

Field coils in series, 0.57 ohm and 0.00075 henry
Field coils in parallel, 0.143 ohm and 0.00019 henry
Resistance of 75-volt range = 2,337 ohms
Resistance of 150-volt range = 4,674 ohms

If the same wattmeter is used with the zero at the left, as it is usually furnished by the manufacturer for ordinary laboratory use, and the current coil is short-circuited while 200 volts is momentarily applied to the 150-volt potential range, a deflection of 3.7 divisions occurs, or 2.47 per cent of full scale. This deflection is only slightly reduced if the current coil is short-circuited through a current transformer having a low ampere-turn value. It is therefore possible when using such a standard wattmeter for measuring small errors, to obtain inaccuracies of from 2 to 3 times as large as the errors to be determined. By placing the coils in the position of zero mutual inductance with the pointer at zero and keeping the coils in this relation by means of the torsion head, the errors just mentioned are eliminated entirely. At the same time, the wattmeter can be used as an ordinary indicating wattmeter.

The author is glad to know that Stanley Green feels that more attention should be given to applying multirange current and potential transformers to laboratory requirements and standardization purposes.

Unfortunately, many engineers are still clinging to the use of instruments of large and small capacity to do the testing and calibrating work. The average accuracy of those instruments usually is very low, and the maintainance cost is high. The economic waste produced by errors in measurements and time spent in tracing errors caused by the changing of instruments and ranges is tremendous.

With a multirange precision instrument transformer it is possible to give an ammeter, voltmeter, or wattmeter 30 or more ranges, if desired, and the transformer is replacing 29 or more instruments. The average accuracy of the equipment will be just as high as the accuracy of the one instrument selected. The errors of the transformer are not discernible on this instrument; moreover, this error can be verified at any time with the "one-to-one" method.

I. M. Stein's objection to the term "turn potentiometer" can be understood readily for he has been designing and building excellent resistance potentiometers and would not like to have the name "potentiometer" used for anything else. The author had no intention of using the name "turn potentiometer"; in fact, all through the paper the term "calibrating transformer" is used; it is used also in the captions of figures 9, 10, and 11. The term "turn potentiometer" was used merely in an effort to show the similarity in the principle of the devices.

The third paragraph of Stein's discussion is correct. The value of the multirange current transformer standard (figures 12 and 14 of the paper) exists primarily at the current values above 2,500 amperes. This will answer, at the same time, C. O. Werres, who felt that the author's standard is useful at the more moderate currents up to 3,000 amperes. The author wishes to emphasize that the present design of the standard transformer, figures 13 and 14, is particularly valuable for large currents and simplifies the method of accurately calibrating such transformers. The author knew, but did not mention in the paper, that the Bureau of Standards has started to use the ampere-turn method at the larger current values at which the orthodox method of using noninductive shunts became somewhat difficult and inaccurate. The author has been advocating, and still advocates after years of experience in calibrating current transformers, that the ampere-turn method could be used to good advantage for calibrating current transformers of any current value up to the largest using the one-to-one method as the principal means for standardization.

Silsbee pointed out that the values of ratio and phase angle physically are dimensionless quantities; therefore, higher accuracy should be possible, using suitable means, to determine the ratio of a transformer without the need of determining quantitatively the current or voltage on the primary and secondary to determine the ratio.

The author does not agree with Stein's statement that several standard high-grade transformers that are standardized by a properly equipped laboratory would offer advantages over standards that provide a a great many ranges. The author, being primarily a user of calibrating equipment and not a manufacturer, appreciates standards that can be checked in themselves instead of a multiplicity of individual standards that should check with each other but frequently do not.

Stein mentions the reassurance provided by duplicate standards. This is true, if they agree with each other, but is aggravating if they do not, whereas if the standard can be checked in itself with the one-to-one method, it is reassuring. These multirange standards are very much less expensive than a multiplicity of standards, and duplicates can be provided, if necessary.

Stein probably misunderstood the statements that the average meter accuracy on the system of the company with which the author is associated has improved noticeably after the use of his testing methods. The test methods referred to are the methods of testing the meters themselves, the portable standards by which these meters are tested in the field and not the instrument transformers used in metering. The instrument transformers are used in connection with the calibrating equipment shown in figures 18, 19, 20, and 22 in the paper. This equipment, because of the multirange precision transformers used in it, makes possible a foolproof usage, giving it remarkable stability of accuracy, which prevents the periodic wrong calibration of thousands of meters in service because of variations in the accuracy of the substandards used on a large system.

Accuracy records are kept for all types of meters and the initial higher accuracy of the more modern type of meter is not involved in the problem at all, nor is the improved accuracy of the newer type of instrument transformers.

Meter engineers, who have been struggling with the problem of maintaining hundreds of thousands of meters uniformly accurate over a large scattered system, will appreciate this problem. Frequently the average meter accuracy is lowered by the use of testing equipment that is temporarily in error.

So far as the author's terminology is concerned, he admits that more careful thought should have been given to it but feels sure no misunderstanding has been caused and that the engineering terms used are fully understood.

C. T. Weller asked how the one-to-one method can verify the accuracy of all the ratios of the multirange transformer. The one-to-one method can verify the accuracy of all ranges only if the proper proportions of the turns have been established at the time of manufacture. In the average current transformer this is usually simple, for the error of one turn can be detected by ordinary commercial instruments, and with a correct construction of the winding the ratio and phase-angle errors of all ranges must be the same. The correctness of this statement has been verified frequently by checking against other transformers that have been calibrated by the Bureau of Standards.

The advantage of being able to determine whether any change has occurred in the transformation ratio is very apparent. If the one-to-one check gives exactly the same results as before, no defect has developed in the transformer and the original ratio and phase angle are still correct. This check is extremely simple, can be made at a moment's notice at the time the multirange standard transformer is to be used for calibration, and is of great value to a laboratory.

The difficulties are greater with potential transformers, of course, but good success has been acquired so far for potential transformers for over 13,000 volts. Such transformers designed for zero ratio and phase-angle error at zero burden and calibrated one-to-one are found to have zero ratio and phase-angle errors even at the highest voltage ranges checked by the Bureau of Standards. There is no question that with the still higher voltages the charging current will become important, but there are means of overcoming this by shielding the various sections of the winding and employing methods for reducing the charging current to zero.

E. C. Wentz wonders whether it would be more practical to replace the auxiliary transformer used in the calibrating transformer (figure 9 of the paper) with taps or extensions in the secondary winding. This, in the author's judgment would be a difficult task, for the ratios are supposed to be within 0.01 per cent proportional to the current rating of the various ranges, which by necessity must be odd values. Since the secondary can have only a few hundred turns to give a practical ampere-turn value, the ratios that can be obtained are limited. The turn value must be divisible by prime factors such as 7, 11, 13, 17, and 19.

The little compensating transformers enable one to obtain almost any ratio without making the capacity of these transformers more than about 2 per cent of the capacity of the main transformer. The difficulties of winding the various sections of voltage transformers to obtain equal coupling with the secondary are not very great, and the winding easily carried out after the problem has been solved. It will be very difficult to make the reactance low enough so that slight differences would be unimportant, unless it is for commercial transformers, not precision standards.

Attempts were made sometime ago to use an ordinary winding to obtain reasonably close equality between the ratios of the various ranges of a potential transformer up to 11,000 volts for recording-wattmeter use in field testing. The results were so unsatisfactory that it was advisable to use the interlaced winding, similar to the one used in precision standards. The expense was only slightly higher and the results were highly satisfactory, notwithstanding that little care was used in the spacing of the windings and insulation. Only 0.01 or 0.02 per cent variation in ratio was found between the ranges and the variation in phase angle was less than one minute.

Measuring Equipment for Oil Power Factor

Discussion and author's closure of a paper by L. J. Berberich published in the March 1936 issue, pages 264–8, and presented for oral discussion at the measurements and selected subjects session of the summer convention, Pasadena, Calif., June 25, 1936.

H. W. Bousman (General Electric Company, Schenectady, N. Y.): The improvement in power factor of a light cable oil after 96 hours of aging at 115 degree

centigrade is interesting. The writer has noticed it several times, and it has been accompanied by deterioration in d-c resistivity.

The more volatile substances in the oil presumably would be the ones of smaller molecular weight, and any charged molecules of that sort contribute to the 60-cycle loss, but might be swept toward the plates by the d-c field sufficiently to modify the field greatly before a galvanometer reading could be taken ("The Conductivity of Insulating Oils—II," June 1931, J. B. Whitehead. AIEE TRANS., volume 50, June 1931, page 692–8). Thus the loss of such material accompanied by the formation of heavy, charged molecules or aggregates could cause a decrease in a-c loss accompanied by an increase in d-c loss. This experience is consistent with Berberich's hypothesis. Unfortunately, the hypothesis, if true, complicates the use of a power-factor aging test for controlling the quality of insulating oil. For example, if an easily ionized volatile material, driven off or combined with heavier molecules by aging is an acid of low molecular weight with a deteriorating effect on organic solid insulation, an oil that shows an improved power factor after aging might be believed to be less satisfactory than one showing an equal lowering of d-c resistivity but no improvement in power factor.

D. W. Roper (retired; Chicago, Ill.): The author states that monel metal is suitable for the cell and that the design of the cell permits easy cleaning, but the only test results submitted (figures 6 and 7 of the paper) give no information on these points; instead they show the sensitivity and accuracy of his Schering bridge and the high grade of the oils tested. If he wishes his statements about the suitability of monel metal and ease of cleaning to be accepted, he should submit some test results bearing directly on these points. One method of securing the information would be to test first one of the high-grade oils, then test an oil of very low grade, one having a power factor of at least 0.10 at the maximum temperature. The cell then should be cleaned by the methods described by the author and the first test on the high-grade oil repeated in the endeavor to secure results approximating those secured on the first test.

Until the author has submitted such data, his statements about the suitability of monel metal and the ease of cleaning of his design of cell are to be regarded merely as allegations—unsupported claims. Other workers who are called upon to test oils having a wide range of quality should await such test results before following the lead of the author.

J. B. Whitehead (The Johns Hopkins University, Baltimore, Md.): The author has referred to the d-c conductivity of oil and its measurement. The importance of the d-c conductivity is not obvious, except perhaps as a rough check on successive samples of the same oil. The d-c conductivity is not a measure of the alternating dielectric loss, which may be 100 or more times greater than that computed from the d-c conductivity. Figures of d-c conductivity for 2 different types of oils or 2 samples of the same oil at different viscosities are particularly misleading. The d-c conductivity is proportional to the number of ions present and inversely proportional to the viscosity; consequently, comparisons of different oils and different samples should always be on the basis of the same value of viscosity.

The product of the initial or short-time conductivity and the viscosity is proportional to the number of free ions present. The ions oscillate in the alternating field and it would appear that they should have a marked influence on the stability of the oil under electric stress. The initial conductivity may be computed approximately from the 60-cycle power factor.

H. H. Race (General Electric Company Schenectady, N. Y.): A few years ago several testing sets for measuring the d-c resistivity of insulating oil were designed and constructed ("Some Electrical Characteristics of Cable Oils," Herbert H. Race. ELECTRICAL ENGINEERING, volume 50, August 1931, page 673). These proved to be considerable improvements over equipment previously employed. Since that time, however, oils of increasingly high resistivity have been produced and submitted for use; therefore, it has become necessary further to increase the sensitivity and improve the design of testing equipment. The cells used during the last 6 years have 2 major disadvantages: first, no guard was provided, so that the equipment indicated the combined resistance of electrode supports and unknown liquid in parallel, and if the resistance of the supports is not very much greater than the unknown, the test results are worthless; second, the electrode insulation was in contact with the liquid to be tested, making it difficult to clean the cell so that no contamination of the unknown liquid was possible. The guarded cell described by the author eliminates both difficulties, and cells of this or similar design should be adopted for both a-c and d-c measurements on oils having resistivities greater than 10^{12} ohms per centimeter.

In addition to these general remarks, the writer would like to comment on several specific points of the paper.

1. The author states that his cell can be cleaned by giving it 2 washings of benzol followed by a washing with petroleum ether. Past experience indicates that if oils of widely varying quality are being tested, this procedure is not sufficient to prevent contamination of a good oil, and that scrubbing with soap and water followed by thorough drying should precede the benzene washing.

2. For dehydrating oil the author suggests heating the oil before admitting it to the degasification chamber. In order to prevent oxidation during heating cold oil should be admitted slowly into the top of a vertical glass tube around which an electrical heater is placed and in which a good vacuum is maintained. Most of the oxygen is thus removed from the oil before it is heated to allow complete degasification.

3. The author suggests an improvement in the test equipment used for measuring the oxidizability of an oil by the change in resistivity with aging. The writer believes that in such tests diffusion is an important unknown factor and that direct quantitative measurement of oxidizability should be obtained as has been done for example by R. W. Dornte ("Oxidation of White Oils," *Industrial and Engineering Chemistry*, volume 28, January 1936, pages 26–30). Such measurements should be more reproducible, should have more theoretical significance and could be performed in the presence of metal catalysts if desired.

O. A. Knopp (Pacific Gas and Electric Company, San Francisco, Calif.): The writer has been confronted for years with the problem of testing transformer oils for a large utility for the purpose of deciding whether certain new oils were satisfactory to be used in a transformer or if others were still fit to be used again after years of service.

The power factor of the dielectric loss of the oil appeared to be a criterion of the oil worthwhile investigating. Such investigation particularly appeals to an electrical laboratory confronted with this problem.

The Schering bridge developed by the author is no doubt an excellent means for achieving this purpose, and it has been developed in a most ingenious manner. Most utility laboratories are not in a position to spend very large sums of money for the testing of oils and therefore the writer would like to ask if before developing the Schering bridge for the purpose, other methods, which might be less expensive, were tried.

The writer had in mind obtaining the power factor by means of a small vibrating rectifier that has been developed in Germany and is excited from a phase shifter so that it is possible to measure with a very sensitive d-c instrument the values of the in-phase and wattless currents. It is claimed that current values of the order of 10^{-10} ampere can be measured and apparently that method might be sufficiently sensitive to obtain measurements of power factor in a simple manner.

Another question about the apparatus for dehydrating the oil suggests itself: Is it possible that, in using this apparatus, certain more volatile constituents of the oil may be carried away in the process, so that the oil is not exactly the same as before the treatment? These constituents, if any, although minute may have an appreciable effect upon the performance of the oil as well as on the power factor.

L. J. Berberich: The author wishes first to indicate an error in the drawing of the cell given in figure 3 of the paper. The guard-ring electrode (5) is shown hatched across under the inner insulating ring (7). There should be a gap just as in the case on the upper side of inner insulating ring (7).

One of the purposes of the paper is to point out not only the difficulty in measuring accurately the electrical properties of good oils, but also to call attention to the poor agreement of interlaboratory measurements made on the same oil. In the first part of the paper the lack of agreement of various laboratories in resistivity measurements is discussed. Since the paper was published agreement in power factor measurements has been found to be just as poor. This may be attributed to one or all of the following: (1) differences in cells used; (2) poor design of cells used; and (3) inadequate sensitivity of measuring equipment.

One large industrial organization already has appreciated the need for standardization of cells and has constructed 3 cells similar to the one described in the paper for use as interlaboratory standards within the organization. Many do not realize the nature of this situation, however, and hence cannot be expected to do much to remedy it. The ASTM is at the present time actively interested in furthering the cause of more accurate resistivity and power-factor measurements and deserves the support of all those interested in such measurements.

D. W. Roper in his comments deplores the absence of data upon which the statements that the cell can be cleaned easily and that monel is a suitable metal for the electrodes may be based. Such data can be supplied and precisely the same method suggested by Roper was used in obtaining it. An oil of high grade first was measured over a range of temperature. After the simple cleaning procedure described, a deteriorated and contaminated oil drawn from a transformer was measured. Finally, after using the simple cleaning procedure again, the measurements on the oil of high grade were repeated. This was done not only with the cell described in the paper but also with 2 other cells of more conventional design. In table I of this discussion cell number 1 refers to the cell described in the paper, cell number 2 refers to a brass cylindrical cell with bakelite insulation between the guard and measuring electrodes, and cell number 3 refers to a cell similar to cell number 2 except that the brass electrodes are chromium plated.

In table I the order in which the measurements were made is given in the first column. The data in the third column show that the cleaning procedure is adequate for the cell described in the paper. A more thorough cleaning, which is discussed later, is recommended at periodic intervals and particularly if badly deteriorated oils are studied. The values for the high-grade oil as determined in the other 2 cells are considerably higher than those for cell number 1, and highest values are obtained in cell number 2 where brass is exposed to the oil; moreover, cells number 2 and 3 show a negative power factor for an air dielectric. The author attributes this to the bakelite interelectrode insulation, which is under oil in the case of these 2 cells. Any porous insulation that may absorb or trap oil should be avoided, particularly if it can come in contact with the oil. Heat-resistant glass has been found to be as satisfactory as fused quartz for this purpose.

Some additional data showing the suitability of monel metal also can be supplied. In this case samples of a certain oil were aged for 4 hours at 110 degrees centigrade in the presence of strips of various metals of the same size. One sample of the same oil was aged under the same conditions, except that no metal was placed in it. This is referred to as the "blank." It was decided that 4 hours would be the maximum time for keeping any oil in the cell at a high temperature. After this aging procedure the resistivity of each sample was measured at 100 degrees centigrade in a nickel-plated cell. The results, which are discussed in a paper by W. G. Horsch and the author ("A Cell for Routine Electrical Measurements on Insulating Oils," *Review of Scientific Instruments*, volume 5, May 1934, page 194–6), are given in table II of this discussion.

Table II

Metal Catalyst	Resistivity, Ohms Per Centimeter Cube x 10^{12}
Blank	29.2
Nickel	32.8
Monel	31.6
KA_2 alloy	29.0
Brass	21.5
Copper	19.9

Table II shows that both copper and brass have a distinct catalytic effect even in 4 hours at 110 degrees centigrade. This is not apparent in the case of the other metals listed. Monel was chosen as the most workable of the 3.

As to J. B. Whitehead's comments, the author did not intend to imply that d-c conductivity measurements can ever wholly replace power factor measurements. Both find extensive use, however. As Whitehead has mentioned, the most important use for the d-c conductivity test is checking successive samples of the same type of oil. As such, this simple test serves the purpose, provided it is carried out properly. On the contrary, however, to measure the d-c conductivity of an oil whose electrical properties are wholly unknown, and from such results to attempt to predict the power factor might produce some misleading results. As Whitehead has shown, the power factor can be computed only from short-time conductivity measurements.

H. W. Bousman's comments are interesting, for the author's experience has been that the oils that show a decrease in power factor on aging have always shown a deterioration in d-c resistivity. This deterio ration in resistivity, however, has alway been found to be small compared with tha of an oil that shows no such improvement i power factor on aging. Perhaps if the agin had been carried out at a lower temperatur on the light oil whose power factor curve i given in figure 7 of the paper this anomal would not have appeared. It was not done however, and no definite further explana tion can be given. It should be mentioned perhaps, that the chemical changes in th oils showing such anomalous behavior wer almost immeasurably small.

The author agrees with H. H. Race tha the cell should be given a more thoroug cleaning after a very badly deteriorated o has been measured in the cell; it should b done particularly after an oil containing polar material has been measured. Suc materials may adhere to the metal and can not be removed by the simple cleaning pro cedure described. In such cases the autho has used very fine carborundum powde moistened with ether with success. O course, such a cleaning procedure cannot b used on a plated cell.

Race also refers to the method of dehydrating oil described in the paper. Th method he has used undoubtedly is superio to the author's where complete degasifica tion is desired. The danger, however, o oxidizing a reasonably stable oil in the shor time required for dehydration is not ver serious. As A. Gemant ("Absorption of Ai by Mineral Oil," *Transactions* of the Fara day Society, volume 32, April 1936, p 694–701), has found, some heat is desirabl in this process and its principal effect is t aid in the removal of moisture.

The author agrees with Race that diffusion is a very important unknown factor i the oxidation of oil. This is particularly tru of white oils, which may oxidize at such high rate that diffusion of oxygen throug the oil is the limiting factor in the oxidatio process. Dornte's method of determinin the oxidizability, referred to by Race, is ad mirably suited to a theoretical investigatio of the oxidation of various types of oils The apparatus described in the paper, how ever, is designed for the routine simul taneous investigation of a number of sample and as such has been found satisfactory Dornte's method applied to several sample would become too complicated for this pur pose. Such fundamental work as Dornt has reported, however, should be en couraged.

O. A. Knopp has inquired whether an method simpler than the Schering bridg method has been tried for measuring o power factor. The answer is that of a commonly known methods, authoritie generally agree that the Schering bridge i capable of most accuracy. In measuring th extremely low power factor of unused oils i particular, all of the sensitivity and accurac available are usually necessary. The autho is not familiar with the small vibrating rec tifier to which Knopp refers and in connec tion with which he unfortunately does no give a reference. Other types of mechanica rectifiers have been tried with high sensi tivity d-c galvanometers as null detectors i bridges and have been found to give con siderable contact trouble. Knopp has i mind measuring the in-phase and wattles components of the current directly. Sinc the ratio of the wattless current to the los

Table I

Material Measured	Temperature, Degrees Centigrade	Power Factor Readings: Cell Number 1	Cell Number 2	Cell Number 3
High grade oil	80	0.000153	0.000406	0.000374
High grade oil	60	0.000042	0.000134	0.000123
High grade oil	40	0.000013	0.000043	0.000040
Air	80	0.000004	−0.000083	−0.000136
Poor oil	80	0.0236	0.0270	0.0226
Air	80	0.000005	−0.000070	−0.000096
High grade oil	80	0.000192	0.000481	0.000430
High grade oil	60	0.000045	0.000159	0.000132
High grade oil	40	0.000013	0.000051	0.000043

component of the current is so large, some shunting arrangement must be provided in order to accommodate the galvanometer to both components of current. Considerable research will be necessary to show the limitations of such a scheme.

As to Knopp's other question regarding the dehydrating of oil, the author agrees that a small percentage of the more volatile constituents of the oil may be carried away; however, it has a negligible effect on unused oils, as can be shown by measuring a dry oil before and after such a treatment. For a badly deteriorated oil, however, this may have more significance, because some of the more volatile acid products of deterioration may be removed and in turn may modify the power factor. In such cases a lower temperature should be used in the treatment of oil, in order that the removal of such volatile material may be a minimum.

In concluding his remarks, the author wishes to qualify the statements made in the paper regarding the straight-line relationship on a semilogarithmic plot of the power factor-temperature curve. It should have been mentioned, perhaps, that these observations were limited to straight hydrocarbon oils essentially free from polar material. It is conceivable that, since the water of highly polar material modifies the mechanism by which loss occurs, other polar materials may have a similar effect. Thus, in using this relationship oils containing polar additions should be avoided.

Effect of Electric Shock on the Heart

Discussion and authors' closure of a paper by L. P. Ferris, B. G. King, P. W. Spence, and H. B. Williams published in the May 1936 issue, pages 498–515, and presented for oral discussion at the measurements and selected subjects session of the summer convention, Pasadena, Calif., June 25, 1936.

Wills Maclachlan (consulting engineer, Toronto, Ont., Canada): There is little in the literature on this subject that would compare with the exact presentation of the great number of experiments carried out and presented in this paper. Those who have knowledge of the elaborate apparatus and extended period of these experiments have been looking forward for some years to a summary of the work that is here presented, and the authors are to be sincerely congratulated.

The main point that the writer wishes to put forward in discussion of the paper is a note of warning in translating to man the data obtained by experimental work on animals.

In the first conclusion it is quite apparent that if one knows the voltage to which the victim is subjected and knows also the average resistance for the type of contact, the current can be obtained.

Against the second conclusion there is no argument, if it is agreed that ventricular fibrillation after electric shock is arrested without outside interference.

Dealing now with the third conclusion, the resistance from hand to hand with wet contact has been given as 1,500 ohms ("Current Passage Through the Human Body," *Bulletin* deL'Association Suisse des Electriciens, July 7, 1929) and as 1,650 ohms (laboratory report number 31 *E* 3971, "Relative Physiological Perception of Alternating and Continuous Currents," Electrical Testing Laboratories, New York, N. Y.). If a current of 0.1 ampere flows through a circuit having a resistance of, say, 1,500 ohms, there will be a potential of 150 volts across the resistance. If the statement that "the current just below the threshold for ventricular fibrillation is about 0.1 ampere in man" is correct, then it should be possible for man to receive a shock of 150 volts with wet hands without danger, but many authenticated cases of fatal shock are recorded in which the voltage of the circuit was 110 volts or lower and the point of contact was not with wet hands but apparently had a higher resistance. One is forced to the conclusion, therefore, that one is not warranted in applying to man conclusions based upon animal data without very careful investigation of field cases in man. Probably the threshold current for man is about 10 or 20 milliamperes.

Dealing now with the fourth conclusion, R. Lutembacher ("Anatomy and Physiology of Heart in Coeur et Vasseaux" number 10, part 1, page 433–4) in speaking of the relative susceptibility to fibrillation of various animals, stated that the ease of producing fibrillation is in the order from dog, cat, guinea pig, rabbit, rat, mouse to man. Guiseppe Ajello of Milan, Italy, in a paper in *L'Energia Elettrica* (February 1928) stated, "the sensibility of organisms to the current, is relative: At the head of the scale of animals most resistant are the frogs and turtles, and following in a descending scale, dogs, rats, horses, and monkeys."

From this, it is evident that other investigators have different views than that the threshold current increases roughly with both body weight and heart weight but that it might be dependent upon variation in structure and a function varying with the species under consideration.

The information in conclusion 4*B* and 4*C* of the paper is corroborated by field data on accidents to man, and this applies also to conclusion 4*E*.

One of the most interesting and notable developments in the whole paper is that set forth in 4*D*, and no doubt will be the means of stimulating further research, not only in connection with electrical shock but also in the whole study of the heart and particularly ventricular fibrillation.

One of the most dangerous features of the whole paper is contained in conclusion 7. Those responsible for remedial measures after electrical shock, if they followed the recommendations contained in this conclusion would be assuming a tremendous responsibility. After a man receives a shock, and while resuscitation is being carried out, are orders to be given to cease the resuscitation and give the man a second shock? If the second shock is not successful in restoring the life, the natural question to ask is: "Which shock killed the man?" The authors advise that in animal work, electrical countershock was successful in about $^6/_{10}$ of the cases. A paper by the writer ("Electrical Injuries: Interpretation of Field Notes II," *Journal of Industrial Hygiene*, January 1934, table 13), shows that in actual field cases over a period of years, 64 per cent of the electric shock victims were resuscitated by the standard technique of artificial respiration according to Schafer. There is no doubt that further investigation is warranted in connection with the value of electrical countershock, but this investigation should be carried out only by those fully realizing the hazards of the situation. At the present time other less dangerous methods are available.

Gordon Thompson (Electrical Testing Laboratories, New York, N. Y.): This valuable paper presents a mass of data bearing on electric shock, painstakingly and intelligently acquired over a period of several years of research. Since experimentation with human beings is impossible, the authors' suggestion that the susceptibility of the human heart to fibrillation be deduced from data on animals on the basis of body weight and heart weight, represents a forward step in this field.

When the results shown in figure 4 and figure 5 of the paper are examined on this basis, the currents derived for a man, 0.26 ampere by body weight and 0.29 ampere by heart weight, are in satisfactory agreement; but the authors correctly indicate that the minimum value and not the average must be used for any safety measures, and a current of 0.1 ampere, or 100 milliamperes is proposed.

The resistance of the human body along this path (right hand to left foot), as along any other path, is determined primarily by skin conditions at the points of contact. With dry surfaces at low voltages the resistance may be 10,000 ohms. With extensive moist surfaces or with breaks in the skin at both contacts the resistance may be less than 1,000 ohms. Under the former conditions the computation indicates 1,000 volts as hazardous, but breakdown of the skin occurs at much lower voltages than that, and the resistance of the path then approaches the lower value given. For a resistance of 1,000 ohms a voltage of 100 volts is indicated as hazardous.

Of course, a considerable body of testimony is now available as to the values of alternating currents at low voltages that have caused death, presumably by fibrillation and suffocation. Zimmern, an authority in the field, in his "Accidents d'Electricite" cited considerably lower values than 100 milliamperes; he feels that currents above 25 milliamperes are hazardous for adults; small children have succumbed to 10 milliamperes or less.

Users of electric devices must guard against interposing body resistance between points with 100 volts difference in potential, without protective series resistance. Although the number of accidents with electric appliances is surprisingly small in view of the number of devices in use and the general disregard on the part of the user of obvious precautions, still it will be wise to continue to emphasize the need for adequate insulation continuously maintained in electrical appliances handled by the general public. Educational campaigns of the "don't" and "watch out" variety are of only temporary value at best. The continuous interposition of a megohm of impedance between the electric circuit of the appliance and its exposed metallic surfaces, supplied by durable insulating materials, is

the best guarantee against starting fibrillation of human hearts.

The authors' success in resuscitation of animals with hearts in fibrillation by applying a strong countershock current is encouraging. One's mind naturally leaps forward at once to the application of countershock to human beings. The authors have indicated some of the areas in which more research will be necessary to make this a safe and effective procedure. They have already made a start on the magnitude of the countershock current and its duration. How promptly it must be applied is not known. The practical details of size of transformer, size and type of electrodes, and places of application probably can be best studied on corpses in the manner indicated by H. Freiberger ("Der elektrische Widerstand des menschlichen Koerpers," Berlin, 1934). The industry should see to it that this work goes forward.

W. B. Kouwenhoven (The Johns Hopkins University, Baltimore, Md.): The authors were fortunate in having sufficient funds available to make possible the study of the effects of electric shock on large animals. Their tests on dogs confirm results ("Resuscitation by Counter Shock," W. B. Kouwenhoven and D. R. Hooker. Electrical Engineering, volume 52, July 1933, page 475), obtained by the writer and an associate, namely that a current of 100 milliamperes is sufficient to throw that animal's heart into ventricular fibrillation.

The writer's records show that for an intact animal he was successful in recovering the fibrillating heart by means of a countershock ("The Effect of Alternating Electric Currents on the Heart," D. R. Hooker, W. B. Kouwenhoven, and O. R. Langworthy. *American Journal of Physiology*, volume 103, 1933, page 444) in 13 out of 22 attempts. Relatively little difficulty was experienced unless the heart had been left in fibrillation for a period of 4 or more minutes. The authors using the same method of resuscitation check those results closely.

The authors' discovery that the heart is more easily thrown into fibrillation during certain phases of the cardiac cycles than others is of great importance, for it provides a possible explanation of certain baffling field cases.

The writer also agrees with the authors' statement that successive shocks are not cumulative in their action. The writer has not found any lowering of the threshold value of the fibrillating current when repeated shocks were used; instead the data indicate that the heart becomes less irritable in some instances.

The problem of applying data obtained in animal experiments to man naturally is difficult. The authors' third conclusion, namely, that 100 milliamperes is the maximum safe current that may pass from hand to feet in man is not believed to be correct. Such a current value ("Electric Shock," W. B. Kouwenhoven. *Engineers and Engineering*, volume 48, 1931, page 137) is extremely dangerous and probably fatal. This conclusion is based not only on experiments made with animals, but also upon certain tests made on humans and on the field cases given in table I of this discussion.

In these cases the current values were measured carefully, neglecting any resistance that might be offered by the body itself. If that were considered the current values given in the above table would be reduced slightly. Of the cases listed, 4 were reported by the Berufsgenvesenschaft der Feinnechanik and Elektrotechnik, of Berlin, and in the other case the writer personally made the measurements.

Table I—Field Cases of Electric Shock

Case	Current Pathway	Current in Milliamperes	Frequency in Cycles per Second	Result
K	Hand to hand	42.5	50	Lived
P	Hand to hand	76	60	Lived
H	Hands to feet	110	50	Death
L	Hands to feet	110	50	Death
M	Hands to feet	230	50	Death

L. P. Ferris, B. G. King, P. W. Spence, and **H. B. Williams:** In reply to Wills Maclachlan's comments first, it goes without saying that great care should be exercised in translating data from animals to man. Referring to our first conclusion, Maclachlan infers it requires but a simple application of Ohm's law with known voltage and average resistances to obtain shock current. The difficulty is that contact and body impedances vary over a wide range and can seldom be ascertained within close limits in particular cases. This is unfortunate because the voltage involved in accidents generally is well known.

Maclachlan would accept our second conclusion only on condition that ventricular fibrillation is arrested without outside interference. The evidence is strongly against the necessity of this condition. Without the use of an electrocardiograph to indicate the difference between co-ordinate heart action and ventricular fibrillation, it is impossible to be certain that victims of electric shock have ventricular fibrillation. From observations on many animals, both large and small, the authors know that fibrillation has occurred without leaving any pathological symptoms in the cardiac tissue. *Postmortem* examinations on most of the large animals employed in the tests and pathological examinations of the cardiac tissue of several of the large and small animals have shown that the hearts of those that died from ventricular fibrillation appeared no different from the hearts of victims of asphyxial death. Fibrillation is characteristically a functional disturbance of heart action. The normal electrocardiograms of numerous large animals that were recovered from fibrillation by counter shocks are further evidence of lack of damage to the cardiac tissue. In observations of fibrillation on over 500 large animals, but one spontaneous recovery occurred in an animal as large as sheep. As shown by table X of the pape countershock, definitely "outside interfe ence," was given to 160 of those anima with fibrillating hearts, resulting in th arrest of fibrillation and recovery in 91 case Perhaps it should be pointed out that th second conclusion does not state that cu rent through the heart, even though abov the threshold of fibrillation, will always pr duce this effect. It is possible for victims c electric shock to be prostrated without ve tricular fibrillation.

Although the authors are of the opinio that the data from experiments on man animals of about the size of man, and bot smaller and larger than man, combine to i dicate that the threshold fibrillating curre for man will be above 0.1 ampere with ra possible exceptions, they do not believe th indicates by any means that it is always sa for a man to receive a shock at 110 volt Too many authenticated fatalities from 110 volt circuits prove otherwise. Unusual con tact conditions will permit 0.1 ampere o more to result from accidental contact wit such circuits; furthermore, it is recognize in the paper that much lower currents ma cause fatalities if long continued, throug stoppage of respiration. The values of 10 o 20 milliamperes suggested by Maclachla seem so far below the evidence of this in vestigation as to be quite improbable as th threshold of ventricular fibrillation for man

Maclachlan cites Lutembacher and Ajell as to conclusion 4*A* of the paper, namely that "Among the different species th threshold current increases roughly with bot body weight and heart weight." No experi mental data are given by Lutembacher i the article referred to by Maclachlan. I rating the ease of producing fibrillation i different animals, it is not evident tha Lutembacher is using the threshold curren as a basis. In physiological laboratorie fibrillation is most commonly induced b current from an induction coil applied wit small electrodes placed close together upo the exposed heart. These conditions are en tirely different from those in accidents or i the experiments described in the paper The authors believe it is a fact generall well known among physiologists that th hearts of the smaller warm-blooded anima are more difficult to maintain in fibrillatio than the hearts of the larger animals. Sma rapid hearts generally recover spontane ously from fibrillation, almost immediatel after it is set up. This may be the basis o Lutembacher's rating of the different an mals except man. It is extremely doubtfu that Lutembacher or any other investigato has ever performed any tests on man to de termine the threshold of fibrillating curren or deliberately to set up ventricular fibrilla tion for any other reason, as this conditio has long been regarded by the medical pr fession as leading to almost certain deat The Ajello article likewise contains no e perimental data on the susceptibility of th species mentioned to ventricular fibrillatio It is not even apparent that Ajello is refe ring to fibrillation in his statement; how ever, it is definite evidence that it is mo important and the paper under discussio summarizes the experimental evidence, upo which conclusion 4*A* is based, in table I an in figures 3, 4, and 5 of the paper. Th authors recognize that there is no precis correlation between minimum fibrillatin

current and body weight and heart weight. As stated in the paper, if the 3 smaller species be considered alone this relationship does not hold, but the data for these species combined with those for the large species conclusively show an over-all picture of minimum fibrillating current increasing as both the body weight and heart weight of the species are increased. The thresholds were found to differ widely for different individuals of the same species; consequently, there is no suggestion that this general relationship be applied within any particular species. That apparently similar biological specimens exhibit different degrees of sensitivity to different diseases and drugs well known, so that it was not surprising to find such differences in the responses of individuals of a species to electric shock.

The seventh conclusion of the paper does not warrant the view that the authors are suggesting without further study, the general use of countershock in accident cases that occur in the field where resuscitative measures must be applied by personnel with no training in this technique and no equipment designed for it. In this conclusion the authors stated in the paper: "Further study is desirable to develop the optimum conditions and practical apparatus for utilizing this method in accident cases." In such cases the use of a simple electrocardiograph also is suggested, so that the condition of the heart may be diagnosed positively. Finally, in the last statement the authors strongly urge the administration of artificial respiration. The seventh conclusion therefore should not lead to a dangerous risk in the handling of accident cases. The belief of the authors in the efficacy of countershock is indicated by the agreement among them that should any one of them receive a serious shock in the course of the experimental work, the others should administer a countershock, provided the conditions indicated fibrillation.

It does not seem that the statistics on recovery of man by artificial respiration cited by Maclachlan can be properly compared with the statistics given in the paper on the recovery of animals from fibrillation by countershock. The large animals that were recovered from fibrillation by countershocks were known definitely by electrocardiograms to be in fibrillation, whereas the heart condition of human victims of shock rarely is known. The respiratory mechanism may, of course, be seriously disturbed by electric shock without any occurrence of fibrillation of the heart. Unfortunately there is seldom an opportunity in electrical accident cases in field to determine the heart condition by an electrocardiogram. Such information is highly desirable in determining the reasons for success and failure of artificial respiration and the field of application of countershock. It is particularly important to know whether recoveries have been obtained by artificial respiration in spite of ventricular fibrillation. As previously stated, observations of ventricular fibrillation on over 500 large animals showed but one spontaneous recovery. Apparatus and technique for the application of countershock can be developed so that under conditions where it is indicated and where it can be promptly applied, lives that would be lost by the administration of artificial respiration alone may be saved.

W. B. Kouwenhoven disagrees with the conclusion that 0.1 ampere is the maximum safe current and would set a lower limit. The field cases cited by Kouwenhoven are not inconsistent with the 0.1 ampere limit; however, it must be admitted that for 2 of those which resulted fatally the current given is but slightly above this limit. More information about such cases would be desirable. One naturally wonders what circuit conditions limited the current to 110 milliamperes in the absence of any body impedance.

With the fullest information usually possible to obtain, it is generally very difficult to reproduce accident conditions closely enough to obtain accurate current values. In New York City a case recently was brought to the authors' attention immediately following the accident; the conditions under which the shock was received seemed unusually simple and well known. Great care was taken to reproduce those conditions and current measurements were made with an oscillograph, which showed that the victim, who lived, might have received as much as 350 milliamperes; but, on the contrary, if other conditions of about equal probability obtained at the time of the accident, the current might have been limited to about $^1/_{10}$ of that value. The voltage in this case was 100,000 and the frequency 60 cycles per second. The duration of the shock probably was not more than 3 seconds.

No information is given by Kouwenhoven on the duration of the shocks in the field cases cited. Duration plays an important part in determining the effect of a shock. Information in this regard is given in the paper. The 0.1 ampere (60 cycle) figure mentioned in conclusion 3 applies only to the production of ventricular fibrillation by shocks of a second or more duration. In the fourth paragraph of the paper is a statement that any currents that prevent voluntary control of the skeletal muscles are dangerous because prolonged application of such currents might cause asphyxial death. Other investigators have shown that voluntary control of the muscles is lost at current values much less than 0.1 ampere. The time required for death in such cases is a matter of minutes instead of seconds; therefore, opportunity may be afforded for the rescue of the victim. The value of current just under the threshold for ventricular fibrillation is taken by the authors as the maximum current to which man may safely be subjected, because from a greater current death is likely to result regardless of rescue or after-treatment. None has ventured to suggest that this criterion is too conservative. If the possibilities of prompt rescue or automatic interruption of the shock cannot be relied upon, then clearly a value of current lower than 0.1 ampere must be considered as dangerous. Conversely, if shocks can be reliably and automatically limited in duration to less than 0.1 second—and this is possible under some conditions—then the tolerable current, as shown by the paper, can be multiplied by about 10 times. These are facts of which general recognition should promote the effectiveness of engineering considerations of safety.

Gordon Thompson has referred to an article by Zimmern. The value of 25 milliamperes mentioned by Zimmern as dangerous is based upon tests by others showing that alternating current passed between the hands becomes intolerably painful above that value and is not based upon any observations of fibrillation.

Thompson has emphasized that users of electric devices must guard against imposing body resistance across 100 volts without protective series resistance, and suggests that the continuous interposition of a megohm of impedance between the electric circuit of an appliance and its exposed metallic surface is the best guarantee against starting fibrillation in human hearts. Such a resistance undoubtedly would limit the current from 110-volt circuits to a safe value, but unfortunately accidental conditions in which occasionally such a resistance is short-circuited without any visible evidence of fault seen to be inevitable. An even better guarantee of safety from low-voltage circuits would seem to be arrangements that would limit the current from such circuits through a ground fault to a value less than the danger limit and to interrupt automatically the circuit within a fraction of a second of the occurrence of such a fault. Practical difficulties and the expense of such arrangements are apparent, but the possibilities and advantages of such safety measures should not be overlooked.

The authors are glad of Thompson's interest in the application of countershocks to human beings in cases of accident. Although Freiberger, whom he cites, has done valuable work on body impedances by employing corpses the authors do not believe that much, if anything, can be learned about the apparatus or application of counter shock by studies on corpses as suggested by Thompson.

The Sparkless Sphere Gap Voltmeter—II

Discussion and authors' closure of a paper by R. W. Sorensen and Simon Ramo published in the May 1936 issue, pages 444–7, and presented for oral discussion at the measurements and selected subjects session of the summer convention, Pasadena, Calif., June 25, 1936.

J. B. Whitehead (The Johns Hopkins University, Baltimore, Md.): The limitations of the sphere gap voltmeter, particularly for large spacings, have long been recognized. The differences of the results of different observers has been attributed principally to the influence of the surroundings and several uncontrolled variables affecting the dielectric strength of air.

The authors have presented an interesting and valuable survey of the possibilities of using the force of attraction between charged spheres as a measure of voltage. At first sight it appears that such a method should be free of those errors arising in the variation of the dielectric strength of air. That it has any advantage in freedom from the influence of surroundings is not so evident, however. This trouble has been encountered by the authors, in fact, and the readings of the sparkless gap are found to vary with its configuration as related to surrounding walls and planes and dimensions of supports. Perhaps the most valuable feature of the work is the analysis of the influence of these factors. The writer wonders whether

the forces between planes, as in the absolute electrometer with guard rings, which has been utilized abroad in the high-voltage range, would not be less subject to disturbances of this character.

A recent paper by Walter Dattan, "Zur Eichung von Kugelfunkenstrecken bei Stobspannungen und Normalfrequez," *Elektrotechnische Zeitschrift*, volume 57, April 2, 1936, p. 377–81 and April 9, 1936, p. 412–14, reported a careful experimental study of the sphere gap with critical comparison of the recent work of Meador, Bellaschi, and Weicker. Working with both direct current and alternating current, he found variations caused by polarity; he stated that a vertical axis with the upper sphere grounded is least subject to outside disturbance, and further that a part of the variation of results for wide gaps is caused by the influence of space charges in the vicinity of the spheres at high stresses.

Unfortunately no convenient and constant standard for high-voltage measurement has as yet been proposed. Some years ago the writer presented an experimental study of the corona voltmeter in which it was shown that the instrument is remarkably constant and accurate and has an inherent calibration in terms of its dimensions. Those results subsequently were confirmed by the National Bureau of Standards. The instrument is the most accurate and reliable method available for high-voltage measurement; however, because of its elaborate construction and auxiliary equipment it has never been utilized in practical service, nor has it been developed into the extra-high-voltage range. The sphere gap therefore, spark or sparkless, remains the most reliable engineering standard and, thanks to such papers as this its limitations, though serious, are now thoroughly understood.

J. R. Meador (General Electric Company, Pittsfield, Mass.): The authors are to be commended for their continued work on this interesting and apparently basic method of measuring high alternating voltages. It is to be hoped that other laboratories soon will be in a position to present substantiating data.

Referring to figure 2 of the paper, the spread of the voltage points emphasizes the fact that in speaking of spark-over in air a "zone" or tolerance must be recognized. The proposed AIEE sphere-gap spark-over curves should be used with the understanding that such a tolerance exists. The recommendations are for a tolerance of ±3 per cent. Average values from the authors' test points appear to agree with the proposed curve for the most part within this tolerance. By limiting the use of the spheres to a spacing of 70 per cent or 80 per cent of sphere diameter, the largest variations from the proposed curve are eliminated.

A comparison between the relative spacings of the 50-centimeter and 100-centimeter sphere gaps taken from the curves obtained by the authors would be interesting. These equivalent spacings often are useful in comparing the results obtained by different investigators.

One of the discussers has referred to a paper by Walter Dattan ("Voltage Calibrations of Sphere Gaps on Impulses and on Standard Frequencies," *Elektrotechnische Zeitschrift*, volume 57, April 2, 1936, p. 377–81 and April 9, 1936, p. 412–14). That paper shows spark-over curves for several sizes of spheres mounted horizontally on both impulse and 50-cycle voltages. His voltages for the 50-centimeter sphere gap at large spacings fall below the proposed AIEE curve by approximately the same amount that those by Sorensen and Ramo on vertically mounted spheres lie above it. These 2 sets of data probably represent the extent of the working zone in which sphere spark gaps will be used and are good indications of the results to be expected with different spacing arrangements. Dattan apparently was working in somewhat cramped space, but Sorensen and Ramo have clearances larger than usually are possible in the average high-voltage laboratory.

The authors show that in their laboratory the vertical arrangement of spheres requires a higher spark-over voltage at large gap spacings than does the horizontal arrangement for the same distance from the ground plane. It appears that under some laboratory conditions this relation may be reversed. With the vertical arrangement and the line-potential sphere 5 diameters above the ground plane, the surrounding walls, fences, and ground-potential structures may have considerably more influence on the field than does the ground plane, even when the surrounding objects are 5 or 6 diameters from the spheres. A reasonable conclusion seems to be that the vertical arrangement is affected more by surrounding objects, and the horizontal arrangement is affected more by the ground plane.

To standardize on some method of specifying clearances seems reasonable. The writer suggests that the distances from the ground plane and from surrounding objects be measured from the arcing point of the line-potential sphere, or to be more exact, from the point at which a line drawn through the centers of the 2 spheres intersects the surface of the line-potential sphere.

R. E. Hellmund and **P. H. McAuley** (Westinghouse Electric & Manufacturing Company, East Pittsburgh, Pa.): The authors are attempting to establish a primary standard for the measurement of high voltages by measuring the forces between spheres; therefore, reference to a device constructed by Harold Smith while doing some work for the Westinghouse Company, by which he successfully utilized an electrostatic repulsion voltmeter as a secondary standard, may be of interest.

The arrangement is shown in figure 1 of this discussion and its principle will be seen to be equivalent to the gold-leaf electroscope; 2 parallel elements, one fixed and one supported on knife-edge bearings, are connected to the voltage to be measured, and with a mirror on the movable element, the angular displacement produced by the applied voltage is determined optically. Cylindrical screen elements enclosed in a cylinder of large diameter and oil damping are refinements necessary at high voltages in a practical instrument. Perhaps the main advantage of such a device is the continuous-reading characteristic. For some types of work this arrangement has been found to be convenient and reliable; however, for high-voltage tests on insulation the maximum voltage usually is more important than the effective voltage measured by arrangement described. Therefore, if vices for measuring the effective volt are used, extreme precautions must taken to prevent any variations in the w form. Such variations may be caused a change in power supply and also by capacity or impedance effects of vari apparatus connected in the testing circ

As an indication of the effects of w form, the writers are familiar with one c

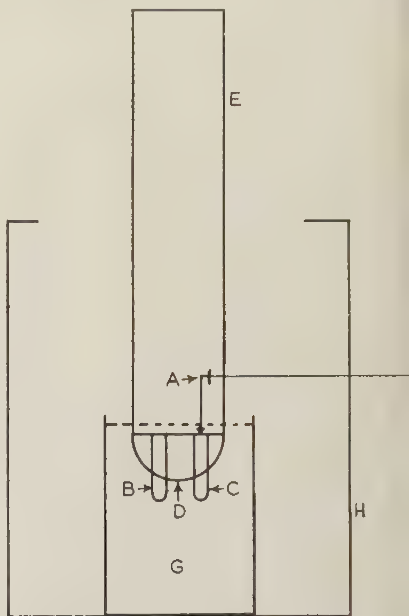


Fig. 1. A typical arrangement of the hi voltage repulsion type of voltmete

A—Mirror
B—Fixed element
C—Movable element on knife-edge bearing
D—Slotted hemisphere
E—Cylindrical electrode
F—Light beam
G—Oil in insulating tank
H—Screen shield

in which 100-centimeter spheres set at kv indicated average tertiary winding v ages of 188, 207, and 221 kv for 3 ferent sources of power supply. For reason a crest-voltage indicator seems most imperative for high-voltage tes even in 60-cycle tests. The recent acti on surge testing also makes a crest-vol indicator indispensable and this fact therefore detracted somewhat from the terest in devices indicating the root m square value. Furthermore, for the pose of correlating 60-cycle and imp measurements and as a universal stand the sphere spark gap is unique in its fiel

It therefore seems that the additio the experimental data on sphere-gap bration presented by the authors is a v able feature of the paper. The prop standard calibration, as may be know the average of results obtained in sev laboratories, and it is gratifying to note the results obtained by the authors c bined with some similar results rece obtained abroad would not modify average appreciably. The data give the paper are valuable also because indicate definitely the influence that rounding objects and distance from gro may have upon the sphere-gap calibra

Table I—Test Conducted by the Westinghouse Company

Test Number	Wave Calibration	Laboratory	Date
1	1.5 x 40	Trafford	April 1934
2	1.5 x 40	Trafford (3 series)	April 1933
3	1.5 x 40	Sharon (2 series)	April 1933
4	1.5 x 40	Trafford	August 1934
5	60-cycle	Trafford	August 1934

for large spacings. All of these contributions are valuable to the knowledge on sphere-gap testing. It is rather regrettable, therefore, that in spite of this excellent work, present-day practice in sphere-gap testing, although satisfactory for most practical commercial purposes, leaves a good deal to be desired in the way of precision measurements. The authors correctly point out that lower flashover voltages are obtained if the voltages are increased gradually; furthermore, figures 2 and 3 of the paper indicate a considerable spread of points obtained over a period of time. This spread is by no means limited to large gap spacings but also obtains for the smaller spacings, where surrounding objects should have practically no influence. In figure 2 of the paper, for instance, at the 15-centimeter setting the spread and also the departure of the extreme points from the proposed standard is nearly 5 per cent. It seems, therefore, that those carrying on investigations along this line should turn their attention to the cause of such spread in the smaller gap spacings, especially since the spread is by no means limited to the investigations of the authors.

Table II and figure 2 of this discussion represent a collection of results obtained over several years in 2 different laboratories for both 60-cycle and surge testing. As figure 2 of the paper indicates, the new AIEE standard curve is an average of the values presented by Meador and by Bellaschi and McAuley. Because of the 6 per cent spread between these values at large spacings, several check tests were made in an effort to improve the agreement or confirm the disagreement. One set of tests made at Trafford resulted in values below the new standard curves but all other tests indicated that the original values were correct. For this reason, it gives the writers a great deal of satisfaction to note that the Bellaschi points of figure 2 of the paper are a very good average curve of the authors' test points at those large spacings. At spacings below $^1/_2$ diameter the writers' points are consistently higher than the authors' tests indicated.

The points of figure 2 of the paper indicate that the spread of the data is from 4 to 4.5 per cent from spacings of 15 to 45 centimeters. At first glance it seems illogical to blame this spread mostly on inconsistency of the spark gap and absolve the method of measurement of appreciable variations. With this point in mind, the writers have made a similar graph (figure 2 of this discussion) of data from the tests indicated in table I of this discussion.

All but 3 points lie between 7 kv below and 14 kv above the new standard curve. A comparison of table II with the Sorensen and Ramo data shows similarity.

Thus the test points from 2 Westinghouse laboratories, obtained mostly on impulse voltages as compared with 60 cycles and measured with the cathode-ray oscillograph as compared to force measurements on a larger sphere, show about the same spread. With such radically different methods of measurement, the variations would appear to come from the sphere spark-over. However, neglecting the one set of divergent Trafford data, the spread for the greater part of the results becomes much less.

Undoubtedly it would be desirable to determine whether the variations are due to various causes of preionization, to irregularities of the sphere surfaces caused by dust particles or otherwise; whether the electrostatic field is possibly disturbed by large dust particles in the air, or whether the humidity and pressure corrections of the atmosphere are not accurate. Possibly some of the variations could be eliminated by air conditioning the test room, by keeping the temperature and humidity constant for extended periods, and by removing dust particles from the air. As a matter of course, it should not be taken for granted that all discrepancies are due to the action of the sphere gaps, for they may also be found in other parts of the circuits and testing arrangements. It is believed that if progress can be made in the explanation of these variations for the smaller gap settings where surrounding objects are of little influence, many of the discrepancies in the larger gap settings can be accounted for in the same way.

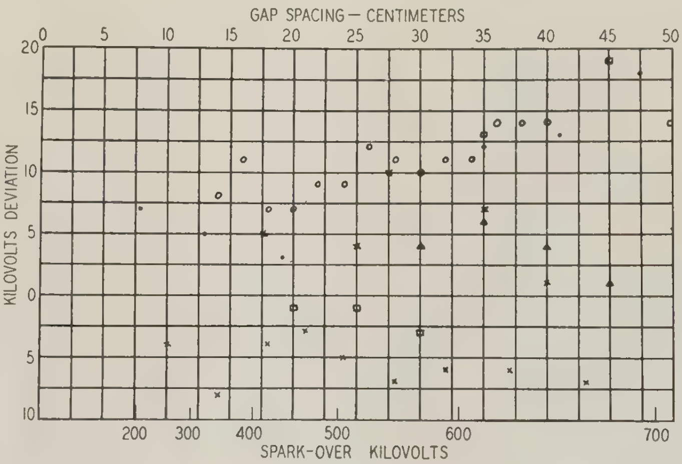


Fig. 2. Deviation of test points from the new AIEE standard cailbrations for a 50-centimeter vertical sphere gap with lower sphere grounded

●—Test series 1
×—Test series 2
○—Test series 3
△—Test series 4
□—Test series 5

P. L. Bellaschi (Westinghouse Electric & Manufacturing Company, Sharon, Pa.): The authors present pertinent data on the effect of walls, the ground plane, and the mounting of the spheres on the spark-over voltage. Experiments by the writer and his associates have produced essentially the same conclusions (reference 4 of the paper).

A 25-centimeter sphere gap mounted vertically in a framework and with the lower sphere 5 diameters above ground, conforming to AIEE requirements, was used. A grounded steel plate 132 centimeters long and 87 centimeters wide was placed vertically facing the gap, thus simulating a wall or ground plane. The plate was spaced 2.5 sphere diameters from the center of the spheres. A positive impulse voltage of $1^1/_2$ x 40 microsecond wave was used. These were the results: At gap spacings 0.2 and 0.4 of the sphere diameter no change in spark-over voltage was observed with or without the plate. At a spacing 0.76 of the sphere diameter the spark-over voltage when the plate was present was 90 per cent of the spark-over voltage with the plate removed.

Next, a grounded steel plate 87 centimeters long and 57 centimeters wide, having a center hole to pass through the shank of the lower (grounded) sphere was mounted

Table II

Centimeters Spacing	Spread—Kilovolts: Sorensen and Ramo	Westinghouse	Westinghouse (Omitting Low Trafford Tests)
15	17	18	5
25	15	16	12
35	25	20	8
45	30	26	18

horizontally 2.5 sphere diameters below the center of the upper sphere. The presence of the plate did not affect the value of spark-over voltage at gap spacings 0.2 and 0.4 of the sphere diameter. At a spacing 0.76 of the sphere diameter the spark-over voltage was reduced to 97.5 per cent of the spark-over voltage without the plate.

The recent contributions on the effect of shields on the spark-over voltage are of direct interest. R. Elsner ("Die Eichung einer 100 cm Kugelfunkenstrecke mit Stobspannung," *Elektrotechnische Zeitschrift*, volume 56, December 27, 1935, p. 1405–7) found that a torus-shaped shield of 2 sphere diameters applied to a 100-centimeter vertical sphere gap and spaced 1.5 sphere diameters above the upper (line) sphere increases the spark-over voltage only 2 per cent for a gap spacing 0.75 of the sphere diameter. Using both line and ground shields to control the electrostatic field between a 6.25-centimeter vertical sphere gap, Vanoni and Di Pieri ("Spinterometri a Sfere con Anelli di Guardia," *L'Elettrotecnica*, volume 22, December 25, 1935, p. 834–6) obtained similar results with shields of the same relative dimensions. In both of those investigations the cross sectional diameter of the torus was from 0.20 to 0.25 of the sphere diameter.

Because of the effects of surroundings and other inherent variations, it is desirable to limit the use of the sphere spark-gap to less than 0.75 of the sphere diameter. With this restriction, and provided the proper spacing (about 6 sphere diameters) is maintained to walls and ground, the authors found (conclusion 3) that the same calibration curves can be used for spheres mounted both vertically and horizontally. This conclusion comprises one of the important contributions of the paper. In this connection Dattan ("Zur Eichung von Kugelfunkenstrecken bei Stobspannungen und Normalfrequenz," *Elektrotechnische Zeitschrift*, volume 57, April 2, 1936, p. 377–81 and April 9, 1936, p. 412–14; tables 6, 7, and 8) and presents 50-cycle and negative-impulse voltage (0.5 x 50 microsecond wave) data, which can be considered to be in practically good agreement with the corresponding proposed new AIEE standard. Wider departures appear between the positive impulse data. Dattan used sphere gaps horizontally mounted, and the distance from spheres to ground was equal to from 3 to 4 times the sphere diameter at least.

For both types of mounting a spacing to ground of about 5 or 6 diameters should be satisfactory; however, for sphere gaps having diameters greater than 100 centimeters in which cases vertical mounting is generally used, a spacing of 5 sphere diameters to ground planes is not always feasible. For larger spheres half of this spacing to ground planes should be permissible and perhaps the sphere gap should then be restricted to 0.5 diameter spacing.

A second important point, presented in figure 2 and figure 3 of the paper is the deviation in voltage measured from the average value or curve. The total spread of the measurements of figure 2, expressed in per cent of the average voltage, is given approximately as follows:

Spacing in per cent of sphere diameter	10	20	30	40	60	80	100
Spread, in per cent	3	2	5	4	3	4	6

If the widely scattered data were neglected however, the values of the spread would be reduced about 1 per cent. Dattan presented a deviation curve for the 50-centimeter sphere which gives values of about the same magnitude.

A comparison of data obtained with a 50-centimeter sphere gap at Sharon is of interest. One set of these data represents 2 calibrations taken about 2 months apart with a negative impulse voltage of a $1^1/_2$ x 40 microsecond wave. The voltage was measured by means of a resistance divider and cathode-ray oscillograph. Each point represented the average of 2 consecutive measurements taken within 5 or 10 minutes. These 2 consecutive values seldom varied more than 1 per cent. With the negative impulse data 60-cycle calibration data obtained about a year later were included. The 50-centimeter sphere gap was at that time calibrated against a 100-centimeter sphere gap as a reference standard using the proposed new AIEE calibration curve. It is of interest to note that the average 60-cycle calibration for the 50-centimeter sphere gap falls in line with the dotted triangles shown in figure 2 of the paper.

The negative-impulse and 60-cycle data represent about 30 average points and well over 100 points when considered on the basis of single measurements. The spread values of these data are:

Spacing in per cent of sphere diameter	20	30	40	60	80
Spread, in per cent	2	2	3	2.5	4

The 2 foregoing tabulations and other experiences indicate that a variation in the measured voltage greater than ±2 per cent is not to be expected. All this assumes some conditioning of the sphere gap, a reasonable number of tests from which the measurements are averaged, and the use of the sphere spark-gap within its proper limits.

The third important point covered in the paper is the question of fundamental methods of measurement. The authors are to be commended for the development of the sparkless sphere gap voltmeter, which essentially provides a fundamental means of measuring the effective values of voltage at industrial frequencies. The wave form of the voltage must be recorded to convert the effective value into the corresponding crest voltage. The simplicity and convenience of the sphere spark gap also should be recognized. In measuring impulse voltages another fundamental method is essential, and in such measurements the resistance-divider cathode-ray-oscillograph method has found wide application both in the United States and abroad.

Undoubtedly the combination of these fundamental methods and of other equally useful methods ("Calibration of the Sphere Gap," J. R. Meador, ELECTRICAL ENGINEERING, volume 53, June 1934, p. 942–8) have made possible the present comprehensive understanding of sphere spark gaps and the extensive data on them.

R. W. Sorensen and Simon Ramo: If the sparkless sphere gap voltmeter is to be compared with other means of measuring high voltages, careful consideration m be given to the problem involved and laboratory conditions governing the m urements being made.

The sparkless gap has been shown in and the first paper, together with its cussions, to have advantages and disadv tages as compared with the sparking depending on the particular electrical cuit, the laboratory, and the accuracy sired. The authors believe the spark gap has great value as a primary stand They have found it entirely free from t fruitful source of erratic readings pres in the spark gap; namely, the variable uncontrollable dielectric strength of J. B. Whitehead's statement that the spa less gap is not free from the influence o surroundings is correct, but the authors thoroughly satisfied that the sparkless v meter, when skillfully used, can be set u the influence of adjacent objects is small subject to computation or perhaps eve experimental determinations. Thus, influence, though undesirable, does eliminate the sparkless sphere gap voltm from the primary standard class. Pla were considered by the authors before c structing the sphere gap meter but t could see no way of avoiding the huge required by such a meter to give precise sults in the higher voltage ranges.

Table III of this discussion gives equi lent spacings of the 50-centimeter and 1 centimeter sphere gaps taken from sparking curves obtained by the author requested in the discussions. The 50-c timeter data are for the standard sphere a recommended mounting with the vertical and about 5 diameters above floor; the 100-centimeter data were

Table III

Voltage, Kv Crest	50-Centimeter Spheres Gap Spacing, Centimeters	100-Centimeter Spheres Gap Spacing, Centimeters
100	4	4
200	7.5	7.5
300	12	12
400	17	16
500	24	20
600	33.5	24
700	44	30

tained with cast aluminum spheres moun in a horizontal position about 5 diamet above the floor.

In discussing the spread of points tained with sparking spheres, attent should be called again to the fact that spreads shown in figure 2 of the paper those observed over a period of a months and not the spread observed on one particular set of readings. It has b stated that the wide spread is not limi to larger spacings and this statement is agreement with the authors' data. accuracy of determining the gap spacin less for small spacings, because it is diffic to set off accurately a few centimeters o 50 or 100 centimeter gap. This condit contributes to the few per cent spread tained.

The authors concur in the opinion t more attention should be given to the ca

of the spread in spark-over voltage of the sphere gap if it is to remain the standard for high-voltage measurement. It may be that learning to control the factors causing the variation will be much more difficult than developing other means more suited to precision work in both the power-frequency and impulse-voltage field. The spark gap with its present faults is sufficiently accurate for most commercial work.

Fields and Charges About a Conductor

Discussion and author's closure of a paper by W. G. Hoover published in the May 1936 issue, pages 448–54, and presented for oral discussion at the electrophysics session of the summer convention, Pasadena, Calif., June 23, 1936.

R. R. Benedict (University of Wisconsin, Madison): The work on space charges in corona is fundamental to the problem, and will be of especial importance in the further development of theories of corona. As mentioned in the paper, Holm has developed a theory of corona loss based on considerations of the movement of space charge Although Holm's theory involves some oversimplified assumptions regarding the space charge, the results calculated by the theory are in fair agreement with experimental results. For example, the condition for the peak of the voltage wave is simplified to an equivalent representation by means of 2 charge distributions; one on the wire itself, and the other distributed in space in a cylindrical shell concentric with the wire. The calculated radius of this space-charge cylinder is 10.7 centimeters for the conditions of Hoover's figure 7. This agrees reasonably well with the author's results, since the extreme excursion of the main body of space charge would be expected to be greater than the average radius calculated according to Holm's theory. That the calculated value of the space charge is approximately 3 times the quantity of charge on the wire at the peak of the voltage wave is an interesting additional fact.

In conclusion, it would seem that a very valuable bit of information would be an oscillogram showing the waveform of the total current collected by the cylinder in the a-c tests. If the author has this information, it should be worth including in his discussion.

J. D. Cobine (Harvard Graduate School of Engineering, Cambridge, Mass.): The author has employed successfully a unique type of probe in studying a corona cylinder. This work is instructive and valuable in adding to the information concerning the processes occurring in corona discharges.

The user of a probe of that type might well recognize that the field may be distorted, especially near the central wire. In an effort to check this point, the writer evaluated the constant k of the relationship $dV/dr = k/r$ at 20 centimeters for curve F of figure 3 of the paper. The value thus found indicated that the experimental value for the point nearest the wire was about 5 per cent too high. This is quite good; however, the author's omission of the theoretical curves for no corona is regrettable. An additional source of error to be expected from the use of this probe in the presence of drifting ions is that which might be caused by the turbulence set up by the rotating wires, especially where the ion-drift velocity is about the same as the linear velocity of the wires; moreover, there is always some uncertainty in interpreting the current to a probe in determining the potential of the probe region if the probe were not there.

In the experimental procedure described in the paper were precautions taken to reduce to a minimum the vibration of the conductor caused by corona?

Derived curves of the net space charge as a function of the distance from the wire for the d-c tests would have been interesting, especially if compared with a similar curve for a-c corona. The data of table I of the paper indicates a maximum of charge density near the positive wire, although the negative wire's space charge seems to be constant in that region. Possibly the space charge of the latter case is much closer to the wire and the probe could not be moved close enough to bring out the maximum.

Attention is called to the fact that the "field wave" at 20 centimeters from the wire figure 7 of the paper, the positive and negative halves of the cycle are different in form. Since oscillograms of corona current have indicated a similar dissymmetry, it would be instructive to compare the 2 quantities under the same conditions.

Victor Siegfried (Worcester Polytechnic Institute, Worcester, Mass): This paper is a valuable contribution toward understanding the field and charge about conductors in gaseous dielectrics. The scope of the material included is such as to collect and generalize the present information on space charge, and to add new data comparing charges existing under alternating and continuous potentials. The author is to be commended for the fine work done in accumulating so much information pertinent to the general problem.

Practical application of material of this character may be made to engineering problems encountered in the use of electrical cable with gaseous dielectrics under high pressures and to problems of corona loss from high-voltage conductors in air. Although the author has gone far in this work, the investigation ceases at the threshold of the latter problem, the generalized explanation of the mechanism of corona loss. By approaching the investigation of corona loss from this point of view, correlation of engineering measurements and space-charge data should lead to a more advanced knowledge and an ability to predict corona loss on conductors in air.

The most significant of Hoover's results in this respect are those of the unidirectional drift of charged ions to the cylinder, as shown in his figures 9 and 11. Those curves show the rectifying effect of corona, and indicate a reversal in sign of the net charge reaching the cylinder as the voltage applied to the center electrode is increased. This phenomenon agrees with results of Carroll and Lusignan ("The Space Charge That Surrounds a Conductor in Corona," AIEE TRANSACTIONS, volume 47, 1928, p. 50–7) and with observations made by the writer at the Ryan Laboratory. The striking thing about these observations is that the reversal in sign of the residual charge is accompanied by a "break" in the shape of the corona-loss curve, indicating a change in the rate of increase of loss with increasing voltage.

The correlation of the residual charge with the break in the curve was observed by the writer in making a study of corona loss, in which the conductor was suspended within a steel tank 5 feet in diameter. The set-up and typical curves of corona loss from the conductor have been described by L. Hegy and G. W. Dunlap ("Corona Loss vs. Atmospheric Conditions," ELECTRICAL ENGINEERING (AIEE TRANSACTIONS), volume 53, February 1934, p. 272–3). The device for measuring the charge reaching the cylinder was similar to that used by Hoover. With a stranded conductor of 0.240 inch maximum diameter, the corona-loss curve shows this break at approximately 50 kilovolts. At the same voltage, the number of negative ions reaching the cylinder wall was exactly equal to that of the positive ions. At a lower voltage the positive ions predominated and at a higher voltage the negative were more numerous.

This break is most noticeable where smooth, polished conductors are used. The rougher stranded cables show less departure from the portion of the curve lying well above the corona starting voltage. The sign of the ion which predominates in reaching the outer edge of the field depends upon the type and surface conditions of the conductor used. For smooth conductors, the positive ions predominate at the higher voltages, while for the rough or stranded conductors, the negative ions appear to be more profuse. From this it would seem that the sign of the residual charge is intimately connected with a balance between the rate of production of the ions leaving the conductor and the relative mobilities of those ions moving in the field to the grounded plate or cylinder. Simultaneous measurements of corona loss and of the charge existing in the space surrounding the conductor or of the charge reaching the cylinder thus offer possibilities for further extension of the understanding of corona loss.

W. G. Hoover: Referring to the discussion of J. D. Cobine, it may be stated that the several possible sources of error mentioned have been recognized, but the results indicate none of them to be of serious consequence for d-c fields. For alternating fields errors were minimized by maintaining the exploring device at a potential approximately corresponding to its position in the field.

The calculations of R. R. Benedict on the basis of Holm's theory are interesting and serve as a fairly satisfactory check between theory and experiment. Unfortunately, oscillograms of the current to the cylinder are not at present available for the apparatus described in the paper.

The author appreciates Victor Siegfried's comments regarding the residual charge

arriving at the cylinder. It should be emphasized that, for the particular case described, charge of only one sign arrived at the cylinder at any one voltage. This indicates that the cylinder was beyond the region in which ions of each sign might be collected by an electrode.

The Magnetic Vector Potential

Discussion and author's closure of a paper by J. W. McRae published in the May 1936 issue, pages 534–42, and presented for oral discussion at the electrophysics session of the summer convention, Pasadena, Calif., June 23, 1936.

J. F. H. Douglas (Marquette University, Milwaukee, Wis.): The magnetic field in conductors, like all field problems, may be attacked by analytical, graphical, and experimental methods. This paper is a valuable contribution to the analytical side of the subject. In the integration of the partial differential equations of fields, the the difficulties imposed by boundary conditions and by the arbitrary functions often lead to devices that reduce the partial differential equations to ordinary differential equations; for example, the use of Ampere's equation, and the use of Fourier series. The problem of end-connection reactance probably will be solved in this way.

The writer cannot let this opportunity pass without calling attention to the graphical approach to this problem by Lehman, or the experimental approach by Mullner. A fascinating truth is that every 2-dimensional vortical field has a conjugate field in which the curl is replaced by divergence; there is an interchange insulation and equipotentials on the boundary, and an interchange of resistivities in the medium itself. It is this principle that makes an experimental approach possible.

It is probable that the solution of problems on the fields in conductors, like those in air and insulation, require for their solution the labors of those skilled in graphical, experimental, and analytical methods. In other words it is probable that parts of the various problems will yield more easily sometimes to one and sometimes to the other mode of attack.

J. W. McRae: J. F. H. Douglas has properly emphasized the necessity for employing several different methods in the attack on practical field problems.

Even the very useful experimental methods he mentions usually cannot be applied without a preliminary mathematical analysis to determine the validity of the proposed experimental procedure. Many new problems might thus be solved by suitable cooperation between analytical and experimental workers, the analytical work providing a basis and guidance for the experimental work. In any investigation of this kind it is necessary to keep clearly in mind the fundamental properties of the magnetic field. Familiarity with the Maxwell field equations, which provide the most compact statement of these properties, therefore is desirable.

Advanced students of electrical engineering should be encouraged to give more attention to these equations and their use in the future, and to devote further study to the use of the magnetic vector potential in relation to practical problems and in the development of electrical engineering theory.

Transformer Circuit Impedance Calculations

Discussion and authors' closure of a paper by A. N. Garin and K. K. Paluev published in the June 1936 issue, pages 717–30, and presented for oral discussion at the electrophysics session of the summer convention, Pasadena, Calif., June 23, 1936.

Gabriel Kron (General Electric Company, Schenectady, N. Y.): The authors are to be congratulated upon presenting a procedure that is not a special method of attack, of interest only to specialists in multiwinding transformers, but is a general method of analysis applicable to any stationary bilateral network. As is usually the case, the physics involved in a universal procedure is always simple and clear-cut. A generalization of the equations given so that they would be applicable to unilateral networks, such as rotating machines or vacuum tube circuits would be interesting.

In recent issues of Electrical Engineering several articles have appeared in which the necessity arose to use summation signs applying to indices varying between different limits. For instance, in appendix IV of this paper the index k is summed up between the limits 1 and s, $s+1$ and n, and 1 and n at various places. There is an accepted mathematical usage which is followed under similar circumstances. Detailed description can be found in any book on differential geometry.

The notation consists of using 3 different sets of indices, each set capable of varying between particular limits. For instance let

$a, b, c \ldots$ assume all values from 1 to s
$p, q, r \ldots$ assume all values from $s+1$ to n
$\alpha, \beta, \gamma \ldots$ assume all values from 1 to n

the indices $a, p, \alpha \ldots$ assuming all values in succession within their respective limits instead of any one. With this set of indices it is not necessary to indicate the limit where the summation starts and ends. Using the equation previous to $D6$ in appendix IV as an example,

$$\sum_{l=1}^{l=s}\left[2\sum_{k=1}^{k=s}\check{Z}_{kl}I_k'+2\sum_{k=s+1}^{k=n}\check{Z}_{kl}I_k'\right]= 2\sum_{l=1}^{l=s}\sum_{k-1}^{k-n}\check{Z}_{kl}I_k'$$

it can be written with the 3 types of indices as

$$2\sum_a \check{Z}_{ab}I_a'+2\sum_p \check{Z}_{pb}I_p' = 2\sum_\alpha \check{Z}_{\alpha b}I_\alpha'$$

Accepting also the customary conventi[on] that 2 identical indices in a term are alwa[ys] to be summed up, the summation sign m[ay] be omitted entirely, and the equation m[ay] be written

$$2\check{Z}_{ab}I_a' + 2\check{Z}_{pb}I_p' = 2\check{Z}_{\alpha b}I_\alpha$$

All the equations in the appendices of t[he] paper and in other papers can be thus wr[it]ten without the summation signs, improvi[ng] their readability.

Incidentally, the indicated notation has far deeper significance than a mere gain [in] readability. Under certain circumstanc[es] the new symbols Z_{ab}, Z_{pb}, and $Z_{\alpha b}$ becom[e] new mathematical entities and one c[an] reason in terms of them as if they we[re] scalars. With the notation used in t[he] paper the only possible mathematical enti[ty] representing a set of impedances is $\check{Z}$ unless an expression such as $\sum_{l=s+1}^{l=n}\sum_{k=1}^{k=n}\check{Z}$ is considered as a different entity fro[m]

$$\sum_{l=1}^{l=s+1}\sum_{k=s+1}^{k=n}\check{Z}_{kl}$$

A. N. Garin and K. K. Paluev: In pr[e]paring this paper the authors were und[er] constant temptation to generalize, to di[s]cuss circuits in general instead of tran[s]former circuits. Gabriel Kron's remar[ks] show that his interest lies in the sam[e] direction. It was decided, however, th[at] the paper would be of greater practic[al] usefulness if its content was limited to [a] treatment of the rather narrow and speci[al] field of transformer impedance problem[s] as complete as possible, with just enoug[h] consideration given to the properties of t[he] general stationary bilateral network to for[m] a background and a foundation.

The notation adopted in the mathematic[al] part of the paper was selected with the id[ea] of making the paper serviceable to a larg[e] group of readers. Opinions may differ [as] to the relative elegance of various metho[ds] of expressing algebraically a given relati[on] but unfortunately only too often the mo[re] elegant a mathematical expression, the le[ss] intelligible it is to the uninitiated.

Quite aside from Kron's remarks t[he] following comments may be of gener[al] interest.

Referring to the "circuit" methods [of] impedance calculation presented in t[he] paper, the familiar method of calculati[ng] self-inductances by geometrical mean d[is]tances, may be considered an example [of] "circuit" methods applied to a "fiel[d"] problem. In the geometrical-mean-distan[ce] method actual windings are subdivided in[to] an infinite number of ideally simple pa[rts] (parallel filaments) for which rigorous se[lf-] and mutual-inductance formulas are ava[il]able, and then the self-inductance of a co[m]plete winding is obtained by a proce[ss] equivalent to the substitution of co[m]ponent self- and mutual-inductances into "circuit" formula.

Referring to the equilibrium conditio[n] of a network, current flow in a network [of] several degrees of freedom is given by equ[a]tion 1 of the paper, which can also be [re]written as equations 2 and 3. Append[ix] IV gives the algebraic proof of these re[-]

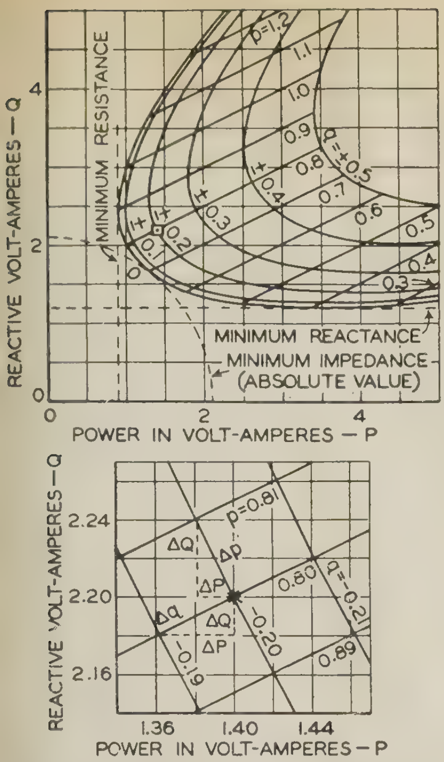


Fig. 1. Graphical representation of the current flow relations for a simple circuit of 2 degrees of freedom

*—Actual current flow

tions for the general case of a network of any number of degrees of freedom. Figure 1 of this discussion gives a graphical illustration of these relations for the simplest circuit of 2 degrees of freedom.

The circuit consists of 2 impedances

$$Z_1 = 1 + j3$$

and

$$Z_2 = 9 + j2$$

connected in parallel. To simplify the calculation, mutual impedance is assumed to be absent. Unit current flows through the 2 parallel-connected impedances. If the current through Z_1 is called

$$p + jq$$

the current through Z_2 is given by the current law of Kirchhoff as

$$(1 - p) - jq$$

the resultant power and reactive input to the circuit become

$$P + jQ = Z_1\,[p^2 + q^2] + Z_2\,[(1 - p)^2 + q^2]$$

Therefore the power and reactive input to the circuit depend on the values of p and q, that is, on the manner in which the unit input current divides between the 2 parallel impedances.

The relation between the current division in the 2 parallel impedances and the resultant power and reactive input to the circuit is shown by families of curves in the center of the figure. For any value of p, equal and opposite values of q give the same value of $P + jQ$. The constant q curves have been marked, therefore, with plus and minus signs. The lowest possible power input to the circuit is obtained by the current flow specified by

$$p + jq = 0.9 + j0$$

giving a power and reactive input of

$$P + jQ = 0.90 + j2.45$$

The lowest possible reactive input to the circuit is obtained by the current flow specified by

$$p + jq = 0.4 + j0$$

giving a power and reactive input of

$$P + jQ = 3.40 + j1.20$$

The foregoing 2 conditions may be called the conditions of "minimum resistance" and of "minimum reactance." One more minimum condition is of interest: the condition of "minimum impedance," for which only the absolute value of impedance is considered.

The current flow giving "minimum impedance" is specified by

$$p + jq = 0.7 + j0$$

giving a power and reactive input of

$$P + jQ = 1.30 + j1.65$$

The actual current flow in the 2 parallel impedances does not satisfy any of the 3 minimum conditions. It is specified by

$$p + jq = 0.8 - j0.2$$

giving a power and reactive input of

$$P + jQ = 1.40 + j2.20$$

In the center of the upper part of the figure this point is enclosed by a little square, which is reproduced on larger scale at the bottom of the figure. The point denoting the actual current flow on the chart is unique, for at this point only the intersecting families of p-constant and q-constant curves form perfect squares, while everywhere else on the chart they form somewhat distorted parallelograms.

To anyone familiar with the theory of dielectric and magnetic field plotting, the theory of conformal representation, or the theory of functions of complex variables, the point representing the actual current flow may be specified by the equations

$$\frac{\partial P}{\partial p} - \frac{\partial Q}{\partial q} = 0$$

$$\frac{\partial P}{\partial q} + \frac{\partial Q}{\partial p} = 0$$

The correctness of these equations may be verified by substituting the small increments ΔP, ΔQ, Δp, and Δq, marked on the large scale chart at the bottom of the figure. For this particular example q must have a negative value.

Referring to the procedure to be followed in calculating the impedance of transformer circuits, item 8 in the body of the paper gives the rules for evaluating the yet undetermined values of currents in circuits of several degrees of freedom. Some of the currents may be found to have negative values, showing that in such windings the actual current flow is opposite to that assumed under item 4. The minus signs must not be overlooked in calculating the impedance in accordance with instructions given under item 6.

Referring to equation 37 of the paper, it will be observed that for zero boost connection

$$p = 0$$

$$X_s = \infty$$

so that the term

$$\left(\frac{p}{1 + p}\right)^2 X_s$$

becomes indefinite.

This indefinite expression is readily evaluated as follows: Let the maximum boost connection be given by $p = r$ and the reactance of the series transformer at the excitation corresponding to the maximum boost connection be given by X_s'.

Then

$$X_s = \left(\frac{r}{p}\right)^2 X_s'$$

and the indefinite term is evaluated as

$$\left[\left(\frac{r}{1 + p}\right)^2 X_s'\right]_{p \to 0} = r^2 X_s'$$

Similar observations apply to equation 38.

Fields Caused by Remote Thunderstorms

Discussion and author's closure of a paper by K. E. Gould published in the June 1936 issue, pages 575–82, and presented for oral discussion at the electrophysics session of the summer convention, Pasadena, Calif., June 23, 1936.

J. F. H. Douglas (Marquette University, Milwaukee, Wis.): The low ratio of horizontal to vertical component shown in this paper is surprising at first glance. In the calculation of zero-sequence reactance of lines, one is made acquainted with Carson's work on image conductors, sometimes 2,000 feet below the earth's surface, due to high earth resistance. The writer should like to inquire, therefore, whether the values in the paper are consistent with those of Carson's theory.

K. E. Gould (Bell Telephone Laboratories, Inc., New York, N. Y.): Derivation of the ratio of the maximum horizontal electric intensity to the maximum vertical electric intensity for a nonperiodic disturbance, such as the field caused by a remote thunderstorm, from the analogous quasi-tilt angle for steady-state single-frequency wave is difficult. The average ratio (231) given in the paper may be shown to be of the order of magnitude to be expected from wave propagation theory.

Figure 20, appendix 3, of a paper by Bailey, Dean, and Wintringham (reference 1 of the paper) shows the relation between quasi-tilt angle and earth resistivity for several frequencies of from 20 kilocycles to 60 kilocycles. The frequencies of most im-

portance in fields due to remote thunderstorms are those below 20 kilocycles, at least in the sense that the insertion of a low-pass filter in the measuring equipment, with cut-off at 20 kilocycles, would not affect greatly the maximum electric intensity recorded for most disturbances. At 20 kilocycles, for example, a value of 1/231 for the ratio of horizontal component to vertical component of the electric field corresponds to an earth resistivity of about 1,500 ohms per centimeter cube; at lower frequencies this ratio corresponds to an earth resistivity higher than 1,500 ohms per centimeter cube. Although independent data are not available regarding the earth resistivity at the test location, the range indicated—somewhat above 1,500 ohms per centimeter cube—seems reasonable in view of the type of earth structure involved.

Power Transformers With Concentric Windings

Discussion and author's closure of a paper by K. K. Paluev published in the June 1936 issue, pages 649–59, and presented for oral discussion at the transformer session of the summer convention, Pasadena, Calif., June 25, 1936.

F. J. Vogel (Westinghouse Electric & Manufacturing Company, Sharon, Pa.): The paper indicates the complexity of the modern power transformer, and in particular the high-voltage concentric-coil shielded transformer, for as many as 1,300 calculations may be required. This is ambiguous, because the nature of what may be termed a calculation will vary, but it is of interest that other forms of construction, being simpler, require less than 1,300 calculations by the design engineer. This is a real advantage in reducing the risk in building large units.

Table I of the paper is of great interest in showing the great improvements in materials and design methods; however, additional attention should be given to some of these. One improvement cited is the item that the "impulse insulation strength—is co-ordinated with the bushing impulse strength." This statement is not necessarily the case, in view of the test procedure proposed by the AIEE subcommittee, because that procedure by neglecting the effects of negative waves does not actually demonstrate co-ordination. The writer has long been in favor of using both negative and positive waves and increasing the severity of the tests to be equivalent to the 60-cycle one-minute tests now specified.

Another item is that of impulse-voltage distribution. A uniform distribution is not always obtained on all types of transformers; yet they can be co-ordinated and may withstand even the severest impulse tests. It is not therefore a prerequisite to a satisfactory design, and in fact obtaining it with shields may be a disadvantage. These statements refer to small transformers of the core type, which can be easily made surge-protected without shields, with very little hazard, and most economically. The idea could be advanced that these same principles might result in the simplest and least hazardous structure even for the largest power units of the core type.

The task of matching reactances in transformers dissimilar in design to his own often confronts the designer, but that the solution is easy perhaps is not common knowledge. Because of the relative ease with which it can be accomplished it has not been the subject of much question or discussion. Any calculation may be in error however, and to insure absolute accuracy it is often desirable to check the calculations with a model for large 3- or 4-winding units. This is the practice of at least one manufacturer, and the practice assures correct results in the completed transformer

The most important advance cited by the author is the elimination of corona on both 60-cycle and impulse tests. This truly eliminates the principle causes of insulation deterioration. Any appreciable disturbance caused by gas bubbles on the surface of the oil is an evidence of corona and is objectionable in the modern transformer. It is undoubtedly the practice of many manufacturers to observe the surface of the oil during dielectric tests and to design so that there is no disturbance during such tests.

P. L. Bellaschi (Westinghouse Electric & Manufacturing Company, Sharon, Pa.): Examined in the light of wide experience in the design and impulse testing of surgeproof transformers of practically all types, the author's generalization on behalf of the shielded core type of transformer can be misleading. Typical practical examples will best illustrate the point.

The 4,500 kva, 144-kv to 12-kv 25-cycle transformers installed by the Pennsylvania Railroad about 4 years ago in connection with the New York-Washington line electrification required a 4 per cent impedance. The Westinghouse company supplied surgeproof transformers of the shell type, having good uniform distribution of voltage stresses and meeting all performance specifications ("Pennsylvania Railroad Substation Transformers," S. S. Cook and W. M. Dann. *Electric Journal*, volume 29, July 1932, p. 341–4). The General Electric Company supplied transformers of the core type which, because of the low impedance requirement, have of necessity a low-high-low winding arrangement. Shielding transformers of the core type is not feasible, which indicates that in transformer design each field application presents inherently its own problems and therefore no rule can safely be set down beforehand as to what constitutes the best type of design.

European practice in transformer design gives another example. Abroad, the core type of transformer is common, yet the simple conventional design is adhered to. Foreign designers have preferred the surgeproof insulation method of construction, judged from the fact that impulse testing is now becoming a common practice (see for example, "Standardization of Impulse-Voltage Testing," T. E. Allibone and F. R. Perry, *Journal of the Institution of Electrical Engineers*, volume 78, March 1936, p. 257–72).

Another example illustrating that generalization can be misleading is shown by the following: Upon development of the methods of surgeproof design 5 years ago
these were applied not only to the shell t
of transformer but to the core type as w
In fact, in 1931 a 132 to 2.3-kv, 1,000-
single-phase core type of transformer
built at Sharon. The design is essenti
conventional in all respects, except
insulation incorporates the surgeproof c
struction. Judged purely from the app
ent impulse voltage distribution, the des
may not appear the best, for the en
voltage of a $1^1/_2$x40 microsecond wave
pears initially practically across $^1/_{10}$ of
line end winding; besides, the voltage
ground at points of the winding 20 per c
and 80 per cent from the grounded end
tains respectively 80 per cent and 120
cent of the applied crest line voltage.
that transformer has been in success
use at the Sharon high-voltage laborat
for the past 5 years. It has been subjec
to not less than 1,000 impulses, ranging f
200 to 1,000 kv in voltage amplitude;
wave form and duration of these impu
ranged from very steep impulses chopped
the front rising at 1,000 kv per microsec
to steep-front surges enduring some 1,
microseconds on the tail. Power volt
excitation was on the transformer in m
of these tests.

Other examples could be cited, but
3 just given illustrate the prime importa
of the surgeproof design irrespective of
construction forms. As to the form of c
struction, the surgeproof shell type of tra
former has fully proved its wide applica
ity with respect to power and voltage r
ings and in many other ways. At the sa
time the surgeproof core type of design
its applications. In fact, judging from
total number of transformers, as many
the core type as of the shell type have b
built and impulse tested by the Westi
house Company. This broad experience
building transformers of both the core
shell types has been particularly valuabl
predetermining the real theoretical
practical advantages of each type and
thus providing the best design suitable
each particular application.

K. K. Paluev: In 3 instances the answ
to P. L. Bellaschi's and F. J. Vogel's disc
sions of the paper are somewhat complica
by what appear to be conflicting opini
expressed either by different members
their engineering group or by the same in
vidual.

First, Bellaschi, in his own paper ("
pulse Characteristics of Core-Type Tra
formers," *L'Elettrotecnica*, volume 21, Ja
ary 5, 1934, p. 1–14) to which he refers
his discussion, expresses confidence that
necessary impulse strength can be secure
core type transformers without shi
(which he calls "simple normal type")
to 138 kv, and even above, in the follow
words:

Comportamento Dei Trasformatori a
clei Nei Riguardi Delle Onde d'Impu
P. L. Bellaschi. . . . "Per quanto riguarda la prac
attualmente seguita dai construttori americ
tutti sono d'accordo sulla opportunit di mante
la costruzione normale per trasformatori a nu
fino a 115 kv per quanto, come si e gia avvertit
prove di cui e oggetto il presente articolo sugg
cano la probabile convenienca della stessa semp
struttura fino a 138 kv senza escludere che il li
possa subire ulteriori incrementi, sul che, alm
dal punto di vista tecnico sebrano non sus
dubbi fondati."

"In regard to the present practice followed by he Americans, everybody agrees to maintain the ormal type of construction of the core type for oltages up to 115 kv. As has been mentioned in he present article, tests have shown that probably his simple normal type of construction can be mainained up to 138 kv. From the technical point of iew, at least, there seems to be no foundation for loubt that this limit can be raised even further."

However, in the paper of his colleagues Cooke and Dann to which he also refers in his discussion, are found statements to the effect that in accordance with Bellaschi's calculations the necessary impulse strength could not be secured in 4,500-kva 132-kv transformers of the core type if all other performance requirements are to be satisfied.

Incidentally, the General Electric Company furnished many transformers of the core type, of identical rating, to the Pennsylvania Railroad Company in 1931–32, which were of proper impulse strength, and which also met all other performance specifications.

Bellaschi's statement that shielding of low-reactance 4,500-kva 144-kv (132-kv operating) to 12-kv 25-cycle transformers having a "low-high-low" winding arrangement "is not feasible," suggests that he may not be familiar with all available methods of shielding.

Second, Vogel, in the first paragraph of his discussion expresses the opinion that the many calculations deemed essential for the proper design of a modern transformer is principally due to radically greater complications of the structure of General Electric transformers in comparison with those of others. Yet, in his and James' paper presented at the same AIEE convention describing their transformer (page 443 of the May 1936 issue of ELECTRICAL ENGINEERING) the following statement is found:

"The authors believe that it may be stated that although the power transformer is often considered a simple piece of static apparatus and hence worthy of relatively little note or attention, it is actually an exceedingly complicated piece of apparatus with design features embracing nearly all branches of engineering."

The number of calculations made during design of a structure does not depend upon its complexity alone, but also on the precision and thoroughness with which the design is carried out.

Third, it is still more difficult to accept Vogel's opinion of relative simplicity of their transformer, for he states in his discussion that the calculation of some of the reactances of some of the transformers is impracticable and therefore models must be built to determine the reactances experimentally. The fact that the General Electric Company does not use models and finds that analytical formulas give entirely satisfactory results for every kind of transformer so far built by that company seems to suggest that either the arrangements and configurations of the windings described in the paper are less complicated, or that company's interest in reactance problems has been greater. The "circuit method" of reactance calculation described in the June 1936 issue of ELECTRICAL ENGINEERING by Garin and the author makes use of models unnecessary.

The other questions raised by Bellaschi and Vogel will be answered here.

The definition of the term "surgeproof" given in the second paragraph of Bellaschi's discussion states that the elements of transformer insulations—turn, coil, and major insulation—should be so selected that they all have the same ratio of their breakdown strengths to the calculated maximum transient voltage stresses imposed by a surge. This definition does not imply any particular level of insulation strength to be secured. For this reason some term other than "surgeproof" would be more suitable. For the same reason that it would be inadequate to call a structure "fireproof" just because in case of a fire the temperature of all its parts would bear the same ratio to their respective melting or burning points.

There is a more important practical objection to designing transformers in accordance with the principle stipulated by Bellaschi's definition, however. Since accuracy of calculation of transient voltage stresses in a transformer without a shield is highest for the major insulation and lowest for the turn insulation, it is advisable to establish a proper design safety factor for the major insulation, a materially higher safety factor for the coil insulation, and still a higher factor for the turn insulation, instead of attempting to have them all the same.

The wisdom of such a policy may be illustrated by the fact that formulas published in Bellaschi's papers, referred to in his discussion, are approximate formulas based on approximate basic formulas published by Blume and Boyajian ("Abnormal Voltages Within Transformers," AIEE TRANSACTIONS volume 38, 1919, pages 577–614). The formulas included in that paper were intended only for the presentation of the general theory of transient voltage phenomena. More accurate methods of calculation have been developed for design purposes. Values obtained by the more accurate methods often are from 4 to 8 times as high as those given by the approximate formulas just mentioned. The error increases with the increase of coil width.

The internal co-ordination of strength of transformer insulation elements with transient voltage stresses is an old principle. It was reduced to practice many years ago. In a paper presented by the author in 1929 ("Effect of Transient Voltages on Power Transformer Design," AIEE TRANSACTIONS, volume 48, July 1929, p. 681–701) the following statement regarding such transformers is found: "In view of the behavior of transformers under transient voltage excitation, and the high magnitude of these voltages, the construction and the distribution of the insulation throughout the transformer winding is dictated, not by stresses set up in the transformer under the standard AIEE potential test, of low frequency, but by the law of distribution of voltages in the winding caused by various transients actually observed in service." The foregoing statement is illustrated by a table of insulation strength of turn, coil, and to-ground that had been used in an unshielded 220-kv General Electric transformer of the core type for many years previous to 1929.

Bellaschi, Vogel, and others contrast a shielded winding with a winding having insulation barriers so constructed that they progressively envelop a larger and larger portion of the total winding and thereby increase the dielectric strength of the structure. They apparently feel that this method of distribution of insulation is a unique feature of one particular type of transformer structure and is only 5 years old. Such insulation structure has been used for years in core type transformers, first without shields and later with shields, and therefore these 2 structural features of transformer windings should not be compared.

The European practice referred to by Bellaschi, cannot be used as criterion for practice in the United States, because of the difference in operating condition and standards of service. The author was pleased to learn a few years ago, however, that shielding of transformer windings had begun to take root abroad.

The paragraph before the last in Mr. Bellaschi's discussion should have been addressed to Cooke and Dann instead of to me, for in their paper mentioned previously they stated that 132-kv transformers of the core type cannot be built strong enough to stand impulse tests. The calculations and tests of the General Electric Company simply indicate that, by proper shielding, safety factors much higher than those possible without shielding may be secured throughout a winding. Furthermore, turn, coil, and major-insulation stresses in a shielded winding can be determined with much higher accuracy than is otherwise possible. It may be of interest to observe that the volume of insulation used in our shielded transformers is the same or greater than in the former type.

The second paragraph of Vogel's discussion must be addressed to the AIEE subcommittee, for nowhere in the paper was a statement made that AIEE impulse tests demonstrate co-ordination of transformer insulation with bushing strength. On the contrary, the author stated in 1933, (AIEE TRANSACTIONS, June 1933, p. 458), when proposed AIEE impulse test rules were submitted by the committee, that he did not believe the means then proposed and now used for detection of damage that might be produced by impulse test are sufficiently sensitive or adequate for the purpose. Any person familiar with the technique of the test and the theoretical considerations of the phenomena will agree that it is quite possible to damage a transformer by impulse without its being detected by any of the methods available at present (including 60-cycle excitation simultaneously with the test), short of complete disassembly and minute examination.

When Peek first suggested that the AIEE subcommittees should consider the possibility of commercial impulse tests (AIEE TRANSACTIONS, September 1932, p. 616), the writer was entirely in sympathy with his suggestion, but both Peek and the writer called attention to the fact that the committee must also find reliable methods for detection of minute damage, which often may be of pin hole magnitude (AIEE TRANSACTIONS, September 1932, page 618).

Regarding the fact that an asymmetrical gap, like that of a bushing, requires higher negative voltage than positive, and that therefore the AIEE subcommittee should have recommended negative impulse wave tests instead of positive, the following can be said: At the time the subcommittee was trying to standardize on the levels of impulse test voltages, the measurement of impulse tests in volts not being sufficiently reliable; therefore, it was agreed to estab-

lish voltage levels by specifying corresponding lengths of rod gap of a definite construction and an impulse-wave of a definite shape and polarity. If it were desired to have a higher level of impulse tests than that proposed, all that was needed was to agree on correspondingly longer rod-gap settings instead of on the change of polarity of the wave, particularly since ratio of arc-over voltages of 2 polarities varies with length of the gap.

The writer never could find any scientific or engineering basis for establishing the equivalency between impulse and 60-cycle one-minute strength of transformers, or any other apparatus, that Vogel repeatedly recommends. Such an artificial equivalency, based by necessity on tests of insulation samples instead of complete structures, because of innumerable possible variations of the latter, may seriously handicap progress in securing greater impulse strength and greater 60-cycle strength. This is because the nature of impulse and 60-cycle breakdowns, even of a piece of insulation, under some conditions are different and independent of each other. Therefore, different means may be developed to increase these 2 types of insulation strength; furthermore, the levels of strength of these 2 types necessary for successful operation in service are also independent of one another. For further data on this question one may refer to a previous discussion (pages 456, 458, 459, AIEE TRANSACTIONS, June 1933).

Vogel implies that the shield might be a hazard. Such a fear might have been understood at the time shielded windings were introduced into practice and therefore, as an inovation, were handicapped by the pessimistic attitude of those not thoroughly familiar with the safety factors employed. However, after ten years of service and manufacturing experience, with shielded transformers having a total capacity of more than 4,000,000 kva in service and without a single failure in this type of winding, it is difficult to understand why anyone should speak of their hazard.

Power Transformers for 287.5 Kv Service

Discussion of a paper by W. G. James and F. J. Vogel published in the May 1936 issue, pages 438–44, and presented for oral discussion at the transformer session of the summer convention, Pasadena, Calif., June 25, 1936.

P. L. Bellaschi (Westinghouse Electric & Manufacturing Company, Sharon, Pa.): The main features of the power transformers for 287.5 kv service described in the paper exemplify the new technique in design and the improved materials and manufacturing processes that are applied to modern power transformers. Progress in design has been directed toward improved methods in predetermining the factors and stresses characterizing the magnetic, electric, thermal, and mechanical performances. These improved design methods have enabled the designer to provide more economically and adequately the type of transformer design suited for the requirements dictated by field experience and actual tests.

A marked improvement in transformer design during the past 5 years has been made possible through the application of surgeproof insulation. The fundamental significance of the surgeproof design becomes apparent only from a consideration of the relative stresses and strengths throughout the entire insulation structure. The insulation parts in a transformer for convenience are classified as: *a.* the major insulation, that is, insulation from high-voltage winding and leads to low-voltage winding, iron core and tank; *b.* the coil-to-coil insulation, and insulation between taps; *c.* the turn-to-turn insulation. The technique of surgeproof insulation consists in proportioning the insulation between coils and to ground, so that each of these insulations is equally strong relative to the stresses imposed by a surge.

As early as 1931–32 practical analytical methods were applied to the design of surgeproof transformers as shown in 2 articles by the writer ("Surgeproof, Shell-Type Transformers," *L'Energia Elettrica*, volume 9, April 1932, p. 335–42, and "Impulse Characteristics of Core-Type Transformers," *L'Elettrotecnica*, volume 21, January 5, 1934, p. 1–14). Since then those analytical processes have been further extended and incorporated in design methods. Extensive tests on a great many insulation structures have accompanied the analytical work, many of these tests having been made on full sized transformers. Besides, experience has been acquired in impulse testing transformers with a total rated capacity of about 1,000,000 kva, covering practically all designs, types, voltages, and power ratings. Such experience amply establishes the fundamental importance of a truly surgeproof transformer.

An Exact Formula for Transformer Regulation

Discussion and author's closure of a paper by J. E. Clem published in the May 1936 issue, pages 466–71, and presented for oral discussion at the transformer session of the summer convention, Pasadena, Calif., June 25, 1936.

William Glendinning (The New York Edison Company, New York, N. Y.): The paper is an excellent treatise on the calculation of transformer regulation; however, the equations hold true for unity and lagging power factors only.

With the same notation used in the paper, and with the same reasoning, the following method is submitted as the proper derivation of the exact equation for the regulation

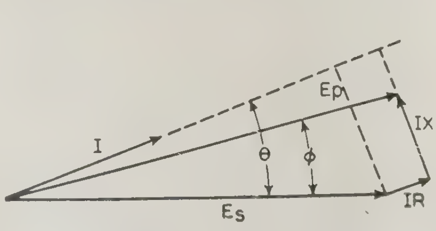


Fig. 1. Vector diagram for a 2-winding transformer supplying a load with leading power factor

of a 2-winding transformer operating leading power factor:
From figure 1 of this discussion,

$$\tan(\theta - \varphi) = \frac{E_s \sin\theta - IX}{E_s \cos\theta + IR} \quad ($$

$$= \frac{n - x}{m + r} \quad ($$

From equation 2,

$$\cos(\theta - \varphi) = \frac{m + 5}{\sqrt{(m + r)^2 + (n - x)^2}} \quad ($$

and from figure 1 of this discussion

$$E_p \cos(\theta - \varphi) = E \cos\theta + IR \quad ($$

$$E_p = \frac{m + r}{\cos(\theta - \varphi)} \quad ($$

Substituting equation 3 in equation 5,

$$\frac{E_p}{E_s} = \sqrt{(m + r)^2 + (n - x)^2} \quad ($$

$$\frac{E_p}{E_s} - 1 = \sqrt{(m + n)^2 + (n - x)^2} - 1 \; ($$

But

$$\frac{E_p - E_s}{E_s} 100 = \text{per cent regulation}$$

definition
So that

per cent regulation =
$$\left\{\sqrt{(m + r)^2 + (n - x)^2} - 1\right\} 100 \quad ($$

in which

$(n + x)^2$ is used for lagging power factor
$(n - x)^2$ is used for leading power factor

J. B. Whitehead (The Johns Hopkins Ui versity, Baltimore, Md.): The auth must be aware that none of the formul discussed in the paper is an "exact" e pression for the regulation of a transform consequently, the use of the word "exac in the title is misleading.

The right-angle vector diagrams of figu 1 and 2 of the paper, upon which all of t formulas are based, are themselves or approximations. That they are good a proximations and have been used for ma years for the computation of the regulati of transformers, synchronous generato and other forms of electromagnetic circu is not a good reason for a claim that th are exact.

The principal error in the diagrams a formulas arises from the omission of the fect on voltage of the magnetizing curre of the transformer. This error is small a may be neglected in computations for mo ern closed-magnetic-circuit transforme In passing, however, it may be noted th even in this case the effect of the magnet ing current would enter in about the four place in decimal expressions of regulati whereas the author has computed values the sixth and even the ninth decimal pla in his numerical calculations. Moreov air gaps are to be found in magnetic circu of modern transformers. In such tra formers the effect of the magnetizing curre on regulation reaches appreciable values.

Perhaps the simplest expressions thus far presented for the secondary and primary voltages of a transformer, and from which the AIEE regulation may be readily determined, are those of the late C. P. Steinmetz in his "Alternating Current Phenomena" as follows:

$$E_1 = E_1' - Z_1 I_1$$

$$E_0 = aE_1' \left\{ 1 + Z_0 Y_0 + \frac{Z_0 Y}{a^2} \right\}$$

in which E_1' is the secondary electromotive force on open circuit, Z_0 and Z_1 the internal primary and secondary impedances, Y_0 the magnetizing admittance, Y the total secondary admittance including coil and load, and a is the ratio of turns.

Although simple in form, these equations are not suitable for engineering computations because neither by computation on measurement is it possible to separate the values of Z_0 and Z_1. They are introduced here because they show the influence of the magnetizing admittance on an exact expression of secondary voltage and regulation.

This comment is in no sense a plea for the use in engineering practice of any exact formula for regulation. All such formulas are subject to the same difficulty as the foregoing method. The approximate methods discussed by the author are simple and entirely justified in practice. They should not, however, masquerade as being exact. The same reasoning holds for the 3-winding transformer.

J. E. Clem: J. B. Whitehead's discussion is frank; he claims that the formulas in the paper are not exact because the magnetizing current has been neglected. He offers some equations from Steinmetz as proof of this, the implication seeming to be that they substantiate his claim. Actually the equation can be shown to prove that the method used to develop the formulas in the paper does lead to an exact expression for transformer regulation as defined in the paper (the ASA definition).

Whitehead's difficulty with Steinmetz's equations as quoted in his discussion may result from a misunderstanding of them. Whitehead defines E_1' as the secondary electromotive force on open circuit, whereas Steinmetz gives it as "the secondary generated electromotive force (see pages 245–6–7 of the fourth edition of Steinmetz's book "Alternating Current Phenomena").

The second of the 2 equations quoted in the discussion is obtained from the equation

$$E_0 = E + Z_0 I_0 \tag{1}$$

by substituting the value of I_0

$$I_0 = \frac{E'}{a^2}(Y + a^2 Y_0) \tag{2}$$

in which

E_0 = primary impressed electromotive force
E' = primary counter electromotive force = aE_1'
Z_0 = primary impedance
I_0 = total primary current
a = ratio of terms, primary to secondary
Y = total admittance of secondary circuit, including internal impedance
Y_0 = primary admittance

$$= \frac{\text{exciting current}}{\text{primary induced electromotive force}}$$

The value of I_0 is given in another equation as

$$I_0 = I_0' + I_0^0 \tag{3}$$

in which

I_0' = primary current corresponding to the secondary current
I_{00} = primary exciting current

could just as well have been used. The result would be

$$E_0 = E' + Z_0(I_0' + I_{00}) \tag{4}$$

The first equation quoted in the discussion is

$$E_1 = E_1' - Z_1 I_1 \tag{5}$$

in which

E_1 = secondary terminal voltage
E_1' = secondary generated electromotive force
Z_1 = secondary impedance
I_1 = secondary current

The conventional manner of carrying reactance calculations is on a "per unit" basis; in other words, as a one-to-one ratio transformer. On this basis

$$E' = E_1' \quad I_0' = I_1 \tag{6}$$

that is, the per-unit voltage induced in the secondary and primary is the same, and the load current values are the same.

Now substitute in equation 5 of this discussion the value of $E_1' = E'$ obtained from equation 4 and remembering that $I_0' = I_1$

$$E_1 = E_0 - Z_0(I_1 + I_{00}) - Z_1 I_1 \tag{7}$$

$$= E_0 - I_1(Z_0 + Z_1) - Z_0 I_{00} \tag{8}$$

But

$$Z_{xs} = Z_0 + Z_1 \tag{9}$$

in which Z_{xs} is the sum of the primary and secondary impedance; that is, it is what is normally called the impedance of the transformer.

Now let E_{sl} be the secondary voltage under normal load and E_{s0} be the secondary voltage at no load. Also let I be the rated load current.
Then

$$I_1 = I \text{ at full load}$$

$$I_1 = 0 \text{ at no load} \tag{10}$$

Substitute equations 9 and 10 into 8

$$E_{sl} = E - IZ_{xs} - Z_0 I_{00} \tag{11}$$

$$E_{s0} = E_0 - Z_0 I_{00} \tag{12}$$

Now subtract equation 11 from equation 10

$$E_{sl} - E_{s0} = -IZ_{xs} \tag{13}$$

which is the change in secondary voltage when the load on the transformer is reduced to zero.

According to the definition of regulation it is this change in voltage expressed as a percentage of the secondary voltage. Therefore, any formulas for regulation correctly derived on this basis is exact; accordingly, the formulas in the paper are exact.

William Glendinning's argument is good, and no further comment is necessary, except that the provision for correct signs has been included in the formulas in the transformer test code.

Sound Projector System Has 10,000-Watt Output

To carry announcements and entertainment to the spectators at the Roosevelt Raceways, Westbury, Long Island, N. Y., where the Vanderbilt Cup races were held October 12, 1936, this system of powerful loud-speakers, mounted on a centrally located 100-foot tower and having an acoustical output of 10,000 watts, has been installed. The installation includes 19 of the so-called "supersound projectors" which are trained upon the distant areas of the field in much the same manner as searchlights are focused upon distant objects, so that announcements may be heard intelligibly above the roar of racing cars over the entire 1/2 square mile area. The group of smaller loud-speakers mounted near the middle of the tower serves the nearby portions of the main grandstand which may be seen in the background. The equipment was designed by the Bell Telephone Laboratories and was built by the Western Electric Company.

News

Of Institute and Related Activities

AIEE Directors Meet at Institute Headquarters

THE regular meeting of the board of directors of the American Institute of Electrical Engineers was held at Institute headquarters, New York, N. Y., on October 20, 1936.

There were present: *President*—A. M. MacCutcheon, Cleveland, Ohio. *Past-Presidents*—E. B. Meyer, Newark, N. J.; J. B. Whitehead, Baltimore, Md. *Vice Presidents*—O. B. Blackwell, New York, N. Y.; L. T. Blaisdell, Dallas, Texas; C. V. Christie, Montreal, Can.; Mark Eldredge, Memphis, Tenn.; R. H. Fair, Omaha, Neb.; C. Francis Harding, Lafayette, Ind.; W. H. Harrison, Philadelphia, Pa.; N. B. Hinson, Los Angeles, Calif.; C. E. Rogers, Seattle, Wash.; A. C. Stevens, Schenectady, N. Y. *Directors*—F. M. Farmer, New York, N. Y.; N. E. Funk, Philadelphia, Pa.; F. Ellis Johnson, Columbia, Mo.; C. R. Jones, New York, N. Y.; P. B. Juhnke, Chicago, Ill.; W. B. Kouwenhoven, Baltimore, Md.; Everett S. Lee, Schenectady, N. Y.; K. B. McEachron, Pittsfield, Mass.; C. A. Powel, East Pittsburgh, Pa. *National Treasurer*—W. I. Slichter, New York, N. Y. *National Secretary*—H. H. Henline, New York, N. Y.

The minutes of the board of directors' meeting held August 4, 1936, were approved.

Resolutions were adopted in memory of Doctor Edward Weston, a charter member, past-president, and Honorary Member of the Institute, who died on August 20. The resolutions appear elsewhere in this issue.

Actions of the executive committee on applications, as of September 28, 1936, were reported and confirmed, as follows: 3 applicants transferred to the grade of Fellow; 6 applicants elected and 26 transferred to the grade of Member; 47 applicants elected to the grade of Associate; 21 Students enrolled.

Recommendations adopted by the board of examiners at its meeting of September 23, 1936, were reported and approved. One applicant was re-elected to the grade of Member, and one applicant was re-elected to the grade of Associate.

A revised form of the application blank for admission to membership in the Institute, prepared by the national secretary's office, was presented and approved.

The finance committee reported monthly expenditures amounting to $12,935.78 in August, $14,672.80 in September, and $26,004.01 in October, the last amount including the first half of the year's appropriation for the Sections; report approved.

Report was made of the affirmative results of a letter ballot of the members of the board of directors in September giving authorization for the sale and purchase of certain bonds and for the renewal of the contract with Young & Ottley, investment counsel of the Institute, for one year, beginning October 10, 1936.

A budget for the appropriation year of the Institute beginning October 1, 1936, submitted by the finance committee, was adopted.

A resolution was adopted directing the transfer of the sum of $7,500 from the general funds to the reserve capital fund, to be invested in accordance with advice of the Institute's investment counsel, and on the approval of the board of directors.

Upon request of the officers of the Southern District, the previously scheduled District meeting to be held in Birmingham, Ala., in December 1936, was canceled.

V. M. Montsinger was appointed a representative of the Institute upon the Standards Council of the American Standards Association for the 3-year term beginning January 1, 1937, and R. E. Hellmund, H. S. Osborne, and E. B. Paxton were appointed alternate representatives for the year 1937.

The following 5 members of the board of directors were selected to serve on the national nominating committee in connection with the nomination of Institute officers for election by the membership in the spring of 1937: O. B. Blackwell, F. M. Farmer, N. E. Funk, A. C. Stevens, and J. B. Whitehead.

A revision of definitions of the scopes of AIEE technical committees was reported by the technical program committee, and was approved.

A report of the committee on co-ordination of Institute activities upon certain suggestions for a revision of the AIEE committee organization, which had been referred t the committee, was presented. The sugges tions were for the consolidation of som committees and fewer members of the larg committees. In effect, the committe recommended that no change be made in th present committee organization; the repo was approved.

It was reported that National Secretar Henline had been elected secretary of th Engineers' Council for Professional De velopment for the year beginning in Octo ber 1936, in accordance with the practice o rotating the secretaryship among the secre taries of the sponsoring societies.

By action of the board of directors o August 4, a special committee, consisting o A. C. Stevens (chairman), Mark Eldredg P. B. Juhnke, L. W. W. Morrow, H. S. Os borne, I. Melville Stein, W. H. Timbie, an J. B. Whitehead, was appointed to conside certain suggestions regarding the broaden ing of Institute activities to cover discus sions of social and economic subjects. Th suggestions looked toward the Institute taking more direct and active part in th public questions of the day which have eng neering aspects, by sponsoring discussion at national conventions and District an Section meetings, thus creating a better un derstanding of such questions on the part o the membership, which might crystalliz into an official attitude by the Institut toward the controversial questions of th day. Such attitude, it was thought, woul have great influence upon the public an upon the public agencies that deal directl with the questions concerned. The speci committee met on the morning of Octob 20, and presented a progress report to th board of directors at this meeting. I brief, the report of the committee was th the constitution and by-laws of the Institu

Membership—

Mr. Institute Member:

It is to the advantage of your associate engineers, professionally and socially, to be members of the Institute. Help them to help themselves by bringing the matter to their attention.

Send any questions to headquarters.

J. Leo Scanlon

Vice-Chairman, District No. 1,
National Membership Committee

contain nothing to prevent the presentation of the types of paper suggested. The Sections and Districts have full authority to schedule such subjects for discussion at their meetings as they desire. So far as national programs are concerned, any papers of the types in question that are presented will receive full and careful consideration by the technical program and publication committees. Realizing, however, that there is a feeling on the part of some members that papers along the lines proposed are not acceptable to the Institute, or that there is some rule against them, the committee recommended that the board adopt a resolution that the Institute should encourage the presentation of papers dealing with the economic basis of electrical-engineering projects, also papers dealing with the sociological effects of engineering developments.

After an extensive discussion, the board passed a motion concurring in the suggestion that the topics under consideration are of general interest to electrical engineers, pointing out that the Institute always has offered opportunity for the discussion of such questions, and further suggesting that suitable papers on these subjects be secured and submitted to the proper committees of the Institute for consideration for national presentation and publication.

The board considered a request of the New York Section of the Institute that the board of directors give further consideration to the "Model Law for the Registration of Professional Engineers and Land Surveyors," which was prepared several years ago by a committee of the American Society of Civil Engineers, on which several national engineering societies were represented, including the Institute. The proposed model law never was officially approved by the AIEE. The board voted to refer this request to the committee on legislation affecting the engineering profession. In this connection, the board discussed the attitude of the Institute toward the general subject of licensing or registration of engineers. Most of the states now have licensing laws and the movement is spreading; therefore, consideration was given to the question of whether or not the AIEE should participate in the discussion and guidance of license laws. It was voted that the board of directors feels that the Institute as an organization is interested in participating in the consideration of licensing laws.

Other matters were discussed, reference to which may be found in this or future issues of Electrical Engineering.

Winter Convention Plans Progressing

The AIEE 1937 winter convention will be held Monday to Friday, inclusive, January 25–29, 1937, with registration Monday morning and the opening session Monday afternoon. This will mark the resumption of a 5-day winter convention schedule, the preceding 3 conventions having extended over a period of only 4 days each. Convention headquarters as usual will be in the Engineering Societies' Building, New York, N. Y. The program will be comprised of technical sessions on timely subjects, entertainment, inspection trips, and social functions, the details of which will be announced in subsequent issues of Electrical Engineering.

Arrangements now being made indicate that 10 technical sessions will be developed to treat subjects as follows: tensor analysis, electrophysics, power transmission, power distribution, protective devices, measurements and oscillography, communication, electric welding, electrical machinery, and a symposium on transformers. In addition, several technical conferences, where small groups can get together and discuss informally their specialized problems, will be arranged. One of these conferences will be under the auspices of the committee on education.

Some of the papers that will be on the program have been published in this and previous issues of Electrical Engineering; others will appear in later issues.

Winter Convention Committee. The personnel of the general committee as appointed by President MacCutcheon is as follows: C. R. Beardsley, *chairman;* T. F. Barton, O. B. Blackwell, E. E. Dorting, A. G. Oehler, C. S. Purnell, S. A. Smith, Jr., George Sutherland, and F. P. West.

Chairmen Meet With President to Discuss Plans for Year's Committee Work

To FACILITATE a more effective coordination of the varied work of the many Institute committees, President A. M. MacCutcheon held a dinner meeting with the chairmen of most of the Institute's committees at the Engineers' Club adjoining the headquarters' office building in New York, September 14, 1936. To 41 busy men scattered among some 16 cities in 10 different states from the Atlantic to the Pacific went a letter, during the latter part of August, inviting each as the chairman of an Institute committee to meet with the president in New York as his guest for the purpose of discussing committee activities. Invitations also were sent to nearby Institute officers. Of this group of leaders of Institute activities, as indicated in the accompanying list, some 14 chairmen of general committees, 13 chairmen of technical committees or representatives thereof, 2 vice-presidents, a past-president, a past-director, and the national secretary attended the meeting.

Statistically, the attendance list reveals other interesting facts incident to the wide geographical distribution attained by President MacCutcheon in his committee chairmanship appointments. For instance, there is a matter of "man miles" represented by those in attendance at the meeting. The 20 men in attendance, whose residence or place of business is elsewhere than in New York City or environs, represent the imposing total of 13,518 "man miles" traveled, an average of more than 675 miles per man.

In emphasizing the extent and importance of committee activity in the Institute, President MacCutcheon in his opening remarks gave other statistics: ..... "I find that there are over 500 members serving on national committees. There are 61 Sections, and it is a reasonable assumption that there are about 30 men working on committees for each of the Sections, totaling 1,840, and making a grand total of approximately 2,400 men active in the association. If each man gives 20 hours of his time a year—many of the committees give vastly more time than that—we find 48,000 hours spent in work of the Institute in a year, the equivalent of 23 trained engineers working for 2 years, day and night. I give these figures to show that we have a tremendous asset at our disposal, and to emphasize that you are gathered together tonight to say how we shall properly make use of that asset." President MacCutcheon then went into a discussion of various suggestions concerning committee activities, and this was followed by a general exchange of comments and suggestions from the others present. The essential substance of the specific suggestions will be embraced in the President's Message scheduled for publication in the December issue of Electrical Engineering.

Attendance at President's Dinner Meeting for Committee Chairmen

General Committees

A. M. MacCutcheon (Cleveland), Executive
W. I. Slichter (New York), Columbia University Scholarships
R. N. Conwell (Newark), Constitution and By-Laws
R. W. Sorensen (Pasadena), Economic Status of the Engineer
Everett S. Lee (Schenectady), Finance
F. M. Farmer (New York), Headquarters
N. E. Funk (Philadelphia), Lamme Medal
G. A. Kositzky (Cleveland), Membership
J. P. Jackson (New York), N. Y. Museum of Science and Industry Advisory Committee
W. R. Smith (Newark), Technical Program
I. Melville Stein (Philadelphia), Publication
H. S. Warren (New York), Safety Codes
W. H. Timbie (Cambridge), Sections
V. M. Montsinger (Pittsfield), Standards
H. Goodwin, Jr. (Wyncote, Pa.), Transfers

Technical Committees

C. B. Jolliffe (New York), Communication
O. W. Eshbach (New York), Education
H. M. Hobart (Schenectady), Electric Welding
J. L. Hamilton (St. Louis), Electrical Machinery
F. O. Schnure (Sparrows Point, Md.), Electrochemistry and Electrometallurgy
H. C. Koenig (New York), Instruments and Measurements
A. L. Powell (New York), Light, Production and Application of
A. Kennedy, Jr. (Schenectady), Marine Work, Applications to
H. W. Rogers (Schenectady), Power Applications, General
G. M. Armbrust (Chicago), Power Generation
C. T. Sinclair (Pittsburgh), Power Transmission and Distribution
F. R. Longley (Springfield, Mass.) (representing J. P. McKearin), Protective Devices
W. B. Kouwenhoven (Baltimore), Research

Others

W. H. Harrison (Philadelphia), Vice President, District 2
A. C. Stevens (Schenectady), Vice President, District 1
A. W. Berresford (New York), Past-President
J. E. Kearns (Chicago), Past-Director
H. H. Henline (New York), National Secretary

Nomination of AIEE Officers for 1937 Election; Members Invited to Submit Suggestions by Dec. 15

FOR the nomination of national officers to be voted upon in the spring of 1937, the AIEE national nominating committee will meet during the winter convention, January 25–29, 1937. To guide this committee in performing its constituted task, suggestions from the membership are, of course, highly desirable. To be available for the consideration of the committee, all such suggestions must be received by the secretary of the committee at Institute headquarters, not later than December 15, 1936.

In accordance with the provisions in the constitution and by-laws, as amended during 1935 and quoted in the following paragraphs, actions relative to the organization of the national nominating committee are now under way.

Constitution

28. There shall be constituted each year a national nominating committee consisting of one representative of each geographical district, elected by its executive committee, and other members chosen by and from the board of directors not exceeding in number the number of geographical districts; all to be selected when and as provided in the by-laws. The national secretary of the Institute shall be the secretary of the national nominating committee, without voting power.

29. The executive committee of each geographical district shall act as a nominating committee of the candidate for election as vice president of that district, or for filling a vacancy in such office for an unexpired term, whenever a vacancy occurs.

30. The national nominating committee shall receive such suggestions and proposals as any member or group of members may desire to offer, such suggestions being sent to the secretary of the committee.

The national nominating committee shall name on or before January 31 of each year, one or more candidates for president, national treasurer, and the proper number of directors, and shall include in its ticket such candidates for vice presidents as have been named by the nominating committees of the respective geographical districts, if received by the national nominating committee when and as provided in the by-laws; otherwise the national nominating committee shall nominate one or more candidates for vice president(s) from the district(s) concerned.

By-Laws

Sec. 22. During September of each year, the secretary of the national nominating committee shall notify the chairman of the executive committee of each geographical district that by December 15 of that year the executive committee of each district must select a member of that district to serve as a member of the national nominating committee and shall, by December 15, notify the secretary of the national nominating committee of the name of the member selected.

During September of each year, the secretary of the national nominating committee shall notify the chairman of the executive committee of each geographical district in which there is or will be during the year a vacancy in the office of vice president, that by December 15 of that year a nomination for a vice president from that district, made by the district executive committee, must be in the hands of the secretary of the national nominating committee.

Between October 1 and December 15 of each year, the board of directors shall choose 5 of its members to serve on the national nominating committee and shall notify the secretary of that committee of the names so selected, and shall also notify the 5 members selected.

The secretary of the national nominating committee shall give the 15 members so selected not less than 10 days' notice of the first meeting of the committee, which shall be held not later than January 31. At this meeting, the committee shall elect a chairman and shall proceed to make up a ticket of nominees for the offices to be filled at the next election. All suggestions to be considered by the national nominating committee must be received by the secretary of the committee by December 15. The nominations as made by the national nominating committee shall be published in the March issue of Electrical Engineering (Journal of AIEE), or otherwise mailed to the Institute membership not later than the first week in March.

Independent Nominations

Independent nominations may be made in accordance with provisions in Section 31 of the constitution and Section 23 of the by-laws, which are quoted below:

Constitution

31. Independent nominations may be made by a petition of twenty-five (25) or more members sent to the national secretary when and as provided in the by-laws; such petitions for the nomination of vice presidents shall be signed only by members within the district concerned.

By-Laws

Sec. 23. Petitions proposing the names of candidates as independent nominations for the various offices to be filled at the ensuing election, in accordance with Article VI, Section 31 (constitution), must be received by the secretary of the national nominating committee not later than March 25th of each year, to be placed before that committee for the inclusion in the ballot of such candidates as are eligible.

On the ballot prepared by the national nominating committee in accordance with Article VI of the constitution and sent by the national secretary to all qualified voters during the first week in April of each year, the names of the candidates shall be grouped alphabetically under the name of the office for which each is a candidate.

(*Signed*) H. H. Henline,
November 1, 1936. *National Secretary*

Edison's Voice to Be Heard at Patent Centennial

Thomas A. Edison's voice will speak to dinner guests at the celebration, in Washington, D. C., of the 100th anniversary of the American patent system on November 23, 1936.

Leading inventors, patent attorneys, industrial leaders, and government officials will participate in the celebration, it is announced by Doctor C. F. Kettering (A '04, F '14), president of the General Motors Research Corporation, in accepting the invitation of the Secretary of Commerce, Daniel C. Roper, to serve as chairman of the national committee being formed to celebrate the event.

The tentative program calls for addresses in the morning of the centennial celebration day at Washington reviewing the progress made in the last century under the present patent system and the significance of the law on the growth of the United States. The present value of the system to raise the American standard of living will be discussed, and the continued growth of the inventive arts in the future will be forecast.

In the afternoon new "invention babies"—industrial developments just ready to take their places in economic usefulness—will be shown in a series of demonstrations.

Climax of the centennial day will be a dinner at the Mayflower Hotel in Was[hington] at which special novelties of intere[st] to inventors will be on the program. Tra[ns]mission of the original Morse telegra[ph] message will be re-enacted between t[he] old Baltimore and Ohio station in Bal[ti]more, Md., and the dinner hall, where [it] will be received on one of the origin[al] Morse instruments loaned by Corn[ell] University. The late Thomas A. Edis[on] also will address the dinner guests in [his] own words from one of his early phon[o]graph recordings.

Ambrose Swasey Awarded Hoover Medal

At the annual dinner of The Americ[an] Society of Mechanical Engineers, to [be] held in New York, N. Y., December [...] 1936, Ambrose Swasey (HM'28) pas[t] president and honorary member of th[at] society, and founder of the Engineerin[g] Foundation, will have the high distinctio[n] of becoming the second recipient of th[e] Hoover Gold Medal. A brief biographic[al] sketch of Doctor Swasey is given in th[e] "Personal" columns of this issue.

The Hoover Gold Medal was institute[d] to commemorate the civic and humanitaria[n] achievements of Herbert Hoover (HM'29) and the first award was made to him i[n] Washington, D. C., on the occasion of th[e] fiftieth anniversary of the ASME in Apr[il] 1930. Inscribed on the Medal is the legen[d] "Awarded by Engineers to a Fellow Engi[]neer for Distinguished Service."

The trust fund creating the award is th[e] gift of Conrad N. Lauer, of Philadelphia, Pa., past-president of the ASME. It i[s] held by the ASME and is administered b[y] a board of award consisting of representa[]tives of the American Society of Civi[l] Engineers, the American Institute o[f] Mining and Metallurgical Engineers, Th[e] American Society of Mechanical Engineer[s] and the American Institute of Electrica[l] Engineers. The personnel of the boar[d] that selected Doctor Swasey as the re[]cipient is as follows: for the ASCE, Ralp[h] Budd, Albert S. Crane (A'04, M'13), an[d] Thaddeus Merriman; for the AIME Scott Turner, Clinton H. Crane, an[d]

Future AIEE Meetings

Winter Convention
New York, N. Y., Jan. 25–29, 1937

North Eastern District Meeting
Buffalo, N. Y., May 5–7, 1937

Summer Convention
Milwaukee, Wis., June 21–25, 1937

Pacific Coast Convention
Spokane, Wash., Date to be determined

Middle Eastern District Meeting
Akron, Ohio, Fall 1937

J. V. W. Reynders; for the ASME, S. F. Voorhees, C. N. Lauer, and W. H. Kenerson; and for the AIEE, F. B. Jewett (A'03, F'12, past-president), Gano Dunn (A'91, F'12, past-president), and H. H. Barnes, Jr. (A'00, F'13).

"Grandfather" Clause in N. Y. License Law Expires Soon

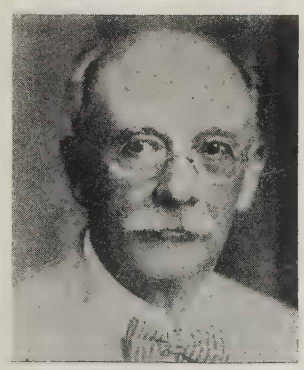
During the year ended June 30, 1936, the New York State Board of Examiners of Professional Engineers and Land Surveyors held 12 meetings and considered 4,424 applications for license, of which 796 were rejected, 1,774 were held for written examination, 426 were held for further consideration, 7 were returned to the State Department of Education as not being eligible to file an application, and 1,421 were recommended for licenses.

The net amount received from fees during the fiscal year ended June 30, 1936, was $88,339.00, and the expenditures were $25,524.68, leaving a surplus of $62,814.32.

During the year no charges have been submitted to the board seeking the revocation of any license heretofore granted.

No amendments were made to the licensing law during the 1936 session of the State legislature. In accordance with existing provisions of the law, prior to January 1, 1937, the board may exempt applicants for a license to practice professional engineering, if graduates of a college or school of engineering in a course registered by the Department of Education as maintaining satisfactory standards, from any part of the examination except the written examination to establish their competency to plan, structurally design, and supervise the con struction of buildings and similar structures. Every person applying subsequent to January 1, 1937, for a professional engineer's license shall be required to pass the examination as prescribed by the board, and to be admitted to such examination the applicant must submit evidence of graduation from a college or school of engineering registered by the Department of Education as maintaining satisfactory standards, or must present evidence of at least 12 years of practical experience in professional engineering work of a grade and character satisfactory to the board. The first examinations under the provisions of the law which become effective January 1937, will be held in June 1937. The January 1937 examinations will be similar to the examinations given during the past few years as all applicants to be admitted to such examinations will have filed their applications prior to January 1, 1937.

In Memoriam



EDWARD WESTON

DR. EDWARD WESTON, a charter member, the fourth president, and an honorary member of the American Institute of Electrical Engineers, died on August 20, 1936, at the age of 86.

Becoming keenly interested as a boy in chemistry, electrometallurgy, and electrical and mechanical investigations, he spent his spare time in these fields while studying medicine at the request of his parents. In 1870, at the age of 20, he decided not to continue in medicine, and went to New York where he worked for 2 years as a chemist and an electrician.

He then established his own nickel-plating business, but on account of the lack of suitable dynamos for his power supply, and his important improvements in such machines, he soon abandoned the electroplating field to form a partnership for the manufacture of dynamos and other electrical equipment.

During the next 11 years he made further improvements in dynamos, invented new devices for starting, controlling, and protecting them, and also engaged in intensive development of arc and incandescent lighting.

Great difficulty encountered in making the necessary electrical measurements caused him to develop and build more suitable instruments, and they marked such an advance in electrical measurements that, in 1888, he relinquished his other interests to establish the Weston Electrical Instrument Company, of which he served as vice president and general manager 1888–1905, president 1905–24, and chairman of the board until his death.

Entering the Institute as a charter member, he was elected a member of the first board of directors for a term 1884–87 was president 1888–89, and vice president 1889–91. He was elected an Honorary Member in 1933, and received the Lamme Medal for 1932. He received many high honors from other organizations.

His many notable contributions to the development and manufacture of numerous types of electrical equipment, and especially his development of precision electrical measuring instruments, brought him international fame as a practical scientist.

RESOLVED: That the board of directors of the American Institute of Electrical Engineers, upon behalf of the membership hereby expresses its deep regret at the death of Doctor Weston, and its sincere appreciation of his many important contributions to electrical-engineering progress, and be it further

RESOLVED: That these resolutions be entered in the minutes and transmitted to his family and his company.

—AIEE Board of Directors, October 20, 1936.

Elihu Thomson Honored by Welding Society

Elihu Thomson (A'84, F'13, HM'28, past-president) one of America's greatest pioneers in the field of electrical science and holder of more than 800 patents, was honored Friday night, October 16, 1936, when the Detroit (Mich.) section of the American Welding Society dedicated its program to the fiftieth anniversary of one of Doctor Thomson's greatest inventions, that of resistance welding. The basic patent on this method of joining metals by putting them in contact with one another and then passing through them an electric current which fuses and unites the pieces was granted in 1886. The anniversary program was presented in the auditorium of the Detroit Edison Company.

Doctor Thomson's health is such that he was unable to make the trip from his home in Swampscott, Mass., but he sent a special message which his son, Malcolm, read to the society. Now in his 84th year, Doctor Thomson is one of the co-founders of the General Electric Company. He is the holder of numerous medals and awards and is the only scientist in the world who possesses the 3 most coveted awards of English scientific and engineering institutions, the Faraday, Kelvin, and Hughes medals.

North Eastern District Executive Committee Meets

A meeting of the executive committee of the Institute's North Eastern District (1) was held at Pittsfield, Mass., October 2, 1936. At this meeting, plans for District activities during the coming year were outlined, which include a District meeting to be held at Buffalo, N. Y., May 5–7, 1937.

To serve as the District co-ordinating committee for the ensuing year, the following were designated: A. C. Stevens, *chairman;* R. G. Lorraine, *secretary;* F. N. Tompkins, *Branch counselor;* R. F. Chamberlain, C. L. Dawes, E. D. Lynch, and K. B. McEachron. This committee, together with J. L. Scanlon and N. E. Brown from the Institute's Niagara Frontier Section, will comprise the general committee for the forthcoming District meeting.

T. H. Morgan was appointed to represent the District on the national nominating committee.

ECPD Holds Fourth Annual Meeting—Accredits 135 Curricula of 35 Engineering Schools

UNDOUBTEDLY the most significant of the annual meetings so far held by the Engineers' Council for Professional Development was the fourth, held in the Engineering Societies Building, New York, October 6, 1936. At the morning session, in addition to the election of officers and chairmen of committees, reports of the Council's committees were presented. Interest centered around the report of the committee on engineering schools, and at afternoon and evening executive sessions formal action on the accrediting of engineering curricula of educational institutions in New England and Middle Atlantic states was taken.

Elections and Appointments Reported

Chas. F. Scott (A'92, F'25, HM'29, past-president) professor of electrical engineering emeritus, Yale University, was re-elected chairman. H. H. Henline (A'19, M'26, national secretary) was elected secretary of the Council. By action of the Council the by-laws were amended to provide for the offices of vice-chairman and assistant secretary. R. I. Rees, assistant vice-president, American Telephone & Telegraph Company, was elected vice-chairman, and C. E. Davies, secretary of the American Society of Mechanical Engineers, was elected assistant secretary. Chairmen of the Council's committees were elected as follows: student selection and guidance, Robert L. Sackett, dean of engineering, Pennsylvania State College; engineering schools, Karl T. Compton, president, Massachusetts Institute of Technology; professional training, R. I. Rees; professional recognition, Conrad N. Lauer, president, Philadelphia Gas Works; ways and means, R. I. Rees; and information, H. C. Parmelee, editorial director, Engineering and Mining Journal.

Executive committee members elected were: J. P. H. Perry, representing the American Society of Civil Engineers; F. M. Becket, representing the American Institute of Mining and Metallurgical Engineers, C. F. Hirshfeld, representing The American Society of Mechanical Engineers; L. W. W. Morrow (A'13, F'25) representing the AIEE; H. C. Parmelee, representing the American Institute of Chemical Engineers; R. I. Rees, representing the Society for the Promotion of Engineering Education; and D. B. Steinman, representing the National Council of State Boards of Engineering Examiners.

Appointments of representatives of the participating societies for the 3-year term 1936–39 were announced as follows: ASCE, Frank E. Winsor; AIME, W. B. Plank; AIEE, L. W. W. Morrow (reappointment); SPEE, R. I. Rees (reappointment); AIChE, H. C. Parmalee (reappointment); and, NCSBEE, D. B. Steinman.

In addition to the report of the committee on engineering schools, reports of the committees on student selection and guidance and on professional training were approved. The report of the committee on professional recognition was received and held over for discussion at a meeting to be called at some later date.

The report of the committee on student selection and guidance, of which R. L. Sackett, dean of engineering, Pennsylvania State College, is chairman, was devoted to the committee's studies of co-operative tests in English and mathematics with which it has been experimenting in an effort to find a reliable means of predicting a young student's ability to complete a course in engineering training. Results of the committee's attempts to encourage local groups throughout the country in intelligent selection and guidance of students were reported.

In the report of the committee on professional training, R. I. Rees told of the year's work with junior engineers recently graduated from college. Successful inauguration of organized classes of study for junior engineers of the Providence Engineering Society was announced. It was also reported that the committee had completed its selected bibliography of engineering subjects for young engineers who wish to continue study in allied fields of engineering

Excerpts from the reports of the committees on engineering schools, student selection and guidance, and professional training are given on this and the following pages. The far-reaching report of the committee on professional recognition is scheduled for publication in the December issue

Report on Engineering Schools Covers Recommended Curricula

Formal action in accrediting 135 engineering curricula in 35 educational institutions in the New England and Middle Atlantic states, taken at the annual meeting of ECPD, was based upon the report and recommendations of the committee on engineering schools. The report was presented by Dr. Karl T. Compton (F'31 chairman of that committee.

Accredited curricula included in this report are for institutions in the following states only: Connecticut, Delaware, Maine Maryland, Massachusetts, New Hampshire, New Jersey, New York, Pennsylvania Rhode Island, and Vermont. The committee is now engaged in reviewing curricula at institutions in the remaining states and the District of Columbia, and a complete and revised list is expected to be ready for

AEC Exhibit at World Power Conference

THIS graphic presentation showing the number, locations, and objectives of the engineering organizations of the United States and their "instrumentalities" formed a part of the exhibit at the Third World Power Conference, held at Washington, D. C., September 7–12 1936. The exhibit was prepared by American Engineering Council, in co-operation with the national, state, and local engineering societies, at the request of the officers of the Third World Power Conference. As may be seen, the central feature of the exhibit was a map of the United States measuring approximately 8 by 12 feet, on which were spotted the locations of the national engineering societies and their local sections, the state engineering societies, and the independent local engineering groups. Grouped around this central feature were panels on which were presented the names and salient features of: (1) the 4 national founder societies; (2) the instrumentalities or functional organizations, which are supported by the founder societies and by other national engineering bodies; and (3) a series of lists of names and locations of the national, state, and local engineering organizations that responded to the invitation to participate in the exhibition.

release in the fall of 1937. As provided by the member societies of the ECPD, the accrediting program was initiated in the New England and Middle Atlantic states and on the basis of experience there obtained is now being extended to include the country as a whole. Excerpts from the report, indicating the basis of the accrediting and the curricula accredited, follow:

Basis for Accrediting

"ECPD at the inauguration of the accrediting program announced the following principles and procedures in accordance with which accrediting has been conducted:

"I. Purposes of accrediting shall be to identify those institutions which offer professional curricula in engineering worthy of recognition as such.

"II. Accrediting shall apply only to those curricula which lead to degrees.

"III. Both undergraduate and graduate curricula shall be accredited. (Accrediting of graduate curricula deferred until a later date.)

"IV. Curricula in each institution shall be accredited individually. For this purpose, ECPD will recognize the 6 major curricula—chemical, civil, electrical, mechanical, metallurgical, and mining engineering—represented in its own organization, and such other curricula as are warranted by the educational and industrial conditions pertaining to them.

"V. Curricula shall be accredited on the basis of both qualitative and quantitative criteria.

"VI. Qualitative criteria shall be evaluated through visits of inspection by a committee or committees of qualified individuals representing ECPD.

"VII. Quantitative criteria shall be evaluated through data secured from catalogues and other publications, and from questionnaires.

"VIII. Qualitative criteria shall include the following:

1. Qualifications, experience, intellectual interests, attainments, and professional productivity of members of the faculty.
2. Standards and quality of instruction: (a) in the engineering departments; (b) in the scientific and other co-operating departments in which engineering students receive instruction.
3. Scholastic work of students.
4. Records of graduates both in graduate study and in practice.
5. Attitude and policy of administration toward its engineering division and toward teaching, research, and scholarly production.

"IX. Quantitative criteria shall include the following:

1. Auspices, control, and organization of the institution and of the engineering division.
2. Curricula offered and degrees conferred.
3. Age of the institution and of the individual curricula.
4. Basis of and requirements for admission of students.
5. Number of students enrolled: (a) in the engineering college or division as a whole; (b) in the individual curricula.
6. Graduation requirements.
7. Teaching staff and teaching loads.
8. Physical facilities—the educational plant devoted to engineering education.
9. Finances: investments, expenditures, sources of income.

"The purpose of ECPD is to substitute a single accrediting for the unco-ordinated methods that have been used in the past. ECPD, representing the national engineer-

List of Undergraduate Curricula of Educational Institutions in New England and Middle Atlantic States Accredited by ECPD (as of 1936 and Subject to Continual Revision)

Institution	Engineering Curricula
1. Brown University	Civil Electrical Mechanical
2. Carnegie Institute of Technology	Chemical (*e*) Civil (*a*) Electrical (*a*) Management (*a*) Mechanical (*a*) Metallurgical (*a*)
3. Clarkson College of Technology	Civil Electrical Mechanical
4. College of the City of New York	Civil (*a*) Electrical (*a*)
5. Columbia University	Chemical (*b*) Civil (*b*) Electrical (*b*) Industrial (*b*) Mechanical (*b*) Metallurgical (*b*) Mining (*b*)
6. Cooper Union Institute of Technology	Civil (*c*) Electrical (*c*) Mechanical (*c*)
7. Cornell University	Administrative Chemical Civil Electrical Mechanical
8. Dartmouth College	Civil
9. University of Delaware	Civil Electrical Mechanical
10. Drexel Institute	Chemical (*d*) Civil (*d*) Electrical (*d*) Mechanical (*d*)
11. Johns Hopkins University	Civil Electrical Mechanical
12. Lafayette College	Civil Electrical Mechanical (Technical Option) Metallurgical Mining
13. University of Maine	Civil Electrical General Mechanical
14. Massachusetts Institute of Technology	Aeronautical Architectural Business and engineering administration Chemical Civil Electrical Electrochemical General Mechanical Metallurgy Mining Naval architecture and marine engineering Sanitary
15. University of New Hampshire	Civil Electrical Mechanical
16. New York University	Aeronautical (*c*) Chemical Civil (*c*) Electrical (*c*) Mechanical (*c*)
17. Newark College of Engineering	Civil Electrical Mechanical
18. Norwich University	Civil Electrical
19. Pennsylvania State College	Architectural Chemical Civil Electrical Electrochemical Industrial Mechanical Sanitary
20. University of Pittsburgh	Chemical Civil Electrical Industrial Mechanical Metallurgical Mining Petroleum
21. Polytechnic Institute of Brooklyn	Chemical (*e*) Civil (*a*) Electrical (*a*) Mechanical (*a*)
22. Princeton University	Chemical Civil Electrical Mechanical
23. Rensselaer Polytechnic Institute	Chemical Civil Electrical Mechanical
24. Rhode Island State College	Civil Electrical Mechanical
25. University of Rochester	Mechanical
26. Rutgers University	Civil Electrical Mechanical Sanitary
27. Stevens Institute of Technology	General
28. Swarthmore College	Civil Electrical Mechanical
29. Syracuse University	Civil Electrical Mechanical
30. Tufts College Engineering School	Civil Electrical Mechanical
31. Union College	Civil Electrical
32. University of Vermont	Civil Electrical Mechanical
33. Webb Institute of Naval Architecture	Naval architecture and marine engineering
34. Worcester Polytechnic Institute	Civil Electrical Mechanical
35. Yale University	Chemical Civil Electrical Mechanical Metallurgical

Note: Following is a list of specialized curricula upon which action has been deferred pending completion of the accrediting program throughout the country:

Building engineering and construction
Gas engineering
Mineral dressing
Paper and pulp technology
Petroleum refining
Public health engineering

ECPD also has under consideration at this time curricula at certain institutions in these states for which the subcommittees of inspection are not yet prepared to submit specific recommendations with regard to accrediting. It is expected that these recommendations will be ready for action by ECPD early in 1937.

Footnotes

(*a*) Accrediting applies to both the day and evening curricula.

(*b*). Accrediting applies to the 4-year and 5-year curricula leading to the Bachelor of Science degree.

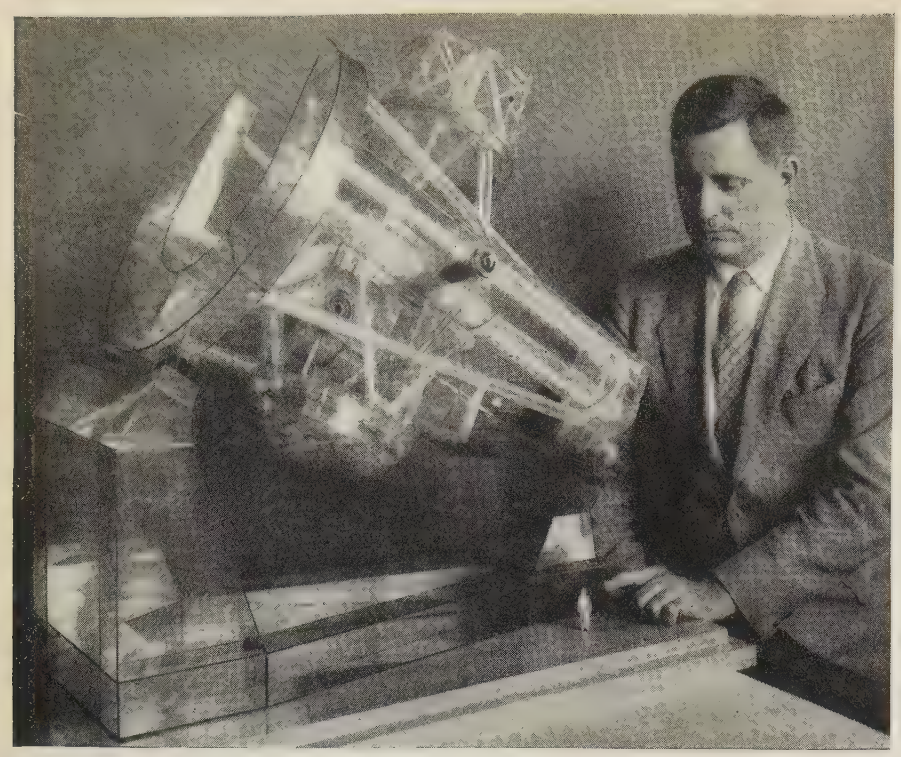
(*c*). Accrediting applies to day curricula only. Action on evening curricula in which the quantitative requirements differ materially from the usual day curricula has been deferred pending further study by a special subcommittee of the ECPD committee on engineering schools.

(*d*). Accrediting applies to the 5-year co-operative curricula leading to degrees.

(*e*). ECPD has not yet received from its subcommittee on chemical engineering any recommendations with respect to evening curricula in chemical engineering.

(*f*). Institutions offering several curricula in engineering and for which ECPD has received recommendations only in respect to chemical engineering have not been included in this preliminary list.

Model of 200-Inch Mt. Palomar Telescope



THIS transparent plastic model of the 200-inch reflecting telescope now being built for installation in a new observatory on Mt. Palomar, Calif., built to $^1/_{32}$ scale, strikingly illustrates the huge size of the instrument; the manikin shows the relative size of a man. The telescope will have an over-all height of 75 feet, when the tube is pointed straight up, and will weigh approximately 1,000,000 pounds. The structure is being built in the South Philadelphia (Pa.) works of the Westinghouse Electric and Manufacturing Company. The 200-inch mirror, which is now being ground in the optical shop of the California Institute of Technology, Pasadena, will be mounted on the lower end of the telescope tube. When this new observatory is complete, observers may see objects 1,000,000,000 light years distant, which is twice the present range.

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ing societies, the state licensing boards, and the colleges of engineering, is the only agency that can accredit colleges of engineering under properly inclusive auspices. As a not unimportant incidental advantage, accrediting by this one agency will avoid the needless duplications of present procedures.

"ECPD is merely authorized by its constituent organizations to publish a list of accredited colleges for use by those agencies which require such a list. It has no authority to impose any restrictions or standardizations upon engineering colleges, nor does it desire to do so. On the contrary, it aims to preserve the independence of action of individual institutions and to promote the general advancement of engineering education thereby.

"As stated in the foregoing paragraph headed 'Basis of Accrediting,' appraisal of institutions will be based upon statistical information as obtained from catalogues and questionnaires, and upon evidences of quality of instruction, adequacy of equipment, and of teaching staff, and other factors not susceptible of statistical analysis, as determined by visits of inspection by committees of qualified representatives of the committee on engineering schools. Emphasis will be given to quality of work rather than to statistical information to a greater degree than in former accrediting procedures. No hard-and-fast prescriptions are laid down for the curriculum, the physical facilities, the investment or expenditures, or other specific points relating to a given institution, though all of these, and others, will be taken into account in appraising the institution as a whole.

"Final decision as to accrediting of each institution rests with ECPD, which will pass upon the recommendations made to it by the committee on engineering schools.

"The general expenses of developing the accrediting program will be borne in part by a grant of funds from Engineering Foundation. Expenses of the visiting committees representing ECPD will be met by a charge made to the individual institutions sufficient to cover cost of travel and subsistence during the inspection. This charge will be on a basis of $100 for the first curriculum to be considered, and $50 for each additional curriculum with a maximum of $400 for any single school. This is in accord with the practice of a number of other accrediting agencies.

"Committees of inspection will comprise both teachers and practicing engineers, the criteria for selection being competenc to judge educational institutions, good judg ment, and availability. For the purpos of organization and administration of th accrediting program, the country has bee divided into 7 geographical regions whic include within their boundaries approx mately equal numbers of engineering co leges: New England, the Middle Atlant States and Maryland, the Southeaster States, the Upper Mississippi Valley, th Lower Mississippi Valley, the Southwes and the Northwest. Committees in the geographic areas will include representa tives of each of the organizations constitu ing ECPD. Alternates will be provide who will serve instead of the regular mem bers when the latter would be called upo to judge a neighboring or rival institutior or the one with which the individual himse is connected. A member of the ECP committee on engineering schools will serv as chairman of each regional committee.

"Accrediting of individual institution and of curricula offered by them will be, c course, upon invitation of the institution.

Personnel of the ECPD committee o engineering schools is as follows:

Karl T. Compton (F'31) president, Massachusett Institute of Technology, Cambridge, Mass., *chair man*, representing the AIEE.

H. P. Hammond, professor of civil engineering Polytechnic Institute of Brooklyn, Brooklyn, N. Y. *vice chairman*, representing Society for the Promo tion of Engineering Education.

G. M. Butler, dean, college of mines and engineer ing, director, Arizona Bureau of Mines, Universit of Arizona, Tucson, Ariz., representing America Institute of Mining and Metallurgical Engineers.

Ivan C. Crawford, dean, college of engineering University of Idaho, Moscow, Idaho, representin American Society of Civil Engineers.

Harry A. Curtis, chief, chemical engineering de partment, Tennessee Valley Authority, Universit of Tenneessee, Knoxville, Tenn., representin American Institute of Chemical Engineers.

P. H. Daggett (A'08) dean, Rutgers University New Brunswick, N. J., representing National Coun cil of State Boards of Engineering Examiners.

A. A. Potter, dean, schools of engineering, Purdu University, Lafayette, Ind., representing America Society of Mechanical Engineers.

Institute members listed as serving o "co-operating committees of inspection' include:

J. W. Barker (M'26, F'30) dean of engineering Columbia University, New York, N. Y.
F. C. Bolton (A'09, M'14) dean of the college, an dean, school of engineering, Agricultural & Mechan ical College of Texas, College Station.
B. M. Brigman (M'29) dean of engineering, Spee Scientific School, University of Louisville, Ky.
H. V. Carpenter (A'03, F'18) dean of engineerin and director of engineering experimental station Washington State College, Pullman.
P. H. Daggett (A'08) dean, Rutgers University New Brunswick, N. J.
R. E. Doherty (A'16, M'27) president, Carnegi Institute of Technology, Pittsburgh, Pa.
A. M. Dudley (A'08, F'13) marine engineer, West inghouse Electric & Manufacturing Company East Pittsburgh, Pa.
O. W. Eshbach, (A'17, M'30) special assistant personnel department, American Telephone an Telegraph Company, New York, N. Y.
H. S. Evans (A'05, M'09) dean and professor o electrical engineering, college of engineering University of Colorado, Boulder.
O. J. Ferguson (A'05, F'13) dean, college of engi neering, University of Nebraska, Lincoln.
W. E. Freeman (A'11, M'27) professor of electrica engineering, assistant dean of engineering, Uni versity of Kentucky, Lexington.
C. F. Harding (A'06, F'14) head, school of elec trical engineering, Purdue University, Lafayette Ind.
D. C. Jackson (A'87, F'12, past-president) profes sor emeritus and honorary lecturer, Massachusett

Institute of Technology, Cambridge.
A. S. Langsdorf (A'03, F'12) dean, schools of engineering and architecture, Washington University, St. Louis, Mo.
A. D. Moore (M'24) associate professor of electrical engineering, college of engineering and architecture, University of Michigan, Ann Arbor.
E. L. Moreland (A'11, F'21) professor, department of electrical engineering, Massachusetts Institute of Technology, Cambridge.
W. S. Rodman (A'07, F'28) dean of engineering and professor of electrical engineering, University of Virginia, University.
W. A. Schmidt (M'22) president and general manager, Western Precipitation Company, Los Angeles, Calif.
R. W. Sorensen (A'07, F'19) professor of electrical engineering, California Institute of Technology, Pasadena.

Report on Professional Training Outlines Progress Toward Objectives

The report of the committee on professional training was presented by R. I. Rees, chairman; excerpts from that report follow:

"The first annual report of ECPD defined the objectives toward which the committee on professional training should work. It seems to the committee that it would be well to review for the Council the accomplishments thus far achieved in realizing these objectives. The objectives are as follows:

1. A survey of junior members of engineering societies to find, among other things, some indication of plans for self-development.
2. The preparation of a personal analysis blank to assist the individual in his program of self-development.
3. Surveys of educational facilities in areas of concentration of junior members.
4. A study of the basic objectives for future independent reading by junior members.
5. Preparation of a bulletin explaining what experience and further intellectual development are demanded by criteria to be set forth by the committee on professional recognition.
6. Development of procedures for participation by joint subsidiary organizations of participating bodies in different localities.

"In connection with the above objectives the following comments are made.

"1. Careful consideration has been given to the possibilities of achieving the first objective, but it was thought that a complete survey of junior members of the engineering societies should grow out of the undertakings of local sections of the several societies in their efforts to develop operating programs for junior engineers in their respective communities. Eventually a complete survey of junior members will be completed and through a study of the needs of individuals, plans for a more complete development program may be determined. The committee feels that it owes a service of this kind to the constituent members of ECPD in order that the aspirations, ambitions, and individual interests of the younger members of the societies may be served. Manifestly, this must be a long-time, continuous effort.

"2. The personal analysis or, as it has been called 'Personal Appraisal,' was prepared and the first edition published October 1, 1934. This appraisal blank was revised in order to correct certain ambiguities, and republished during the past year, on March 2, 1936. The broad purpose of this self-appraisal was to encourage the young engineer to analyze himself, particularly in his attitudes and relationships to his occupation and to his professional and personal status. Such an analysis, it was hoped, would stimulate him to develop a well-rounded program of professional, social, and economic development. While the committee lacks definite information on the success of this plan for stimulating personal development, the demand for the appraisal has been large and some reports have been received indicating that it has been a valuable contribution. As organized programs developed under the leadership of senior engineers become operative throughout the country, this appraisal blank will satisfy a basic need for the development of such programs.

"3. In fulfilling the obligation to undertake surveys of educational facilities, in 1935 the junior committee on professional training completed a survey on a nationwide basis, of university extension courses which are available for young engineers desiring to carry on educational work individually and on a somewhat formal basis. It will be noted that the facilities listed are mainly in nontechnical fields and also designed to satisfy the desire of the young engineer for a broader nontechnical culture which is so necessary to the true professional engineer. As the operating program slowly develops this broad survey should be supplemented by surveys in each community of the educational offerings available.

"4. Out of the study of basic objectives for future reading by junior engineers have developed 2 major efforts. In "Suggestions to Junior Engineers," along with the self-appraisal blank published in October 1934 was a 'Reading List for Junior Engineers.' [This list was published serially in ELECTRICAL ENGINEERING, beginning in the December 1934 issue and extending through the June 1935 issue.] This list had as its objective the encouragement of individual effort for exploration in the broader fields of knowledge and provided a basis on which the nontechnical culture of the young engineer could be continued. The original list was subjected to much criticism and on March 2, 1936 a thorough revision was published, embodying the valuable suggestions which had been received from many members of the profession. Your committee is determined to keep abreast of developments in fields of general knowledge and incorporate in this list outstanding works of recent publication.

"For assistance to the young engineer in continuing his more strictly professional development, the committee decided that it would be desirable to prepare a brief bibliography of outstanding books on technical subjects. This developed into a more serious task than was anticipated and it has taken nearly 2 years to develop a list which is reasonably satisfactory to our contributing critics, of whom there were more than 100, all members of the engineering societies . . . Recognition must be given to Mr. O. W. Eshbach, AIEE, for his constructive work in compiling and editing this bibliography, which the committee considers its major accomplishment during the past year. . .

"5. The preparation of a bulletin explaining what experience and further intellectual development are demanded by criteria set forth by the committee on professional recognition has not yet been accomplished and should be one of the major undertakings of the committee during the coming year. While the committee has given much consideration to this subject, it was thought desirable to defer action until the conclusions of the committee on professional recognition have been more definitely resolved. The committee has before it, of course, the minimum qualifications for an engineer, which gives a generalized objective.

"There are 2 major compulsions which now rest upon most junior engineers. One is achieving competence within his specified occupation, and the other is preparing for licensure within those states having registration laws. One of the fears entertained by the committee is that preparation in both these fields compels development along purely technical and professional lines. When a bulletin is prepared, objectives should be broadened and training criteria defined which would urge young engineers to meet the objectives so well defined by our former chairman, Dr. C. F. Hirshfeld, in his article 'The Engineer of Today and Tomorrow.' [See *Mechanical Engineering*, August 1936, pages 475–8.] Development toward an understanding of and participation in the solution of many of the complex problems of our civilization is as compelling a motivation toward high professional status as the more strictly technical objective.

"6. With reference to the organization of field, operating programs, your committee felt that it should move slowly and cautiously in order that whatever was accomplished in the way of experiment in a few centers would form a dependable foundation for development throughout the country. On page 22 of the annual report of 1935 there appears a suggested operating program which involves the active co-operation of the profession as a whole and the organized effort of the constituent bodies forming the ECPD. On the basis of this suggestion, committees on professional training were formed in several communities, but active operations have been undertaken in but few.

"On invitation of the committee, the Providence Engineering Society which coordinates the activities of the local sections of the major societies, undertook to organize a program for the professional development of the young engineers of Providence and vicinity. The sequential steps which were taken were as follows:

1. An informal gathering of senior engineers to discuss the general aspects of the problem.
2. The organization of a committee on junior engineers' development.
3. Consultation between this committee and a group of junior engineers.
4. Formation of a junior engineers' committee.
5. Survey by junior committee and the listing of all young engineers eligible to undertake the program.
6. A brief questionnaire to determine the training interests of each individual.
7. A general meeting at which the junior committee laid before the members the plans for the development of a training program.
8. The formulation of such a program and preparation to begin active operations in the early fall of 1936.

"This is a fine demonstration of enthusiastic interest and harmonious effort between seniors and juniors to further the program

of professional development. Their method of developing a program can easily be used with adaptation in a helpful manner in other communities throughout the country. . . [A report of this activity was published in the July 1936 issue of ELECTRICAL ENGINEERING, page 839.]

"A second major undertaking for the committee during 1936 and 1937 will be to give helpful stimulation to programs similar to that at Providence in an increasingly large number of centers having local sections of the constituent bodies and a sufficient number of junior engineers to form the basis of operations.

"In considering an operating program for professional development of postcollegiate character, there must also be made an effort to co-ordinate these field operations with the work of the committee on student selection and guidance. The chairman of your committee, and Dean R. L. Sackett, chairman of the committee on student selection and guidance, have already conferred on this problem and by joint effort will endeavor to work out a satisfactory co-ordinated program. Such co-ordination will prevent confusion in the field and bring to the attention of all local sections the desirability of comprehending the complete program of ECPD.

"During the past year the junior committee on professional training has given much attention to a number of fundamental considerations having to do with the development of a program for junior engineers which will be of maximum benefit. This committee, whose members are all located in the metropolitan section of New York, has also studied the possibility of developing a program for greater New York and vicinity. . .

REPORT OF JUNIOR COMMITTEE

The report of the junior committee on professional training was presented by J. C. Arnell, chairman, who said that the work of the junior committee during the past year had consisted primarily of an analysis and evaluation of the factors included in the program of professional training as it has developed in the past 3 years. Now that the components have been determined, the committee feels confident that great and effective strides can be taken in pushing the work of ECPD training itself. Excerpts from the report of the junior committee follow:

"In considering the field of professional training during the past year, the junior committee found itself circumscribed by the following problems: (1) stimulation of junior interest in the entire ECPD program; (2) organization of groups for training in local areas; (3) development of training material; and (4) development of means of qualifying for professional recognition.

"Stimulation of interest we believe is primarily a matter of advertising ECPD to the juniors, beginning in the colleges. Informational material and pamphlets such as 'Suggestions to Junior Engineers' should be brought to the attention of engineering students through the faculty, student chapters of the national societies, and the national engineering fraternities such as Tau Beta Pi, Eta Kappa Nu, and Phi Lambda Upsilon. Particular attention should be given to the establishment of a positive means of maintaining contact with the young engineer during the transitional stage between graduation and establishment in engineering work. Doing this will certainly develop that active interest and ferment in the juniors which will enlist the aid of the seniors in organizing the work of the local areas.

"The local-area program may be organized in a manner suggested in last year's report. The problem of starting ECPD work in the Metropolitan New York area is now being worked upon by the committee.

"The crux of the entire program of professional training lies in the development of training material for the groups after they are organized. While it is conceded that immediate focus of interest of the junior engineer is his job, the committee feels very serious consideration must be given to the long-range job of developing professional engineers in the broadest sense of the term. The programs concerning such subjects as heat transfer, distribution networks, arch dams, etc., we regard as being definitely a responsibility of the local sections of the national societies. The assistance on problems of the job would generally be derived from this class of program. Moreover, it is recognized that in a large number of states, the young engineer is confronted with a state licensing law with which he must comply if he wishes to practice as a professional engineer. A very tangible measure of assistance can be given by the local operating groups of ECPD in acquainting the juniors with the provisions of the law as early as possible and in organizing means of preparing qualified individuals to pass examinations or otherwise meeting the legal requirements.

"Recognizing our present civilization and society as a product of technology, the junior committee suggested that ECPD develop a number of syllabi which would be available for the guidance of local training groups. One series would be concerned with the history and development of various types of industry in order to help the young engineer orient himself and his industry with respect to the entire industrial picture. Another series would be based on the functional aspect of industry covering such phases as production, marketing, accounting, design, research, etc. A third series would help develop in the junior that strong pride in his profession which is held by every true engineer. This would be attacked from the basis of familiarizing the juniors with the history of engineering and the accomplishments of the great engineers. The preparation of such syllabi is recommended to the senior committee as being worthy of the efforts of the best men available. While this recommendation has been set aside for the more practical and realistic immediate work of grooming individuals for the state examinations, we nevertheless regard it as being in absolute harmony with the true spirit of professional development that ECPD is trying to promote. For this reason, it is hoped the coming year will see more attention directed to this work.

"The development of means of qualifying for professional recognition we thought should include: (1) preparation of a standard personnel record, issued and certifie[d] by ECPD, to give a complete picture [of] each individual's professional growth; an[d] (2) establishment of outlets for the publica[-]tion of junior papers. The personnel recor[d] would be a tangible evidence to everybod[y] concerned of a junior's development an[d] would also be of definite help to senio[rs] called upon to advise the young man a[s] well as those certifying him for professiona[l] recognition. Outlets for publication ar[e] also essential as it would be a physical im[-]possibility to publish in the existing societ[y] publications all the material that would b[e] presented by the individuals seeking certi[-]fication. Perhaps ECPD itself would is[-]sue its own publication as one outlet, per[-]haps issuance of material by a local sec[-]tion would help, but further thought an[d] study of this problem is essential. . ."

Report of Committee on Student Selection and Guidanc[e]

In presenting the report of the committe[e] on student selection and guidance, R. L. Sackett, dean of engineering, Pennsylvani[a] State College, chairman of the committee said that the committee is making a definit[e] attempt to interest practicing engineers i[n] the student engineers, through engineerin[g] clubs and local societies. He said that th[e] guidance studies are taking on adde[d] significance as the period of applicatio[n] lengthens. He said further that th[e] figures regarding the students dropped b[y] the faculty (some 15 per cent of those en[-]rolling in the freshman year) indicate [a] problem emphasizing the desirability for [a] program of student selection and guidance— a problem of social, economic, and profes[-]sional aspects, and of significant importanc[e] to practicing engineers.

Excerpts from the report follow:

"The committee has pursued the follow[-]ing guidance services during the past year

1. The Use of "Engineering—a Career, a Culture."
2. The organization of Committees for vocationa[l] counsel.
3. Edited a Manual for the aid of guidance com[-]mittees.
4. Has continued the study of tests of engineerin[g] aptitude.

"The committee has continued to urg[e] the use of "Engineering—A Career, [A] Culture" by engineering schools, hig[h] schools, and vocational counselors in orde[r] better to inform prospective engineers o[f] the objectives of college education and th[e] aptitudes desirable for the largest measur[e] of achievement. The results have bee[n] encouraging. At the same time a sub[-]committee consisting of O. J. Ferguso[n] [A'05, F'13] and W. B. Plank has bee[n] working on a revision of the pamphlet as [a] result of criticisms and suggestions so that [a] second edition may be published when tha[t] edited by The Engineering Foundation i[s] exhausted.

"Guidance committees have been or[-]ganized through joint efforts of local sec[-]tions of the founder societies in a number o[f] cities. Various programs have been pro[-]vided in co-operation with high schools universities, and service clubs designed t[o] inform those who are considering an engi[-]

neering education. The main purpose is to provide experienced engineers who are interested in education to whom boys may go for group and individual conference.

"Of the numerous guidance committees which have been organized or have been in operation for some time, the program provided for Detroit, Mich., may be of help to others.

"Under the leadership of the Associated Technical Societies, with the co-operation of the Detroit Board of Education and the active participation of Doctor W. K. Layton, supervisor of vocational councilors, Dean C. J. Freund of the University of Detroit as chairman of a planning committee of three, brought together representatives of . . . [12] technical societies. . . .

"The work of this organization culminated in the adoption of a program of 3 principal activities, as follows:

1. Stimulating interest in the project among the technical societies of Detroit and securing from these societies the names of counselors to represent the respective societies in the guidance undertaking.

2. A preliminary meeting of committeemen representing the various societies in order to explain in detail the purposes and scope of the guidance work and to prepare a joint meeting of students and counselors.

3. A final meeting for high school students interested in the study of engineering and allied professions with counselors representing the various societies. This final meeting was to be the culmination and objective of the year's work.

"The work of the year culminated in a joint meeting of the high school students and guidance committeemen held on the evening of May 8 in the auditorium of the [Detroit] Central High School. About 220 people were present, including 123 boys . . . [A report of this meeting was published in the August issue of ELECTRICAL ENGINEERING, page 936.]

"A manual for guidance committees was prepared with the assistance of M. B. Richardson of New York, Professor C. E. Bullinger of The Pennsylvania State College, Robert Hoppock of The National Occupational Conference and many others who submitted criticisms and suggestions for the improvement of the earlier copies which were distributed for editorial and critical study. . .

"Copies of the manual were sent to chairmen of all guidance committees and to a large number of interested high school, college, preparatory school teachers, and others. . .

"Summer guidance conferences have been continued by Stevens Institute of Technology, Lafayette College, and Worcester Polytechnic Institute. Experience has proved the worth of such programs which might well be studied by other institutions which desire to stimulate sound guidance and improve selection.

"Co-operative tests in English and mathematics were given to [some 1,800] entering engineers by a group of small and large institutions in 1934. An enlarged list gave the same tests to entering freshman engineers in 1935.

"The average grades of the same individuals at the end of one and two years were requested and the correlation between scores in the tests and average grades in all subjects taken have been charted, studied, and submitted for comment to the institution giving the tests and providing the grades."

The report closes with the following conclusions:

"1. The committee believes that the results obtained establish the value of the test in mathematics for entering freshman engineers.

"2. The test in English is probably predicative of success in any collegiate curriculum. The feature of importance to the teacher of English to engineers is that diction is of more interest, and the engineer displays more aptitude in the correct choice of words than in the spelling or vocabulary sections of the test.

"3. The organization of guidance committees composed of professional engineers has proceeded; the extent of their activities and their effectiveness in guidance cannot yet be determined. There is a definite need for more selective measures in order that those who are advised to take up engineering should do so with more exact knowledge of what the requirements are for real achievement in college and what the career of an engineer really is. The . . . report shows the large losses in numbers during the first college year. Half-hearted ambition, sleazy application, ignorance of the necessary spade work that must be done lead to withdrawal, transfer to some other course for one reason or another, or to failure. It should be noted that those who do not return for some other reason outnumber those who have been dropped.

"This latter fact should be given more emphasis than it usually receives. The charge that one-half or more college students fail (in scholarship) is often made. It has been obvious to engineering educators for years that this charge is untrue. The condition is none the less serious, but some of the responsibility for the discouragement, lack of perseverance, or other reasons for the voluntary discontinuance must be assumed by parents, our public school authorities, and the engineering profession.

"There is ignorance of or misinformation concerning the scholarship required and the ambition and determination necessary to succeed in engineering education. The same qualities which bring satisfactory achievement in college or university are necessary on the whole for success in professional life. There is an absence of clear objective on the part of some who drop out.

"The engineering schools are becoming more concerned about correct guidance for entering freshmen and the influence of ECPD is being felt in arousing the engineering profession to a sense of its responsibility and its opportunity in counseling young men who are considering an engineering career.

"Many state institutions find it difficult or impossible to select those who are better fitted; some have found a way; various devices are in use, such as accepting only those from the top fraction of their high school class or taking only those who passed college-board examinations. In addition to these devices there is need for personal counsel to the boy and groups of sympathetic, informed engineers have an opportunity to serve their profession, the student, and the college.

"Selection, even though the basis be only partially satisfactory, is proving its value in obtaining students of greater fitness. The percentage of those who voluntarily withdraw after a year or more can be reduced and the percentage of those who meet high standards of accomplishment can be increased. The futures of engineering education and of the profession are dependent in no small measure on the effective pursuit of the purposes incorporated in selection and guidance."

ASME to Meet November 30-December 4. The annual meeting of The American Society of Mechanical Engineers has been scheduled to be held November 30 to December 4, 1936, at the Engineering Societies Building, New York, N. Y. One feature of this meeting will be a symposium on corrosion-resistant metals in the design of machinery and equipment. This symposium will feature discussions of the engineering uses of such metals as aluminum, nickel and nickel-base alloys, zinc, lead, copper and copper-base alloys, and corrosion-resisting steel.

American Engineering Council

Annual Meeting Scheduled for January 14–16, 1937

Programs for the conference of secretaries of engineering organizations of the United States and the annual assembly of the American Engineering Council are being arranged for the second week in January, 1937; the meetings will be held in the Mayflower Hotel, Washington, D. C. Schedules for both meetings have been arranged to conserve the time of engineers who wish to attend important meetings of the Founder societies, and a most interesting list of subjects dealing with issues of vital importance to the organization and welfare of the engineering profession and its public service is being prepared for constructive deliberation by what promises to be an unusually large attendance.

Upstream Engineering Conference

The upstream engineering conference held in Washington, D. C., September 22–23, 1936, was attended by representatives of AEC (an item concerning this conference appeared in the September 1936 issue, pages 1044–5). In the current AEC "News Letter" Council reports as follows:

"In the unavoidable failure to accomplish the impossible task set in the program for the Upstream Engineering Conference, the conference was not without value to interested parties in federal and private service. It did not effect a consolidation of all available information concerning soil and water conservation and pertinent techniques; but encouraging emphasis was placed upon the need for a more general use of sound engineering based upon complete hydrological data, and much evidence was submitted supporting the necessity for united engineering action and co-ordinated enabling legislation by the citizens of political

subdivisions lying within drainage areas or river systems to overcome hindrances frequently imposed by state and county boundaries.

"No basic differences were disclosed between upstream and downstream engineering. On the contrary this meeting of minds seemed to dramatize their related values, and to crystallize appreciation of the necessity for united technical, economic, and political action, in each watershed with reference to the conservation and utilization of soil, water, and other natural resources. One of the chief values of the meeting was the presentation by several departments of the government—Agriculture, Army, Forestry, Geological Bureau, Power, represented by TVA and Rural Electrification Administration, and the Weather Bureau—of their specific observations and opinions to a common audience. After the several viewpoints had been presented and discussed, Mr. Morris L. Cooke made a strong case for the reconsideration of the whole program of water control on the basis of co-ordinated knowledge and activities of the various government agencies and recommended the creation of a group of interstate regional areas constituting the drainage basins for our river systems. No action was taken but comment seemed to favor the interstate regional proposition as a practical means of providing enabling co-ordinated action on water control and water use from the viewpoint of areas as a whole rather than independent communities.

"Similar concepts of approach to the practical solution of such conservation and utilization problems and to the inauguration of workable programs are to be found in the minds of a number of prominent engineers, some of whom had rather definitely defined ideas prior to the Upstream Engineering Conference. As a matter of fact, recognition of natural areas in dealing with water power policy, flood control, water resources, etc., has been a part of earlier reports on these subjects by AEC committees. Council's committee on conservation and utilization of natural resources already has a study under way which is expected to provide practical suggestions for engineers who are interested in or may be engaged in the organization and administration of regional entities involving more than one state."

Engineering Foundation

Officers Elected at Annual Meeting

The annual meeting of The Engineering Foundation was held October 8, 1936, in the Engineering Societies Building, New York, N. Y.

The Engineering Foundation is a department of United Engineering Trustees, Inc., organized in 1904, and now joint agency of the 4 national societies representing the civil, mining and metallurgical, mechanical, and electrical engineers. The Engineering Foundation, founded by Ambrose Swasey (HM'28) in 1918, is entrusted with the expenditure of the income from endowments and other funds, its present preferred activity being engineering research.

One of the principal features of the annual meeting is the election of officers. Those elected to serve for the year 1936–37 are as follows:

Chairman—F. M. Farmer (A'02, F'13, director) vice president and chief engineer, Electrical Testing Laboratories, New York, N. Y.

Vice-Chairman—Dr. Robert Yarnall (re-elected) member of firm and chief engineer, The Yarnall-Waring Company, Philadelphia, Pa.

The executive committee, the members of which were elected or re-elected, is composed of these 2 members, together with the following:

O. E. Hovey, consulting engineer, New York, N. Y.

A. L. Queneau, metallurgist, United States Steel Corporation, New York, N. Y.

W. I. Slichter (A'00, F'12, national treasurer, member for life) professor of electrical engineering, Columbia University, New York, N. Y.

Doctor A. D. Flinn was re-elected director and secretary.

These officers and the executive committee of The Engineering Foundation are elected by The Engineering Foundation board from among its own members. The Engineering Foundation board is itself elected by the board of trustees of United Engineering Trustees, Inc. The complete membership of The Engineering Foundation board for the year 1936–37 is as follows:

Name		Term Expires
Four Trustees of U.E.T., Inc.		
Otis E. Hovey	ASCE	1937
A. L. Queneau	AIME	1938
D. Robert Yarnall	ASME	1939
H. P. Charlesworth	AIEE	1939
Eight Members Nominated by Founder Societies		
George E. Beggs	ASCE	1939
Langdon Pearse	ASCE	1938
George D. Barron	AIME	1940
F. F. Colcord	AIME	1938
Albert E. White	ASME	1939
W. H. Fulweiler	ASME	1940
F. M. Farmer	AIEE	1939
W. I. Slichter	AIEE	1940
Three Members-at-Large		
Frederick M. Becket		1938
John V. N. Dorr		1939
Edwards R. Fish		1939

Ex-Officio, President, U.E.T., Inc.

G. L. Knight (A'11, F'17)

At this annual meeting, members of the research procedure committee were appointed or reappointed as follows:

R. D. Yarnall, *chairman*, representing The Engineering Foundation.

F. M. Becket, vice president, Electrometallurgical Company, New York, N. Y., representing The Engineering Foundation.

Thaddeus Merriman, consulting engineer, New York, N. Y., representing The American Society of Civil Engineers.

W. H. Fulweiler, chemical engineer, Wallingford, Pa., representing The American Society of Mechanical Engineers.

L. W. Chubb (A'09, F'21) director, Westinghouse Research Laboratories, Westinghouse Electric & Manufacturing Company, East Pittsburgh, Pa., representing AIEE.

Sam Tour, Lucius Pitkin, Inc., New York, N. Y., representing American Institute of Mining and Metallurgical Engineers.

A. D. Flinn, *secretary*.

The personnel of other committees as appointed or reappointed at this meetin[g] is as follows:

Iron Alloys

G. B. Waterhouse, *chairman and director;* pr[o]fessor of metallurgy, Massachusetts Institute [of] Technology, Cambridge, Mass., representing Ame[r]ican Institute of Mining and Metallurgical Eng[i]neers.

Director, National Bureau of Standards, Lyman [J.] Briggs, represented by Dr. J. G. Thompson, Chi[ef] of Section on Chemical Metallurgy, Nation[al] Bureau of Standards, Washington, D. C.

Director, United States Bureau of Mines, J. W[.] Finch, represented by R. S. Dean, chief enginee[r] Metallurgical Division, Washington, D. C.

J. T. MacKenzie, metallurgist and chief chemis[t] American Cast Iron Pipe Company, Birminghan[m] Ala., representing American Foundrymen's Ass[o]ciation.

John Johnston, director of Research, United Stat[es] Steel Corporation, Kearny, N. J., representin[g] American Iron and Steel Institute.

Bradley Stoughton, dean of engineering, Lehig[h] University, Bethlehem, Pa., representing America[n] Society for Metals.

Jerome Strauss, vice president, Vanadium Co[r]poration of America, Bridgeville, Pa., representin[g] American Society for Testing Materials.

T. H. Wickenden, metallurgical engineer, Inter[national] Nickel Company, New York, N. Y[.] representing The Society of Automotive Engineer[s].

J. H. Critchett, vice president, Union Carbide an[d] Carbon Research Laboratories, Inc., New York[,] N. Y., representing American Electrochemica[l] Society.

Wilfred Sykes (A'09, F'14) assistant to the presi[dent], Inland Steel Company, Chicago, Ill., and [a] director; member-at-large.

F. T. Sisco, *editor.*

Welding Research

C. A. Adams (A'94, F'13, past-president, membe[r] for life) *chairman;* director, American Bureau o[f] Welding, Edward G. Budd Manufacturing Com[pany], Philadelphia, Pa.

D. S. Jacobus (A'03, member for life) advisor[y] engineer, The Babcock and Wilcox Company[,] New York, N. Y.

H. M. Hobart (A'94, F'12, member for life) con[sulting] engineer, General Electric Company[,] Schenectady, N. Y.

J. H. Critchett, vice president, Union Carbid[e] and Carbon Research Laboratories, Inc., Ne[w] York, N. Y.

F. T. Llewellyn, research engineer, United State[s] Steel Corporation, New York, N. Y.

G. F. Jenks, commanding officer, Watertow[n] Arsenal, Mass.

J. J. Crowe, engineer-in-charge of apparatus re[search] and development department of Air Re[duction] Company, Jersey City, N. J.

C. L. Eksergian, chief engineer, Budd Whee[l] Company, Detroit, Mich.

Secretary, William Spraragen (A'17, M'26) con[sulting] engineer, New York, N. Y.

The membership of the welding researc[h] committee is to be increased by addin[g] representatives from the following 1[0] industries: railroads, public utility interests[,] nonferrous metals, automotive and aircraft[,] electric welding apparatus manufacturers[,] resistance welding manufacturers, ship[building], machinery manufacturers, struc[tural] steel fabricators, and oil industry.

Among other business transacted at the annual meeting was the adoption of th[e] condensed annual report of The Engineerin[g] Foundation for the period October 1, 193[5] to September 30, 1936, for transmitta[l] to United Engineering Trustees, Inc., fo[r] the annual meeting of said corporatio[n] which was held October 22, 1936. Furthe[r] details will be given in succeeding issues o[f] Electrical Engineering.

Letters to the Editor

CONTRIBUTIONS to these columns are invited from Institute members and subscribers. They should be concise and may deal with technical papers, articles published in previous issues, or other subjects of some general interest and professional importance. ELECTRICAL ENGINEERING will endeavor to publish as many letters as possible, but of necessity reserves the right to publish them in whole or in part, or reject them entirely.

ALL letters submitted for consideration should be the original typewritten copy, double spaced. Any illustrations submitted should be in duplicate, one copy to be an inked drawing but without lettering, and other to be lettered. Captions should be furnished for all illustrations.

STATEMENTS in these letters are expressly understood to be made by the writers; publication here in no wise constitutes endorsement or recognition by the American Institute of Electrical Engineers.

Two Methods of Mapping Flux Lines

To the Editor:

We read with great interest the discussion by Mr. P. L. Bellaschi, published in the July 1936 issue of ELECTRICAL ENGINEERING, page 826, in which he advises the use of the graphical method for plotting fields. He imputes this method to M. G. Leonard (*Electric Journal*, December 1934, page 471).

We are very surprised by the fact that neither Mr. Bellaschi, nor Mr. Leonard named Doctor Th. Lehmann who is the true creator of the said method, in 1909, and whose very valuable papers are well known to many American engineers. . .

Yours truly,

EDOUARD ROTH (A'18)
Chief Electrical Engineer,
Societe Generale de Constructions
Electriques et Mecaniques,
Paris, France

JOSEPH BETHENOD (A'18)
Consulting Engineer,
Paris, France

Dyadic Algebra Applied to 3-Phase Circuits

To the Editor:

In the nonian form of impedance dyadic $\mathbf{Z}$ given in the paper "Dyadic Algebra Applied to 3-Phase Circuits" by A. P. Sah, which was published on pages 876–82 of the August 1936 issue of ELECTRICAL ENGINEERING (equation 3 at left bottom of page 877) the unit vector $\boldsymbol{k}$ of dyad in first row and second column should be $\boldsymbol{j}$. After correction, this dyad would become $z_{ab}\boldsymbol{ij}$ instead of the $z_{ab}\boldsymbol{ik}$ therein written.

Referring to the author's remarks (right bottom of page 877) on his introduction of the unit vectors $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$, also e_a, e_b, e_c and i_a, i_b, i_c as the projections of $\boldsymbol{E}$ and $\boldsymbol{I}$, respectively. I believe these introductions are fully justified.

The cross products of $\boldsymbol{u}$ (isoclinic unit vector) with $\boldsymbol{E}$ and $\boldsymbol{I}$ for star-delta and delta-star transformations (equations 9 and 12) together with the connection diagrams (figures 6 and 7) that apply for measuring the coefficients of impedance and admittance dyadics, respectively, comprise an outstanding contribution. The author deserves our thanks therefor. Equations 9 and 12 are simple, but their scope is large. The writer has not a clear understanding of what was intended by the few lines immediately following equation 12. Nevertheless, aiming to be helpful as regards accuracy, he thinks "a relation similar to that of" equation 9 was intended instead of the equation 10 therein specified (left top of page 879).

The author has made a strong case for use of the dyadic method and shows that it is at least on even terms with the tensor method of analysis for (3 phase) network, synchronous generator or motor, and squirrel-cage or phase-wound induction motor. Impedance dyadic 33 for symmetrical machines and the 4 dyadics covered by equation 34 for a salient-pole alternator having a single winding on the rotor (right top of page 882) look powerful as analytical tools.

Very truly yours,

O. A. HAVILL (A'11, F'16)
Technical Assistant, H. O. Schundler, New York, N. Y.

Modern Machines in Verse

To the Editor:

For some years I have been reading various degrees of sense and of asininity. Most of the sense on the subject that I have happened to see has been in technical journals; perhaps I have been prejudiced. I am offering herewith the following versified summary of my reading:

MODERN MACHINES

The Complaint

Despots of misery, demons of might,
Monster machines, all too dismal a blight,
Cutting cold work means you're crushing warm men,
Mercy you scorn! Can no right cleanse your den?

Why do you live? For how long will you spurt?
What is the gain when good men must be hurt?
Why take the bread from more worn, weakened slaves?
Why starve the life that each one of us craves?

The Reply

Ages ago the tense toil of the slave
Brought but the lash and more chains, till the grave,
Then for a time man and child with a hoe
Sunrise found weary, and moonrise found slow.

We brought you food—although heartless we're called—
Comfort and care; then where ignorance walled
Sickness and death we fought hard for your life,
Guarding, with science, your children and wife.

Leisure we offer instead of the lash,
Drudgery flees where our motors may flash
Into the midst of your milder day's fray;
Just close the switch, then we do as you say.

More of our kind, more machines built for toil,
Bring more to life, bring more yield from the soil,
Products made well, and a hope that is new;
We freed you salves, yet we still are too few.

Hope we created has brought your unrest,
We broke your chains, you need more of our best,
Much we have given yet more we shall grant,
Truth that we tender will wisdom implant.

Makers of freedom, we're molders of fate,
Pleasure to proffer, your word we await,
Watching your welfare, by time proven true,
Monarchs in service, 'tis you we renew.

HENRY HENDRICKS KETCHAM (A'13, M'23, Life Member)
Bethlehem, Pa.

Solving for Components of Complex Propagation Constant

To the Editor:

I submit herewith a method of solving for the attenuation and wave length components of a complex propagation constant given either its hyperbolic sine or cosine. It will be evident from the final results obtained that this method provides means for obtaining quickly and accurately the desired results.

$$\check{\theta} = \theta_1 + j\theta_2$$

$$\text{Given } \sinh\check{\theta} = \check{A} = a + jb \tag{1}$$

$$\text{Since } \cosh\check{\theta} = \sqrt{1 + \sinh^2\check{\theta}} \tag{2}$$

$$\cosh\check{\theta} = \sqrt{1 + \check{A}^2} = c + jd \tag{3}$$

$$\sinh\check{\theta} = \sinh\theta_1\cos\theta_2 + j\cosh\theta_1\sin\theta_2 \tag{4}$$

$$\cosh\check{\theta} = \cosh\theta_1\cos\theta_2\, j\sinh\theta_1\sin\theta_2 \tag{5}$$

From equations 1, 3, 4, and 5,

$$\sinh\theta_1\cos\theta_2 = a \tag{6}$$

$$\cosh\theta_1\cos\theta_2 = c \tag{7}$$

$$\cosh\theta_1\sin\theta_2 = b \tag{8}$$

$$\sinh\theta_1\sin\theta_2 = d \tag{9}$$

Combining equations 6 and 7,

$$\tanh\theta_1 = \frac{a}{c} = \frac{\epsilon^{2\theta_1} - 1}{\epsilon^{2\theta_1} + 1}$$

$$\epsilon^{2\theta_1} = \left[\frac{1 + \left(\frac{a}{c}\right)}{1 - \left(\frac{a}{c}\right)}\right]$$

and

$$\theta_1 = \frac{1}{2}\log_\epsilon\left[\frac{1 + \left(\frac{a}{c}\right)}{1 - \left(\frac{a}{c}\right)}\right]$$

Combining equations 6 and 9,

$$\tan\theta_2 = \frac{d}{a} \text{ and } \theta_2 = \operatorname{Tan}^{-1}\frac{d}{a}$$

Yours very truly,

D. LAWRENCE JAFFE (A'36)
Laboratory Instructor,
College of the City of New York, N. Y.

Personal Items

Ambrose Swasey (HM '28, John Fritz Medalist '24) chairman of the board, The Warner & Swasey Company, Cleveland, Ohio, has been awarded the second Hoover Gold Medal, which will be presented to him at the annual dinner of The American Society of Mechanical Engineers, December 2, 1936. Details of the award are given elsewhere in this issue. Doctor Swasey was born at Exeter, N. H., December 19, 1846. During the period 1869–80 he was with the Pratt and Whitney Company, Hartford, Conn., where he gave special attention to problems of gearing. In 1880 he formed a partnership with W. R. Warner, the firm being incorporated in 1900 as The Warner & Swasey Company. The Company engaged in the manufacture of machine tools and astronomical instruments at Chicago, Ill., moving later to Cleveland, Ohio. Doctor Swasey has been chairman of the board since the death of Mr. Warner. In 1914 Doctor Swasey established the Engineering Foundation, research agency of the 4 national societies of civil, mining and metallurgical, mechanical, and electrical engineers. He has contributed gifts totaling more than $750,000 to the Foundation. The John Fritz Gold Medal was awarded to Doctor Swasey in 1924; he has also received the Franklin Medal, highest award of the Franklin Institute. In addition to honorary membership in the AIEE, which was awarded him June 27, 1928, he is an honorary member of The American Society of Mechanical Engineers (president 1904), American Society of Civil Engineers, Institution of Mechanical Engineers (Great Britain), Institution of Mining Engineers (Great Britain), and the Society of Civil Engineers. He is a chevalier and officer of the French Legion of Honor, and has received several honorary degrees in the United States.

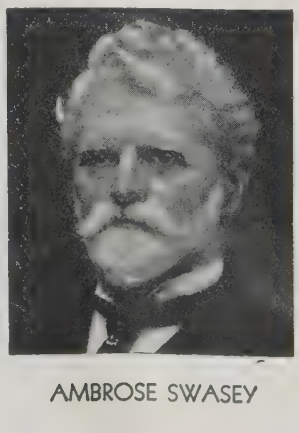
H. S. Osborne (A'10, F'21) transmission engineer, American Telephone and Telegraph Company, New York, N. Y., has been appointed chairman of the Institute's technical program committee for the year 1936–37, and thereby becomes a member of the publication committee and chairman of the committee on award of Institute prizes. Doctor Osborne was born at Fayetteville, N. Y., August 1, 1887, and studied electrical engineering at Massachusetts Institute of Technology, receiving the degrees of bachelor of science (1908) and doctor of engineering (1910). In 1910 he was employed in the office of the transmission and protection engineer in the engineering department of the American Telephone and Telegraph Company, where he assisted in many developments, and in 1914 was made assistant to the transmission and protection engineer. Since 1920 he has held his present position as transmission engineer. Doctor Osborne is the author of several technical papers. He served previously on the technical program committee of the Institute 1924–25, 1927–29, and 1931–34, and is now a member of the Edison Medal committee, the communication committee, and representative on the standards council and the electrical standards committee of the American Standards Association, and on the U.S. national committee of the International Electrotechnical Commission. From 1928 to 1931 he was a member of the committee on education, from 1917 to 1928 a member of the standards committee (chairman 1923–26). During 1931–34 he was chairman of the communication committee, on which he has served since 1929. He is a member of the American Physical Society and the Institute of Radio Engineers.

C. A. Adams (A'94, F'13, past-president, member for life) Lawrence professor of engineering, graduate school of engineering, Harvard University, Cambridge, Mass., has retired from teaching and has become affiliated with The Edward G. Budd Manufacturing Company, Philadelphia, Pa. Doctor Adams was born November 1, 1868, at Cleveland, Ohio, and was graduated in electrical and mechanical engineering from the Case School of Applied Science in 1890. Following his graduation, he became a member of the Reid Expedition to Alaska, making magnetic and meterological observations, and upon his return to Cleveland he obtained a position as draftsman with the Brush Electric Company. In 1891 he received an appointment to the faculty of Harvard University, where he has remained since, first as an instructor and later as assistant professor, 1896–06; professor, 1906–16; Lawrence professor of engineering since 1914; and dean of the engineering school, 1919–21. In 1905 he received the degree of electrical engineer, and in 1925 the university conferred upon him the honorary degree of doctor of engineering. Doctor Adams has carried on an extensive consulting engineering practice in addition to his regular teaching duties, and has conducted numerous investigations of electrical machinery. During the World War he served as chairman of the general engineering committee of the Edison Commission of the Council of National Defense, chairman of the welding committee of the Emergency Fleet Corporation, and chairman of the electrical engineering section of the National Research Council. He has served the Institute as manager (1912–15), vice president (1915–17), president (1918–19), and as member or chairman of many of the technical and administrative committees. Doctor Adams has been active in several technical societies and recently was reappointed a member of the welding research committee of The Engineering Foundation for the year 1936–37.

V. M. Montsinger (A'14, F'29) research engineer, General Electric Company, Pittsfield, Mass., has been appointed chairman of the Institute's standards committee for the year 1936–37. Mr. Montsinger was born at High Point, N. C., April 2, 1884, and received the degree of bachelor of science from the University of North Carolina in 1909. In the following year he entered the testing department of the General Electric Company, and subsequently became engaged in transformer design. Since 1914 the work in which Mr. Montsinger has been engaged has included the investigation of heating and cooling problems relating to large power transformers, dielectric characteristics of insulating materials with particular reference to high-voltage transformers, impulse voltage strength of transformers, and co-ordination of transformers and line insulation. Many papers by him on these subjects have been published, and some of the information gained has been incorporated in the AIEE Standards. Mr. Montsinger has been a member of the Institute's electrical machinery committee since 1926, and was its chairman 1934–36. Since 1928 he has been a member of the standards committee, and as a technical committee chairman has served on the technical program committee since 1934. He has also served on advisory committees of the U.S. national committee of the International Electrotechnical Commission.

C. B. Jolliffe (M'34) engineer in charge of the RCA Frequency Bureau, Radio Corporation of America, New York, N.Y., has been appointed chairman of the Institute's technical committee on communication for the year 1936–37. Doctor Jolliffe was born at Mannington, W. Va., November 13, 1894, and attended West Virginia University, from which he received the degrees of bachelor of science (1915) and master of science (1920), serving there also as an instructor in physics. From 1920 to

C. A. ADAMS



AMBROSE SWASEY

H. S. OSBORNE

1922 he was an instructor in physics at Cornell University, from which he received the degree of doctor of philosophy in 1922. From that year until 1930 he was associate physicist in the radio section of the Bureau of Standards, Washington, D. C., and during this period served as technical adviser for the International Radio Conference at Washington in 1927 and for the International Consulting Committee on Radio at The Hague in 1929. In 1930 Doctor Jolliffe was made chief engineer of the Federal Radio Commission, and when this was replaced in 1934 by the Federal Communications Commission, which is charged with regulation of all wire and radio communications, he became chief engineer of the new commission. In 1932 he was American delegate to the International Telecommunications Conference at Madrid, and in 1933 was technical adviser for the regional radio conference at Mexico City. Since 1935 he has held his present position with the Radio Corporation of America. Doctor Jolliffe has been a member of the Institute's committee on communication since 1934. He is a fellow of the Institute of Radio Engineers and the American Association for the Advancement of Science, and a member of Phi Beta Kappa and Sigma Xi.

G. M. ARMBRUST

H. W. ROGERS

G. I. WRIGHT

G. M. Armbrust (A'11, F 33) assistant chief electrical engineer in charge of the plant engineering department of the Commonwealth Edison Company, Chicago, Ill., has been appointed chairman of the Institute's technical committee on power generation for the year 1936–37. Mr. Armbrust was born at Omaha, Neb., April 27, 1885, and received his technical training from Lewis Institute and Armour Institute of Technology. In 1902 he entered the employ of the Commonwealth Edison Company as a draftsman, and in 1912 was made chief draftsman in charge of engineering and design of the transmission and distribution systems. In 1919 he became an engineer in the electrical engineer's office, acting as chief of staff and reporting to the chief electrical engineer, a position which he held until he was appointed system development engineer in 1932. Later in the latter year Mr. Armbrust was appointed assistant electrical engineer with charge of the design of all electric plant facilities. In February 1936 his title was changed to assistant chief electrical engineer with supervision over the plant engineering division, which had been expanded to include field engineering, line installation, service investigation, and research. Mr. Armbrust was a member of the power generation committee of the Institute during the year 1928–29, and has served on it continuously since 1932. He is a member of the Western Society of Engineers, Edison Electric Institute, and Electric Association of Chicago.

H. W. Rogers (A'12) executive engineer in the industrial engineering department of the General Electric Company at Schenectady, N. Y., has been appointed chairman of the Institute's technical committee on general power applications for the year 1936–37. Mr. Rogers was born at Hartford, Conn., April 28, 1883, and received his technical education at Worcester Polytechnic Institute. In 1906 he entered the testing department of the General Electric Company at Schenectady and 2 years later was transferred to the industrial engineering department. Since 1919 Mr. Rogers has been section head in the department directly responsible for all application engineering and developments in paper, rubber, lumber, machine tool, printing, and other industries. An a-c newspaper press drive was developed by him in 1909, and from 1908 until 1922 he was responsible for all engineering and developmental work on electric shovels. Other work in which he has been associated includes the development of the sectional paper machine drive and high-speed newspaper press. Several patents, including control and regulating devices, have been issued to him. He is the author of many technical articles, and has delivered a number of lectures before various organizations. Mr. Rogers has been a member of the general power applications committee of the Institute since 1922, and is also chairman of the committee on power and lighting of the Technical Association of Pulp & Paper Industry.

V. M. MONTSINGER

C. B. JOLLIFFE

F. O. SCHNURE

G. I. Wright (A'17, M'28) transportation sales and division manager, Westinghouse Electric & Manufacturing Company, East Pittsburgh, Pa., has been appointed chairman of the Institute's technical committee on transportation for the year 1936–37. Mr. Wright was born at East Orange, N. J., July 12, 1889, and was graduated from the electrical engineering course at Stanford University with the class of 1913. Following graduation he was employed by the Southern Pacific Company, Portland, Ore., on railroad electrification work until he entered the U.S. Navy in 1918. In 1919 he became superintendent of construction for the McDougall-Duluth Shipbuilding Company at Duluth, Minn., and in 1922 accepted a similar position with the Duluth Edison Electric Company. Later in 1922 he entered the employ of the Illinois Central Railroad Company and was engaged in electrification successively as assistant engineer, office engineer, and assistant electrical engineer. In 1927 he became engineer of electric traction for the Reading Company at Philadelphia, Pa., where he had charge of the design and installation of the Philadelphia suburban electrical system of the railroad. Mr. Wright was made chief electrical engineer of the system in 1931 and later was given jurisdiction of the Central Railroad of New Jersey. Since September 1936 he has been connected with the Westinghouse Electric & Manufacturing Company. His appointment to the transportation committee of the Institute was made in 1929, and he has served continuously since that year.

F. O. Schnure (A'23, M'35) electrical superintendent for the Bethlehem Steel Company at Sparrows Point, Md., has been appointed chairman of the Institute's technical committee on electrochemistry and electrometallurgy for the year 1936–37. Mr. Schnure was born at Milton, Pa., August 16, 1890, and received the degrees of bachelor of science (1914) and electrical engineer (1919) from Bucknell University. During 1914–15 he was employed by the firm of D. C. and Wm. B. Jackson, Boston, Mass., and then for a short time by the

Pennsylvania Railroad at Altoona, Pa. In 1916 he entered the employ of the Bethlehem Steel Company and was engaged in testing, drafting, and electrical layout work in connection with the plant extension at Sparrows Point. In 1918 he became general foreman of electrical construction, and 2 years later was made general foreman of the electrical repair shop. In 1921 he was appointed assistant electrical superintendent, and since 1923 he has been electrical superintendent. Mr. Schnure has been a member of the Institute's committee on electrochemistry and electrometallurgy since 1935, and has served on the committees on power generation (1931–33), technical program (1932–34), and applications to iron and steel production (1925–34: chairman 1932–34). In 1930 he was elected president of the Association of Iron and Steel Electrical Engineers (now Association of Iron and Steel Engineers) for the year 1930–31. Mr. Schnure has been active in the work of that organization and is the author of a number of papers dealing with the application of electricity to the steel industry.

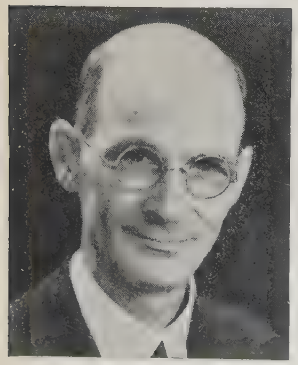
H. M. Hobart (A'94, F'12, member for life) consulting engineer, General Electric Company, Schenectady, N. Y., recently received the Samuel Wylie Miller Memorial Medal of the American Welding Society as "pioneer and leader in a welding research movement which in 19 years has spread to the far corners of the world and vitally affected industry." Mr. Hobart was born November 29, 1868, at Boston, Mass., and was graduated from Massachusetts Institute of Technology in 1889. For 5 years following his graduation he was associated with the Thomson-Houston Company and later with the British company of the same name, resigning in 1899 to become consulting engineer for the Union Elektricitäts Gesellschaft, Berlin, Germany. In 1902 he established his own consulting engineering offices in London, England, continuing until 1911, when he accepted a position with the General Electric Company. Mr. Hobart recently was reappointed a member of the welding research committee of The Engineering Foundation for the year 1936–37. He has been active in Institute affairs, having been manager, 1922–26, and a vice president, 1926–28; in addition, he has served on many of the Institute's technical committees, and is active in promoting international and national electrical standardization, being a member of the U.S. National Committee of the International Electrotechnical Commission. Mr. Hobart is the author of numerous scientific textbooks and papers. He is a member of the American Association for the Advancement of Science, of the Institution of Civil Engineers, Institution of Mechanical Engineers, and Institution of Electrical Engineers (all of Great Britain), and of other technical and scientific societies.

J. L. Hamilton (M'15, F'21) chief engineer, Century Electric Company, St. Louis, Mo., has been appointed chairman of the Institute's technical committee on electrical machinery for the year 1936–37. Mr. Hamilton was born at Morrisville, Mo., September 24, 1883, and received the degree of bachelor of science in electrical engineering from the University of Missouri in 1904. He then entered the employ of the Emerson Electric Manfacturing Company, St. Louis, where for many years he had responsible charge of the design and manufacture of motors. In 1915 Mr. Hamilton accepted the position of chief engineer of the Century Electric Company, a position which since then has brought increasing responsibilities. He is the author of 2 papers on induction motors presented to the Institute, and for many years has been active in the work of the National Electrical Manufacturers Association, having served as secretary and as chairman of the motor and generator section. For 2 years, 1924 and 1925, Mr. Hamilton was president of the Engineers' Club of St. Louis, a period during which the club acquired permanent quarters and doubled its membership. For the following 2 years he was representative of the club and the Associated Engineering Societies of St. Louis on the American Engineering Council. He was a member of the Institute's committee on electrical machinery during 1926–27, and now has been serving on it since 1932, being chairman of the induction machinery subcommittee for 2 years, and also chairman of the committee on preparation of a test code for induction machinery. Mr. Hamilton is a member also of the American Society for Testing Materials.

J. P. McKearin (A'11) chief engineer of the Western Massachusetts Companies, Springfield, Mass., has been appointed chairman of the Institute's technical committee on protective devices for the year 1936–37. Mr. McKearin was born a[t] Proctor, Vt., March 2, 1884, and receive[d] his technical education at Dartmouth Col[-] lege. From 1902 until 1906 he was em[-] ployed by the Vermont Marble Company then from 1906 until 1908 was employed b[y] the General Electric Company at Schenec[-] tady, N. Y. In 1908 he was transferred t[o] the New York office of the General Electri[c] Company, and in 1910 to the Boston, Mass. office. He was made assistant distric[t] engineer at Boston in 1914. Three year[s] later he accepted the position of electrica[l] engineer with the United Electric Compan[y] at Springfield, and in 1927 he became elec[-] trical engineer for the Western Massachu[-] setts Companies. Since 1933 he has hel[d] his present position of chief engineer. M[r.] McKearin has been a member of the pro[-] tective devices committee of the Institut[e] since 1929.

Alexander Kennedy, Jr. (A'10) elec[-] trical engineer in the federal and marin[e] department of the General Electric Com[-] pany, Schenectady, N. Y., has been ap[-] pointed chairman of the Institute's techni[-] cal committee on applications to marin[e] work for the year 1936–37. Mr. Kenned[y] was born at Pittsfield, Mass., June 4, 1883 and received his technical education at Cor[-] nell University. In 1907 he entered th[e] employ of Westinghouse, Church, Ker[r] Company at West Springfield, Mass., an[d] in the following year joined the Genera[l] Electric Company. Mr. Kennedy wa[s] transferred to the induction motor engineer[-] ing department in 1910, and remained ther[e] until joining the U.S. Army engineers i[n] 1917. He returned in 1918, and in th[e] following year was transferred to the ma[-] rine engineering department (the name o[f] which was changed in 1926 to federal an[d] marine department) where he has been con[-] cerned with application engineering in con[-] nection with ship propulsion. Mr. Ken[-] nedy has been a member of the Institute'[s] committee on applications to marine wor[k] since 1924.

R. C. Mason (A'26) research enginee[r] for the Westinghouse Electric & Manufac[-] turing Company, East Pittsburgh, Pa., ha[s] been appointed chairman of the Institute'[s] technical committee on electrophysics fo[r] the year 1936–37. Doctor Mason was bor[n] at Bentonville, Ark., September 6, 1903, an[d] received the degree of bachelor of electrica[l]

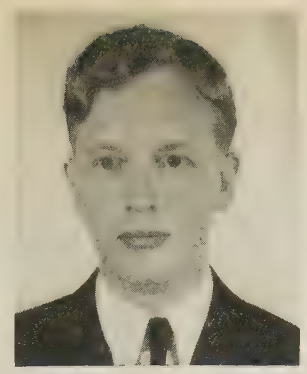

ALEXANDER KENNEDY, JR.


J. L. HAMILTON


H. M. HOBART


J. P. McKEARIN

J. H. EDWARDS



H. C. KOENIG

R. C. MASON

H. C. DEAN

engineering from the University of Arkansas in 1924. The degrees of master of arts and doctor of philosophy later were conferred upon him by Princeton University. In 1924 he entered the student course of the Westinghouse Electric & Manufacturing Company, and a year later was transferred to the materials and process engineering department. Since 1929 he has been in the research department where he has been engaged in research on gaseous discharges and arcs, making contributions to the development of the air circuit breaker. A paper by him on arcs was presented to the Institute in 1933, the year in which he was appointed to the electrophysics committee. Doctor Mason is a member of the American Physical Society and Tau Beta Pi.

O. C. Merrill (M'24) director of the Third World Power Conference, Washington, D. C., has been appointed vice president of the permanent organization of the World Power Conference. Mr. Merrill is a native (1874) of Manchester, Me., and received the degrees of bachelor of arts (1899) and bachelor of science (1905) from Bates College and Massachusetts Institute of Technology, respectively. Upon his graduation in 1905 he received an appointment as an instructor in civil engineering at the University of California, Berkeley, where he remained for a year before becoming assistant engineer in charge of design and construction for a contracting company. In 1909 Mr. Merrill entered the employ of the U.S. Forest Service, serving as hydroelectric engineer until 1912 when he was appointed chief engineer of the service. He was appointed executive secretary of the Federal Power Commission, Washington, D. C., in 1920, and served in that capacity until he was appointed chairman of the American committee of the World Power Conference in 1929, with offices at New York, N. Y. He is a member of the American Society of Civil Engineers.

J. H. Edwards (M'20) associate editor of *Coal Age*, New York, N. Y., with headquarters at Huntington, W. Va., has been appointed chairman of the Institute's technical committee on applications to mining work for the year 1936–37. Mr. Edwards was born at Williamsburg, Iowa, October 2, 1890. He was graduated from the University of Iowa in 1913 with the degree of bachelor of engineering, and in 1917 was granted the degree of electrical engineer in recognition of special work done in illumination and equipment testing while employed by a railroad company. During school vacations he was engaged in electrical installations, and following graduation entered the shops at Silvis, Ill., of the Chicago, Rock Island & Pacific Railway Company, where he was made a foreman a few months later. In 1914 he was made chief electrician of the shops, and 3 years later became supervisor of stationary power in the fuel and mining department at Chicago, Ill. Mr. Edwards accepted the position of chief electrical and mechanical engineer of coal mines of the Elkhorn Piney Coal Mining Company and of coal mines of the Steel & Tube Company of America at Huntington, W. Va., in 1918. Both companies later were taken over by other interests. Since 1924 he has been associate editor of *Coal Age* at Huntington. He has been a member of the Institute's committee on applications to mining work since 1929.

H. C. Dean (A'12, F'30) vice president, New York and Queens Electric Light and Power Company, Long Island City, N. Y., has been appointed chairman of the Institute's board of examiners for the year 1936–37. Mr. Dean was born at Canton, S. D., March 25, 1888, and was graduated from the University of Illinois in 1909 with the degree of bachelor of science in railway electrical engineering. After 3 years with the Public Service Company of Northern Illinois he entered the employ of the city of Chicago, and in 1914 became electrical engineer in charge of the bureau of engineering and construction in the department of gas and electricity. In 1916 he accepted the position of assistant to the vice president of the New York and Queens Electric Light and Power Company, and the following year Mr. Dean was appointed general superintendent in charge of the engineering, construction, operation, and transportation departments of the company, continuing in this position until his appointment as vice president in 1934. In 1935 he was elected a director of the company and also became a member of its executive committee. Mr. Dean has been a member of the Institute's board of examiners since 1933, and is also a member of the committee on legislation affecting the engineering profession, on which he has served since 1934. He was a member of the committee on power transmission and distribution 1929–32.

H. C. Koenig (A'18, M'30) engineer in charge of the electrical department of the Electrical Testing Laboratories, New York, N. Y., has been appointed chairman of the Institute's technical committee on instruments and measurements for the year 1936–37. Mr. Koenig was born at Hoboken, N. J., November 28, 1893. In 1915 he received the degree of bachelor of science in electrical engineering from Cooper Union Institute of Technology, New York, and entered the employ of the Electrical Testing Laboratories as technical assistant in the electrical department. In 1919 he was appointed to his present position in charge of the department, where he is responsible for tests and inspections on a wide variety of apparatus. Since 1916 Mr. Koenig has also been an instructor in electrical engineering at Cooper Union. He has been a member of the Institute's committee on instruments and measurements since 1927, serving as its secretary 1927–31 and 1933–36, and vice chairman 1931–33. He has also been chairman of several subcommittees. Since 1933 he has been secretary of the sectional committee on electrical measuring instruments of the American Standards Association, and since 1935 a member of the sectional committee on dry batteries. From 1929 to 1933 Mr. Koenig was a member of the advisory committee on electrical measuring instruments of the U.S. national committee of the International Electrotechnical Commission.

Wray Dudley (A'12, M'15) superintendent of the electrical department, National Tube Company, McKeesport, Pa., has been appointed chairman of the Institute's committee on applications to iron and steel production for the year 1936–37. Mr. Dudley was born at Montgomery, Mo., March 6, 1881, and was graduated from the University of Missouri with the degree of bachelor of science in electrical engineering in 1905. From then until 1908 he was employed by the General Electric Company in the testing department at Schenectady, N. Y., and for the following year was electrical construction foreman for the Philadelphia, Pa., office of the company. In 1910 he was transferred to the Pittsburgh office of the General Electric Company as steel mill engineer, and later was local engineer. In 1916 he became electrical superintendent of the operating department of the American Rolling Mill Company at Ashland, Ky., and in 1927 he returned to

Pittsburgh as electrical engineer on special problems with the National Tube Company. Recently he was transferred to McKeesport as superintendent of the electrical department. Mr. Dudley was a member of the Institute's committee on applications to iron and steel production from 1931 to 1934, and was reappointed in 1935. He was a member of the committee on electrochemistry and electrometallurgy during 1935–36.

WRAY DUDLEY

Daniel Silverman (A'33) has resigned his position as a research engineer for the Arma Engineering Company, Brooklyn, N. Y., to accept an appointment to the faculty of the department of mechanical engineering, University of California, Berkeley. Doctor Silverman is a native (1905) of Montreal, Que., Canada, and received the degree of bachelor of science at the University of California in 1927; he entered Massachusetts Institute of Technology in the following year and received the degrees of master of science and doctor of science in 1929 and 1930, respectively. Following the completion of his graduate work, Doctor Silverman spent 4 years in the research laboratories of the Westinghouse Electric & Manufacturing Company, East Pittsburgh, Pa. He became associated with the Arma Engineering Company in 1934.

L. S. Wing (A'27, M'34) formerly chief engineer of the national power survey, Federal Power Commission, Washington, D. C., recently was appointed regional director of the Commission, with offices at Denver, Colo. Mr. Wing was born at Blue Earth, Minn., in 1892, and is an engineering graduate of the South Dakota State College of Agricultural and Mechanical Arts. Following his graduation, he held several engineering positions for short periods before becoming associated with the California Farm Bureau Federation, as engineer in charge of rate studies and valuation, in 1921. Mr. Wing was appointed to the Federal Power Commission in 1934. He is a member of the American Society of Agricultural Engineers.

P. L. Bellaschi (A'29, M'34) development and research engineer, Westinghouse Electric & Manufacturing Company, Sharon Pa., recently was awarded the Westinghouse order of merit "for the high order of research and investigation of the nature of lightning; for his pioneer work in production of artificial lightning of simultaneously high voltage and high current; for his measurement of these high impulse voltages and currents; for his discoveries and explanation of the physical nature of the current path of lightning; and for his contribution to the literature of electrical engineering within the province of his activity."

A. C. Streamer (A'10) formerly sales manager of the transportation department, Westinghouse Electric & Manufacturing Company, East Pittsburgh, Pa., has been appointed manager of the switchgear department of the company. Mr. Streamer received the degree of bachelor of science in electrical engineering at the University of Colorado in 1907, and entered the employ of the Westinghouse Company immediately following his graduation.

G. H. Bucher (M'24) vice president, Westinghouse Electric & Manufacturing Company, New York, N. Y., recently was appointed executive vice president of the company, with offices at Pittsburgh, Pa. Mr. Bucher was born July 24, 1888, at Sunbury, Pa., and is a graduate (1909) of Pratt Institute. Following his graduation, he was employed by the Westinghouse Company at East Pittsburgh as a graduate student. In 1911 he was transferred to the export department of that company in New York, N. Y., and in 1920 was appointed assistant to the general manager of the Westinghouse Electric International Company. In the following year Mr. Bucher was made assistant general manager, which position he held until 1932, when he was elected vice president and general manager. In 1934 he was elected president and general manager of that company, and in 1935 was elected vice president of the Westinghouse Electric & Manufacturing Company.

G. H. BUCHER

R. P. Crippen (M'36) formerly assistant to the general manager, Tennessee Public Service Company, Knoxville, has been transferred to New York, N. Y., as assistant electrical engineer for the Phoenix Engineering Corporation. Mr. Crippen has been actively engaged in public utility work since 1928.

S. H. Helmsley (A'26, M'26) formerly assistant transformer design engineer, Bruce Peebles and Company, Edinburgh, Scotland, has accepted a position as senior design engineer for the Asea Electric Company London, England. Mr. Helmsley is a 1923 electrical engineering graduate of Victoria University and has contributed freely to the technical literature in England. He is a member of the Institution of Electrical Engineers.

F. M. Farmer (A'02, F'13, director) vice president and chief engineer of the Electrical Testing Laboratories, New York, N. Y., has been elected chairman of The Engineering Foundation for the year 1936–37. A biographical sketch of Mr. Farmer appeared in the October 1936 issue of Electrical Engineering, page 1157, in connection with his appointment as chairman of the Institute's committee on headquarters.

L. A. Griffith (A'31) who has been an electrical engineer for the Emerson Electrical Manufacturing Company, St. Louis, Mo., recently accepted a position as an electrical design engineer for the Minneapolis-Honeywell Regulator Company, Minneapolis, Minn.

F. V. Andreae (A'17) formerly electrical engineer, Monsanto Chemical Company, Anniston, Ala., now is employed by the Ferro Alloys Corporation, Philo, Ohio. Mr. Andreae is a member of Institute's committee on electrochemistry and electrometallurgy.

C. A. Tudbury (A'35) recently was appointed an instructor in the electrical engineering department of Fenn College, Cleveland, Ohio. Mr. Tudbury is a native (1913) of Warwick, R. I., and a 1934 electrical engineering graduate of Massachusetts Institute of Technology.

Augustin Frigon (A'20) president, Quebec Electricity Commission, Montreal, has been appointed assistant general manager of the recently created Canadian Broadcasting Corporation. Until recently Doctor Frigon was principal of l'Ecole Polytechnique, Montreal.

A. C. Farmer (A'35) assistant manager of transformer sales, Westinghouse Electric & Manufacturing Company, Sharon, Pa., recently was awarded the Westinghouse order of merit "for distinguished sales promotion capacity and energy. . . ."

A. G. Olson (A'27) formerly assistant engineer, Public Service Company of Northern Illinois, Maywood, has accepted a position as distribution foreman, water and electric department of the village of Winnetka, Ill.

Frank Worel (A'29) who has been an engineering assistant in the transmission engineering department, Michigan Bell Telephone Company, Detroit, has been transferred to Ann Arbor as chief toll testman.

Vladimir Yagodkin (A'25) formerly electrical engineer Bureau of Yards and Docks, Navy Department, Washington, D. C., has accepted a position as electrical engineer for the Tennessee Valley Authority, Knoxville.

W. I. Slichter (A'00, F'12, national treasurer, member for life) professor of electrical engineering, Columbia University, New York, N. Y., has been elected a member of the executive committee of The Engineering Foundation for the year 1936–37.

R. L. Bertolacci (A'19) formerly a valuation engineer for the Utah Power and Light Corporation, Salt Lake City, now is employed by the consulting engineering firm of Jackson and Moreland, Boston, Mass.

E. E. Frenzel (A'35) formerly an engineer for the Freeport Sulphur Company, Port Sulphur, La., now is employed as a process engineer for the Standard Oil Company of Louisiana, Baton Rouge.

C. T. Koerner (A'35) formerly an engineer for the Mackay Radio and Telegraph Company, New York, N. Y., recently was employed as an engineer for Hearst Radio, Inc., Redwood City, Calif.

S. H. Munson (A'36) chief testboard man, American Telephone and Telegraph Company, Kalamazoo, Mich., has been transferred to the Cleveland, Ohio, offices of that company.

Alex Becker (A'30) formerly assistant electrical engineer, Sperry Gyroscope Company, Inc., New York, N. Y., now is employed by the Underwriters' Laboratories, New York.

R. H. Rankin (A'28) formerly an electrical draftsman for the Tennessee Eastman Corporation, Kingsport, now is assistant electrical engineer, Eastman Kodak Company, Rochester, N. Y.

W. R. Uffelman (A'34) formerly with the Goodyear Tire and Rubber Company, Akron, Ohio, now is employed in the sales engineering department of the Clark Controller Company, Cleveland, Ohio.

V. R. Parrack (A'25, M'31) formerly assistant electrical engineer, Tennessee Valley Authority, Knoxville, now is employed by the North Carolina Power and Light Company, Raleigh.

W. M. Richardson (A'35) who has been an assistant engineer for the Platte Valley Public Power and Irrigation District, North Platte, Neb., now is employed by the Eggers Pole and Supply Company, Chicago, Ill.

H. G. Barnett (A'33) formerly an assistant, U.S. Bureau of Fisheries, Portland, Ore., now is employed by the Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa., as a student engineer.

R. M. Wyatt (A'26) industrial supervisor, Westinghouse Electric and Manufacturing Company, Baltimore, Md., has been transferred to the Cleveland, Ohio, offices of that company.

Joseph Bronaugh (A'29) engineer, Allis-Chalmers Manufacturing Company, Milwaukee, Wis., has been transferred to the Richmond, Va., offices of that company.

H. A. Rickwood (A'15, M'21) formerly assistant works manager, W. T. Glover and Company, Manchester, England, recently accepted a position as works manager for Johnson and Phillips Ltd. London.

H. B. Day (A'25) formerly a power sales engineer Metropolitan Edison Company, Reading, Pa., now is an industrial power engineer for the Pennsylvania Power and Light Company, Allentown, Pa.

H. R. Pretty (A'34) formerly an electrician for the Oklahoma Gas and Electric Company, Shawnee, has been transferred to the Ardmore offices of that company as an electrical engineer.

P. H. Williams (A'35) who has been plant supervisor, American Telephone and Telegraph Company, St. Louis, Mo., has been transferred to the Chicago, Ill., offices of that company.

R. B. Vaile, Jr. (A'35) formerly an instructor in electrical engineering, Iowa State College, Ames, is now an instructor in electrical engineering at the University of Missouri, Columbia.

H. C. Cunningham (A'28) has left the employ of the Frederick Snare Corporation, Grafton, W. Va., to accept a position as engineer and manager of the Barbour Power Company, Philippi, W. Va.

F. B. Mead (A'13, M'33) general engineer, Westinghouse Electric & Manufacturing Company, Syracuse, N. Y., has been transferred to the Buffalo, N. Y., offices of that company.

H. H. Henline (A'19, M'26) national secretary of the AIEE, New York, N. Y., has been elected secretary of the Engineers' Council for Professional Development.

P. A. Terrell (M'30) district manager, Copperweld Steel Company, Memphis, Tenn., has been transferred to the Atlanta, Ga., offices of that company.

L. H. Connell (A'22, M'28) formerly a sales engineer, Detroit (Mich.) Edison Company, now is with the Keystone Coal Company, Denver, Colo.

H. H. Houston (A'27) electrical engineer, Federal Power Commission, Washington, D. C., has been transferred to the San Francisco, Calif., offices of the Commission.

E. M. Tyler (A'32) formerly with the Muzak Corporation of Ohio, Lakewood, now is with Wired Radio, Inc., Ampere, N. J.

A. E. Ruckgauer (A'28) electrical design engineer, Foster Wheeler Corporation, Cartaret, N. J., has been transferred to the New York, N. Y., offices of that company.

J. C. Fryer (A'35) formerly assistant electrical engineer, Schweitzer and Conrad, Inc., Chicago, Ill., now is employed by the Foxboro Company, Chicago.

A. L. Hopper (A'36) formerly a junior material engineer, U.S. Navy Yard, Brooklyn, N. Y., now is with the Bell Telephone Laboratories, Inc., New York, N. Y.

S. C. Diehl (A'32) now is employed as an electrical draftsman for the New Jersey Zinc Company, Palmerton, Pa.

Obituary

Samuel Montgomery Kintner (A'02, M'03, F'36) vice president in charge of engineering, Westinghouse Electric & Manufacturing Company, East Pittsburgh, Pa., died September 28, 1936. Doctor Kintner was born December 11, 1871, at New Albany, Ind., and was graduated in electrical engineering from Purdue University in 1894. He also attended Cornell University, and received the honorary degree of doctor of science from the University of Pittsburgh in 1930. After spending a year on telephone construction and operation he accepted an appointment to the faculty of the Western University of Pennsylvania, now the University of Pittsburgh, as a laboratory assistant. Later he held the positions of instructor in mathematics, associate professor of electrical engineering, and professor of electrical engineering. Doctor Kintner resigned in 1903 to accept a position with the Westinghouse Electric & Manufacturing Company in its research department. After 4 years of research on high-voltage phenomena he was made design engineer in charge of a-c railway motor design. In 1911 he resigned to join the National Electric Signaling Company as general manager, becoming successively vice president and president. Doctor Kintner was appointed vice president of the International Radio Telephone Company, a new company formed by the National Electric Signaling Company, in 1920, and after 2 years negotiated a sale to the Westinghouse company of the patent rights of his company. Following that negotiation he was engaged by the Westinghouse company to undertake research in radiotelephony and later he was made manager of the research department of that company. He was made assistant vice president in charge of engineering in 1930 and vice president in charge of engineering in 1931. Doctor Kintner was active in Institute affairs, having served as member and chairman of several technical committees. He was a member of the American Physical Society, Institute of Radio Engineers, National Research Council, and The Franklin Institute.

Nicholas Snowden Hill, Jr. (A'96, member for life) president, Hackensack (N. J.) Water Company, died October 18, 1936. Mr. Hill was born June 18, 1869, at Baltimore, Md., attended Georgetown University, and was graduated from Stevens Institute of Technology in 1892. Following his graduation he became an inspector for the Chicago and South Side Elevated Railway Company, Chicago, Ill.; later he became engineer-secretary of the Sewerage Commission, Baltimore, Md., engineer for the Baltimore Electric Subway Commission, and chief engineer of the Charleston (S. C.) Consolidated Railway, Gas, and Electric Company. Beginning in 1901, Mr. Hill maintained consulting engineering offices at New York, N. Y., and was associated with several municipal water companies. In 1926 he was appointed president of the Hackensack Water Company. He was a past-president and honorary member of

the American Water Works Association and a member of the American Street Railway, Association, American Geographical Society, American Institute of Consulting Engineers, American Society of Civil Engineers, The American Society of Mechanical Engineers, and American Society for Testing Materials.

William Albert Edward Doying (A'24) inspecting engineer, The Panama Canal, Washington, D. C., died August 3, 1936. Mr. Doying was born June 13, 1867, at Danville, Que., Canada, and attended Stevens Institute of Technology. After serving an apprenticeship of one year (1889–90) in a machine shop, he became a partner in an electrical contracting firm under the name of Doying Brothers, at Summit, N. J. The firm was dissolved in 1896, and Mr. Doying obtained a position as a draftsman on telephone and switchboard work for the Western Electric Company, where he remained until he accepted a position with the Metropolitan Street Railway Company, New York, N. Y., in the following year. In 1905 he was appointed assistant inspecting engineer of the Isthmian Canal Commission and Panama Railroad Company, with offices at New York, N. Y., and in that position was in responsible charge of the inspection of materials and machinery used in the construction of the Panama Canal and Panama Railroad. In 1908 Mr. Doying was appointed inspecting engineer, and was transferred to Washington, D. C., where he remained continuously.

Robert Ellis Orr (M'19) sales engineer Los Angeles, Calif., died August 27, 1936. Mr. Orr was born August 7, 1883, at Winfield, Kan., and was graduated in electrical engineering from the University of Kansas in 1909. Upon graduation he entered the employ of the Metropolitan Street Railway Company, Kansas City, Mo., and after serving for one year, accepted a position in the office of the electrical engineer of the Los Angeles (Calif.) Pacific Company. In 1911 he accepted a similar position with the Pacific Electric Railway Company, Los Angeles; later he was appointed assistant engineer in responsible charge of electrical maintenance work of that company. In 1920 Mr. Orr went to Buenos Aires, Argentine, and established his own business as an importer and electrical engineer, continuing until 1926, when he returned to the United States and established a sales engineering office at Los Angeles. Recently he became an associate in a firm known as the Orr Calculating Bureau, with offices at Los Angeles.

Stephen Q. Hayes (A'03, M'05, F'12) retired general engineer, Westinghouse Electric & Manufacturing Company, East Pittsburgh, Pa., died April 4, 1936, according to word just received at Institute headquarters. Mr. Hayes was born December 17, 1873, at Washington, D. C., and received the degree of bachelor of arts (1892) and a certificate of proficiency in electrical engineering (1894) at Georgetown University and The Johns Hopkins University, respectively. Following his graduation in 1894 he was employed by the Westinghouse Electric & Manufacturing Company, East Pittsburgh, Pa., as a tester, and his association with that company was uninterrupted for a period of 39 years. He served successively as a switchboard draftsman (1895–98), switchboard engineer (1898–1910), electrical engineer (1911–23), and general engineer from 1923 until he retired in 1934. Mr. Hayes served as a member of the Institute's committee on protective devices, 1914–16. Following his retirement, he served for a time as vice consul of Ecuador.

Jesse B. Millard (A'24) supervisor of electrical maintenance and construction, Hartford (Conn.) Electric Light Company, died August 27, 1936. Mr. Millard was born February 21, 1881, at Birmingham, Mich., and received his education in the public schools of that city. He served with the Michigan Bell Telephone Company from 1898 until he became engaged in electrical contracting work in 1902. In 1905 he became associated with the Westinghouse, Church, Kerr Company in electrical construction work, and continued until 1909, when he entered the construction and maintenance department of the Hartford Electric Light Company.

Frederick Herbert Barrington (A'24) general manager, Moloney Electric Company, St. Louis, Mo., died September 17, 1936. Mr. Barrington was born in 1878 at Waterloo, Que., Canada, and received the degrees of bachelor of arts (1901) and bachelor of science (1906) at McGill University. Following his graduation, he entered the test and engineering department of the Allis-Chalmers-Bullock Company, Montreal, remaining with that company until he was appointed to the Hydro Electric Power Commission of Ontario in 1909. In 1911 Mr. Barrington became associated with the Moloney Electric Company of Canada, Ltd., as an electrical engineer, and after serving for 4 years in that capacity was transferred to St. Louis as chief electrical engineer. He was appointed general manager in 1932.

William Herbert Dieringer (A'29) general superintendent of The South Norwalk (Conn.) Electric Works, died September 26, 1936. Mr. Dieringer was born at Norwalk, Conn., and received his education in the public schools of that city. In 1909 he entered the employ of The South Norwalk Electric Works in the meter department, and his association with that organization was continuous for 27 years. He was made clerk and cashier in the administrative department in 1910; in 1916 he became assistant to the general superintendent; and in 1925, general superintendent.

Edwin Franklin Lawton (A'03) consulting electrical engineer, Hartford Conn., died August 28, 1936. Mr. Lawton was born July 28, 1870, at Cheshire, Conn., and was graduated in electrical engineering from Trinity College in 1891. After a year of postgraduate work at Trinity College, he was employed briefly as a dynamo tender for the Farmington River Power Company before associating himself with the Hartford Electric Light Company in 1895. He was appointed superintendent of that company in 1900 and general manager in 1911, serving in the latter capacity until he retired in 1921. After his retirement he served as a consulting engineer for the Hartford Electric Light Company, Ensign-Bickford Company, Simsbury Electric Company, and the Manchester Electric Company.

Membership

Recommended for Transfer

The Board of Examiners, at its meeting on October 28, 1936, recommended the following members for transfer to the grade of membership indicated. Any objection to these transfers should be filed at once with the national secretary.

To Grade of Fellow

Howe, K. L., district engineer, Westinghouse Electric & Manufacturing Company, Seattle, Wash.
Sinclair, C. T., electrical engineer, transmission and distribution, Byllesby Engineering and Management Corporation, Pittsburgh, Pa.
Weil, Joseph, chief engineer, radio station WRUF; professor and head of department of electrical engineering, University of Florida, Gainesville.

3 to Grade of Fellow

To Grade of Member

Baños, Alfredo, Jr., research fellow in physics, John Simon Guggenheim Memorial Foundation, Massachusetts Institute of Technology, Cambridge.
Boyer, Glenn C., Burns and McDonnell, Kansas City, Mo.
Buck, Omar D., superintendent of operation, Texas Power Corporation, Seguin.
Comly, James M., assistant engineer, Brooklyn, (N. Y.) Edison Company, Inc.
Dobson, George, information manager, Electrical Research Products, Inc., New York, N. Y.
Fowler, Wendell Charles, sales engineer, Sangamo Electric Company, Fort Worth, Texas.
Garnhart, Gordon E., division engineer, Brooklyn (N. Y.) Edison Company, Inc.
Garth, Robert M., electrical engineer, 14th Naval District, Pearl Harbor, Hawaii.
Johnson, Paul L., engineer, chief engineer's office, Southern California Telephone Company, Los Angeles.
Kendall, E. W., technical employee, American Telephone and Telegraph Company, Philadelphia, Pa.
Rorden, Harold L., development engineer, The Ohio Brass Company, Barberton.
Rugge, Raymond A., municipal engineer, Larned Electric and Water Departments, Larned, Kans.
Sah, Adam Pen-Tung, professor of physics, Tsing Hua University, Peiping, China.
Titus, Olcott W., chief electrical engineer, Canada Wire and Cable Company, Ltd., Toronto, Ontario.
Trickey, Philip H., design engineer, Diehl Manufacturing Company, Elizabethport, N. J.
Waldorf, Sigmund K., test engineer, Pennsylvaian Water and Power Company, Baltimore, Md.
Winkler, Edwin W., instructor in electrical engineering, University of North Carolina, Chapel Hill.
Wolfe, William A., relay engineer, Kansas Gas and Electric Company, Wichita.

18 to Grade of Member

Applications for Election

Applications have been received at headquarters from the following candidates for election to membership in the Institute. If the applicant has applied for direct admission to a grade higher than Associate, the grade follows immediately after the name. Any member objecting to the election of any of these candidates should so inform the national secretary before November 30, 1936, or January 31, 1937, if the applicant resides outside of the United States or Canada.

Armstrong, H. D., Brooklyn (N. Y.) Edison Company, Inc.
Beale, R. L., Texas Electric Service Company, Big Spring.
Belden, J. L., General Electric Company New York, N. Y.
Black, J. H., Westinghouse Electric & Manufacturing Company, Wilkes Barre, Pa.
Booker, C. J., Louisville (Ky.) Gas and Electric Company.
Burton, M. S., Texas Telephone Company, Sherman.
Caldwell, B. H., Jr. (Member), General Electric Company, Schenectady, N. Y.
Charton, P. W., National Union Radio Corporation, Newark, N. J.
Crowell, J., Brooklyn (N. Y.) Edison Company, Inc.
Elworthy, B. C., Chambers & Elworthy Ltd., North Vancouver, B. C., Canada.
Farr, J. W., General Electric Company, Pittsfield, Mass.
Flegal, B. C., Oklahoma Gas and Electric Company, Enid.
Fischer, W. C., Globe Union Inc., Milwaukee, Wis.
Gilbert, R. W., Weston Electrical Instrument Corporation, Newark, N. J.
Haywood, W. E. (Member), American Telephone and Telegraph Company, Dallas, Texas.
Heinze, G. M. (Member), New York, New Haven & Hartford Railroad Company, New York, N. Y.
Hicks, I. C., Westinghouse Electric & Manufacturing Company, St. Louis, Mo.
Hoard, B. V. (Member), The New York (N. Y.) Edison Company, Inc.
Hury, E. R., Allis-Chalmers Manufacturing Company, San Antonio, Texas.
Hutchins, J. H., General Electric Company, Schenectady, N. Y.
Kissel, J., Connecticut Light and Power Company, New Britain.
Kochendorfer, A. G. (Member), The New York (N. Y.) Edison Company, Inc.
Larsen, W. P., Brooklyn (N. Y.) Edison Company, Inc.
MacKenzie, J. M. (Member), c/o Andersen & MacKenzie, New York, N. Y.
Maness, A. P., Wilson Dam Hydro Plant, Florence, Ala.
McCarty, W. R., 6400 Plymouth Avenue, St. Louis, Mo.
McCrumm, J. D., Swarthmore College, Swarthmore, Pa.
Michael, D. T., Cincinnati (Ohio) Gas and Electric Company.
Miller, M. V., Building No. 7, Navy Yard, Philadelphia, Pa.
Mollerus, F. J., Andes Copper Mining Company, Potrerillos, Chile.
Morgan, O. J., Delaware, Lackawanna & Western Railroad Company, East Stroudsburg, Pa.
Morgan, G., Jr. (Member), Public Service Electric and Gas Company, Camden, N. J.
Nonken, G. C., General Electric Company, Pittsfield, Mass.
Nutt, F. E., General Electric Company, Pittsfield, Mass.
Oldham, W. J., Brooklyn (N. Y.) Edison Company, Inc.
Osborne, S. C., Wilson Welder and Metals Company, Inc., North Bergen, N. J.
Parks, C. O., Western Electric Company, Inc., New York, N. Y.
Peek, H. L., Westinghouse Electric & Manufacturing Company, East Pittsburgh, Pa.
Phelan, L. T., c/o C. D. Ehret, Philadelphia, Pa.
Ramsay, J. W., University of Texas, Austin.
Randolph, H. J., 410 Atlantic Avenue, Rochester, N. Y.
Rasco, A. J., Brooklyn (N. Y.) Edison Company.
Rath, H. C., General Electric Company, Pittsfield, Mass.
Reed, M. B., University of Texas, Austin.
Robb, H. W. (Member), General Electric Company, Schenectady, N. Y.
Rubanoff, O. M. (Member), Amtorg Trading Corporation, New York, N. Y.
Ruese, W. H. (Member), Kansas Electric Power Company, Lawrence.
Savage, G. D., Philadelphia (Pa.) Electric Company.
Shapiro, L., New York (N. Y.) Edison Company, Inc.
Skinner, L. V., University of California, Berkeley.
Skipton, G., Westinghouse Electric & Manufacturing Company, San Francisco, Calif.
Smith, A. MacD., Public Service Electric Company, Irvington, N. J.
Snell, T. A., Public Service Company of Northern Illinois, Maywood.
Sokop, H. G., General Electric Company, Pittsfield, Mass.
Southerland, J. W., Tennessee Valley Authority, Chattanooga.
Sweitzer, G. W., Brooklyn (N. Y.) Edison Company, Inc.,
Talbott, F. H., Reading Company, Philadelphia, Pa.
Tebo, G. B., Hydro Electric Power Company of Ontario, Toronto, Can.
Thaxton, G. W. (Member), c/o Rural Electrification Administration, Washington, D. C.
Tower, L. W., Habirshaw Cable and Wire Corporation, Houston, Texas.
Tyler, R. A., Oklahoma Gas and Electric Company, Enid.
Varenhorst, F. G., Illinois Bell Telephone Company, Chicago.
Wahlquist, H. W. (Member), Edison Electric Institute, New York, N. Y.
Weber, A. W., Corning Glass Works, N. Y.
Westbee, R. L., Minnesota Mining and Manufacturing Company, St. Paul.
White, W. M., Brooklyn (N. Y.) Edison Company, Inc.
Williamson, R. A., General Electric Company, New York, N. Y.
Wilms, G. O. (Fellow), Allen Bradley Corporation, Milwaukee, Wis.
Winterroth, W. C., Louis Allis Company, Rochester, N. Y.
Zervigon, M. G., Tulane University, New Orleans, La.
Zysk, S., General Electric Company, Schenectady, N. Y.

71 Domestic

Foreign

Asakawa, S., Shibaura Engineering Works, Tsurumi, Yokohama, Japan.
Dertonio, H., Sao Paulo Tramway Light and Power Co., Ltd., Brazil.
Dunlap, B. M., Compania Minera Choco Pacifico Andagoya, Colombia, South America.
Miller, H. S. (Member), British Municipal Council, Tientsin, North China.
Neley, M. E., Water Works, Ambala City, India.
Shah, V. B., Lonawala Khandalla Electric Supply Company Ltd., Nadiad (Gujarat), India.
Unno, K., Shibaura Engineering Works, Tsurumi, Yokohama, Japan.

7 Foreign

Addresses Wanted

A list of members whose mail has been returned by the postal authorities is given below, with the addresses as they now appear on the Institute record. Any member knowing of corrections to these addresses will kindly communicate them at once to the office of the secretary at 33 West 39th St., New York, N. Y.

Burns, Arthur E., 1958 E. 29th St., Brooklyn, N. Y.
Challgren, Fenton, 4306—47th St., Long Island City, N. Y.
Collins, Ogie B., Minimum, Mo.
Eiler, E. E., 101 Brookline Court, Upper Darby, Pa.
Godoy, Ernesto R., Cia. Tel. y Tel. Mex., 16 de Septiembri No. 13, Mexico, D.F., Mex.
Jacobs, David, 316 W. 97th St., New York, N. Y.
Jones, Harry Kenneth, 5511 Kenmore Ave., Chicago, Ill.
Koch, Joseph Stanley, 11 Howe Ave., New Rochelle, N. Y.
Ludwig, Leon R., 434 Burling Road, Forest Hills, Pittsburgh, Pa.
Millheisler, Charles A., 1417 Catalpa Ave., Chicago, Ill.
Miyota, Nath S., 916½ Howell St., Seattle, Wash.
Moore, Everett, 821 Sunset Blvd., Los Angeles, Calif.
Pollastro, John B., Helper, Utah.
Sawyer, Fred E., 811 E. Wisconsin Ave., Milwaukee, Wis.
Wong, Harry Y. L., 771 Broadway, West New York, N. J.

15 Addresses Wanted

Engineering Literature

New Books in the Societies Library

Among the new books received at the Engineering Societies Library, New York, recently, are the following which have been selected because of their possible interest to the electrical engineer. Unless otherwise specified, books listed have been presented gratis by the publishers. The Institute assumes no responsibility for statements made in the following outlines, information for which is taken from the preface of the book in question.

CATHODE RAY OSCILLOGRAPHY, v. 2. (Monographs on Electrical Engineering). By J. T. MacGregor-Morris and J. A. Henley. Pittsburgh, Instruments Pub. Co., 1936. 249 p., illus., 9x6 in., cloth, $6.00. Aims to supply a reasonably concise source of information on cathode-ray oscillography which will meet the needs of electrical engineers. Many types of commercial oscillographs are described and illustrated, and the principal applications and methods of using hot-cathode and cold-cathode instruments are explained.

ELECTRON DIFFRACTION. By R. Beeching with preface by G. P. Thomson. N. Y., Chemical Pub. Co., 1936. 106 p., illus., 7x4 in., cloth, $1.25. Reviews the theory of electron diffraction, describes the apparatus and technique of the method, and points out its applications in the study of surfaces, molecular structure, and crystal growth.

ELECTROPLATING. By J. V. Alfriend, Jr. Scranton, Pa., International Textbook Co., 1935. 82 p., illus., 8x5 in., lea., $1.00. A brief manual of practical directions.

STUDIEN über AUFGABEN der FERNSPRECHTECHNIK. By M. Langer. Munich and Berlin, R. Oldenbourg, 1936. 352 p., illus., 10x7 in., 8 rm. Presents a collection of studies of special problems in telephone engineering, involving various problems of local and long distance telephony, both of an engineering and economic nature.

ELEKTRISCHE MESSUNGEN. By W. Skirl. 2 ed. Berlin and Leipzig, Walter de Gruyter & Co., 1936. 802 p., illus., 8x6 in., cloth, 15 rm. A practical reference book on electrical measurements covering all ordinary requirements, and including diagrams to illustrate the use of measuring apparatus.

KOMPLEXE ZAHLEN und ZEIGER in der Wechselstromlehre. By M. Landolt. Berlin, Julius Springer, 1936. 185 p., illus., 10x6 in., cloth, 15.60 rm. An introductory text upon complex quantities and vectors as applied to the study of alternating currents, in which the mathematical principles and the fundamental electrical concepts and laws are presented, and the treatment of a wide variety of calculations explained.

Die PHOTOELEMENTE und IHRE ANWENDUNG. Pt. 2. Technische Anwendung. By B. Lange. Leipzig, J. A. Barth, 1936. 94 p., illus., 9x6 in., paper, 6.75 rm. Discusses technical applications of photoelectric cells as light meters, exposure meters, etc.

PHYSIK und TECHNIK der ULTRAKURZEN WELLEN. v. 1. By H. E. Hollmann. Berlin, Julius Springer, 1936. 326 p., illus., 9x6 in., cloth, 36 rm. Treats the production of ultra-short waves and describes the different methods and the apparatus used.

PRINCIPLES of ELECTRIC and MAGNETIC MEASUREMENTS. By P. Vigoureux and C. E. Webb. N. Y., Prentice-Hall, 1936. 392 p., illus., 9x6 in., cloth, $5.00. A presentation intended for students of physics or electrical engineering, in which the general principles of the various measurements are discussed, and the various instruments and methods described.

RELAYS and ELECTROMAGNETS. By B. W. Jones and I. C. S. Staff. Scranton, Pa., International Textbook Co., 1935. Illus., 8x5 in., lea., $1.50. Gives a simple, concise description of the various types of relays and their uses and treats of the design of magnets and electromagnets, giving the essential data for various types.

TABLES of PHYSICAL and CHEMICAL CONSTANTS and Some Mathematical Functions. By G. W. C. Kaye and T. H. Laby. 8 ed. N. Y., Lond., Toronto, Longmans, Green & Co., 1936. 162 p., tables, 10x6 in., cloth, $4.75. A collection of the more reliable and recent determinations of some of the more important constants.

Industrial Notes

Building Recovery Here?—The much needed recovery in residential building, believed by observers as necessary to support the wide gains in business generally, is here, according to a late report of F. W. Dodge Corp., New York. For the first 9 months of 1936 this class of construction totaled 23 per cent more than was reported for the entire year 1935. The full year 1936 promises to reach a volume three times the size shown for either 1933 or 1934 the low-points of the depression. The volume of residential building in the 37 eastern states during the first 9 months of 1936 amounted to $588,030,600 as against only $338,907,500 for the same 9 months of 1935, a gain of 73 per cent.

Byllesby Firm Changes Name.—Superseding the change in name of the Byllesby Engineering & Management Corp., previously announced as Public Utility Service Corporation, a name later found to be unavailable, the organization will henceforth be known as Public Utility Engineering and Service Corporation. The change of name was made to more clearly indicate the existence of the company as a separate and distinct organization from H. M. Byllesby & Co., the investment banking house.

G-E Reports Increased Sales.—Sales billed by the General Electric Co. during the first 9 months of 1936 amounted to $189,263,156 compared with $149,173,275 during the corresponding period last year, an increase of 27 per cent. Orders received during the first 9 months of 1936 amounted to $211,891,038 compared with $158,943,765 for the same period last year, an increase of 33 per cent. Orders received during the third quarter and first 9 months of this year were larger than for any corresponding period since 1930.

Cutler-Hammer Opens Plant on Coast.—Cutler-Hammer, Inc., pioneer manufacturers of control apparatus, Milwaukee, Wis., announce the extension of their manufacturing facilities to the West Coast. A new plant at 970 Folsom St., San Francisco, recently began operation. Special control constructions, dead front switchboards, fuse cabinets, panel boards, transformer cabinets and wiring troughs are being produced. Other C-H factories are located in New York City and Milwaukee.

New Capacitors.—The power factor division of the Cornell-Dubilier Corp., South Plainfield, N. J., recently announced a change in the design of their box type power factor correction capacitors. These new units are compact, flexible, and easy to install. It is possible to mount this equipment on the ceiling, wall or floor in single units or compact groups up to 100 kva.

New Circuit Breakers.—For indoor service, type F-122 solenoid operated, oil circuit breakers rated at 25,000 kva, are announced by the Westinghouse Elec. & Mfg. Co. Manually and electrically operated for service up to 600 amperes, 7500 volts and 800 amperes, 2500 volts, two and three poles single throw, these breakers have numerous distinctive features embodied in the design, including the installation of operating levers inside the breaker chamber which removes them entirely from the vicinity of live contact terminals, thus greatly increasing the electrical clearance to grounded parts outside the breaker. The enclosed arrangement of operating levers leaves the outside of the breaker structure with a neat, trim appearance; free from moving parts and easily cleaned. Many other improvements have been incorporated in these breakers.

Trade Literature

Lightning Arresters.—GEA-93K, 20 pp. Describes pellet valve-type lightning arresters and other means of protection for distribution systems. General Electric Co., Schenectady, N. Y.

Resistance Welding.—Bulletin, 12 pp. Describes theory and application of resistance welding. Contains valuable information on proper welding procedure and the correction of faults encountered when using this method of fabrication. The Electric Controller & Mfg. Co., 2700 E. 79th St., Cleveland, Ohio.

Portable Cables.—Bulletin SS2, 32 pp. "Super Service Portable Cables." Describes the construction of a heavy duty, rubber compound insulated cable used in the operation of large portable equipment such as shovels, dredges, as well as minor apparatus. Many applications are illustrated. It is said to be the only portable cable that is vulcanized like a tire. General Cable Corp., 420 Lexington Ave., New York.

Cellular Steel Floor.—Bulletin, 52 pp. Describes a floor, consisting of steel units of 4 cells each, providing a series of potential raceways which carry electrical wiring. A unique system of electrical fittings has been designed to distribute the wiring throughout the floor area. It is claimed that the possibility of electrical obsolescence of buildings is eliminated through this flooring method. H. H. Robertson Co., Grant Bldg., Pittsburgh, Pa.

Street Lighting Control.—Bulletin, 12 pp. Describes "Polatrol," a new method for the control of multiple street lighting. This system was developed by engineers of one of the larger electric utilities to meet the requirements for a simple, reliable method of control for street lights with low first cost, low maintenance costs and freedom from operating difficulties. United Electric Controls Corp., 95 River St., Hoboken, N. J.

Radio Test Instruments.—Folder. Describes the complete line of Weston radio servicing instruments, including the nev model 772 analyzer which operates at a sensitivity of 20,000 ohms per volt. Specifications for 16 fundamental test instruments each designed to meet the requirements o some particular phase of service work, ar included. These instruments not only permit analysis of modern receivers and thei component parts, but extend dependabl direct-reading measurement to many othe types of electronic circuits and equipment—television, public address system, talkin picture recording, and the like. Westo Electrical Instrument Corp., Newark, N. J.

Voltage Regulators.—Bulletin 1183, 24 pp Describes type DFR distribution feede voltage regulators designed for regulatin voltage under load on feeders or busses similar to applications where the induction typ of feeder regulator has heretofore been used as well as for larger ratings than heretofor found economical. As distinguished fron the motor principle involved in the induction type of regulator, this bulletin deal with feeder regulators operating on th transformer and tap-changing-under-loa principle, capable of being furnished i single or three phase units, for any fre quency, for indoor or outdoor application with oil-immersed current limiting reactor built integral, if desired, for automatic o non-automatic control, and with or withou line drop compensation. Allis-Chalmer Mfg. Co., Milwaukee, Wis.

Rubber Mountings for Industrial Use.—Manual. Intended to serve as a practica reference for engineers and others, this bulletin deals with the application of rubbe mountings to modern industrial use for absorbing vibration, cushioning impact, an eliminating noise. Complete engineerin data is included on the various characteristics of rubber, graphs showing its deflectio curves, and its safety load limits. Descriptive illustrations show specific uses to whic these mountings have been put, includin the recently installed rubber mounted floo in the new physics laboratory at M.I.T. also the streamlined trains manufactured b the Pullman Car Mfg. Co. and E. J. Bud Co. The use of rubber in street car and railway equipment is shown to be increasin rapidly. United States Rubber Products Inc., 1790 Broadway, New York.

Electric Machinery Catechism.—Bulleti E100B, 48 pp. Designed to answer thos questions that are likely to arise in th minds of individuals who use electrica equipment, and who do not have an exten sive knowledge of electrical phenomena o terminology. The book is divided into fou sections: simple electric circuits, direct current generators and motors, alternator and alternating-current motors, and electrical machinery in general. A few of the 14 topics discussed indicate the book's scope What is a multipolar machine? What i meant by inductance, capacitance and impedance? How does a synchronous moto operate? What are slip-ring motors used for There are over a hundred drawings, photo graphs and sectional views that show the design of simple machines, the flow and characteristics of electricity, the operation of electrical apparatus, wiring diagrams, et Fairbanks, Morse & Co., 900 S. Wabas Ave., Chicago, Ill.